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Learning in the Oil Futures Markets: Evidence and Macroeconomic Implications*

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Abstract

We show that a model where investors learn about the persistence of oil-price movements accounts well for the fluctuations in oil-price futures since the late 1990s. Using a DSGE model, we then show that this learning process alters the impact of oil shocks, making it time-dependent and consistent with the muted impact oil-price changes had on macroeconomic outcomes during the early 2000s and again over the past two years. The Spring 2008 increase in oil prices had a larger impact because market participants considered that it was likely driven by permanent shocks.

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[*] The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Governing Council of the Bank of Canada, the Board of Governors of the Federal Reserve System, or any other person associated with the Bank of Canada or the Federal Reserve System.
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1 Introduction

The large fluctuations in the price of oil over the past 15 years have renewed interest in the usefulness of the oil futures market in anticipating future price movements (Alquist and Kilian, 2010; Alquist, Kilian, and Vigfusson, 2013). However, the futures market appeared to have only slowly recognized that events since the early 2000s, from the growing importance of China in the world economy to the onset of the financial crisis and the more recent shale revolution, would radically change the outlook for oil prices. Whereas the resulting forecast errors over this period have often been attributed to speculation or to time-varying risk, this paper provides an explanation of the movements in oil-price futures based on learning and shows through a DSGE model that this learning process has important macroeconomic implications.

In particular, our analysis indicates that the developments in the oil futures markets since the late 1990s are largely consistent with investors learning about the persistence of underlying shocks. Using the Kalman filter to infer the permanent and transitory components of shocks to spot oil prices, we show that this simple form of learning can explain the observed behavior of futures prices reasonably well. Our simple framework accounts for the relatively slow increase in oil-price futures at the beginning of the past decade and their unprecedented run-up between 2004 and 2008. Even during the first half of 2008, a period during which oil prices reached historic highs, the model predicts a level of futures prices that is broadly in line with the data. Our estimates suggest a significant and steady increase in the variance of the permanent component of shocks to spot oil prices from 2002 to 2008, accompanied by a similarly steady decline in the variance of the temporary component. In turn, these varying estimates translate into a growing contribution of permanent shocks to the variance of oil prices over this period. More recently, our analysis indicates that investors largely perceived the drop in oil prices during the second half of 2014 as being transitory.[1]

[1] Although the Kalman filter has the advantage of being a straightforward approach to model learning, it is also somewhat restrictive since it assumes that the parameters of the model are constant. In robustness exercises, we present results for two alternative models that allow for time-varying parameters: a constant-gain learning model and a variant of Stock and Watson's unobserved components model (2007) with stochastic volatility, with the unobserved components estimated via the particle filter. We find that our baseline results derived using the Kalman filter are little changed under either alternative model.

Our findings have important implications regarding the macroeconomic impact of oil shocks. Using a DSGE model in which oil is storable and used in production, we show that agents' learning in the form suggested by our empirical results alters the effects of oil shocks by making them time-varying. Consistent with our empirical results, we calibrate three scenarios that capture market participants' perceptions regarding the persistence of oil prices: first, in 2003, when changes in oil prices were largely thought to be transitory; next, in 2007, when oil-price changes were expected to be much more persistent; and finally in 2014 during the recent oil-price collapse. Compared with a framework with full information, we show that the recessionary effects of permanent oil-price increases are roughly halved in the year following the rise in the price of oil under the 2003 scenario. In contrast, the increase in oil prices in the spring of 2008 had a larger impact on output, in part because market participants were more likely to view the rise in oil prices as driven by permanent shocks than earlier in the 2000s.[2] Our analysis also indicates that the large drop in oil prices during the second half of 2014 did not translate into a correspondingly large boost in economic activity partly because investors initially viewed the price declines as being fairly transitory. We show that part of this dampening effect is due to an interaction between learning and storage. In particular, believing that the oil shock is transitory leads agents to draw down inventories, which initially mutes the rise in oil prices and the associated fall in economic activity.

A more muted dampening effect occurs under our 2007 and 2014 scenarios. We show that part of this dampening effect is due to an interaction between learning and storage. In particular, believing that the oil shock is transitory leads agents to draw down inventories, which initially mutes the rise in oil prices and the associated fall in economic activity.

The more muted effects of permanent oil shocks under learning and storage partly account for the relatively weak impact on growth from the run-up in oil prices in the mid-2000s (see Blanchard and Galì, 2009). As such, our results complement Kilian's (2008) emphasis on the importance of the source of oil shocks in understanding their effects on GDP. In particular, Kilian finds that the run-up in oil prices starting in 2003 was largely driven by an increase in world aggregate demand and thus had a muted effect on U.S. economic growth. Our results indicate that the effect on growth was also muted because market participants initially failed to correctly assess the persistence of

[2]Hamilton (2009) analyzes the contribution of the oil shock of 2007-08 to the Great Recession.

the increase in oil prices. Similarly, our model simulations are consistent with the view that the steep decline in oil prices in the summer of 2014 did not translate into a large boost to economic activity.

1.1 Related literature

A growing literature seeks to understand the large movements in the spot price of oil and its changing relationship with the futures market. In particular, our paper relates to some of the recent work on the role of financial markets in driving oil prices.[3] For instance, Hamilton and Wu (2014) argue that increased participation by index-fund investing has reduced oil futures premiums since 2005, accounting for the smaller gap between spot and futures prices observed in the data between 2005 and 2008. Similarly, Buyuksahin et al. (2008) argue that increased market activity by commodity swap dealers, hedge funds, and other financial traders, has helped link crude oil futures prices at different maturities. Acharya et al. (2013) emphasize limits to arbitrage and their effects on spot and futures prices in commodity markets. In their environment, speculators face capital constraints in commodity markets, which limits commodity producers' ability to hedge risk and is reflected in commodity prices. As such, these papers attribute developments in the futures markets to the increased financialization of commodity markets or to speculators' risk appetite, while we show that part of these movements can be attributed to learning. Our emphasis on learning in the futures markets is compatible with Kellog's result (2014) that conditioning on oil futures better explains drilling decisions than assuming that oil prices followed a random walk.

Our paper also complements the work of Alquist and Kilian (2010). They emphasize the presence of a convenience yield associated with oil inventories and its role in accounting for the large and persistent fluctuations in the oil futures spread. Using a theoretical model, they argue that, under greater uncertainty about future oil supplies, the presence of a convenience yield may underlie the poor predictive performance of oil-price futures. We add to this literature by highlighting the interaction between learning and oil storage.

Our approach also shares with Milani (2009) the emphasis on learning. In a DSGE

[3] This literature is large and growing. Some of the many papers discussing this issue include Irwin and Sanders (2012); Fattouh, Kilian and Mahadeva (2013); Kilian and Murphy (2014); Alquist and Gervais (2013); and Singleton (2014).

model in which oil is used in production, Milani (2009) studies the effect of learning on the changing relationship between oil prices and the macroeconomy since the 1970s, assuming that agents in the model learn about the parameters of the model and the underlying shocks through constant-gain learning. He shows that learning is important to account for the changing effects of oil shocks on output and inflation over time. In contrast, our approach highlights that the futures markets also provide valuable information about the learning process. We show that a simple model of learning about the persistence of underlying shocks tracks the evolution of oil futures prices since the late 1990s reasonably well, and that this type of learning can alter the impact of oil shocks on the macroeconomy. Our work on showing the importance of learning in a DSGE model and its implications for economic outcomes is also related to work by Slobodyan and Wouters (2012), Fornero and Kirchner (2014) and Ormeno and Molnar (2015).

Lastly, throughout our analysis we focus on the role of learning, abstracting from other possible important factors that can also influence futures prices. For instance, we abstract from risk-premium variations, which have been shown to be important in understanding futures prices during particular episodes, for instance at the height of the oil-price boom in the beginning of 2008 (see, Baumeister and Kilian, 2015).

The rest of the paper is organized as follows. After describing in more details movements in oil prices over the past 15 years, we lay out our empirical framework, estimating a time-series model of permanent and temporary shocks to oil prices. We then develop a theoretical framework to examine the impact of learning about the persistence of oil-market developments on the economy, calibrating the learning process to match our empirical findings in 2003, 2007, and 2014. After describing our theoretical findings, we conclude in the last section.

2 Oil prices during the 1990s, 2000s, and beyond

We start our analysis by presenting some evidence of the oil market's evolving views regarding the persistence of the shocks hitting the world economy. To do so, consider the movements in the spot and futures prices of oil since the early 1990s, depicted in the three panels of Figure 1. In each panel, the solid line shows the evolution of the spot price, while the dotted lines depict the path of futures prices at a given point in time. The futures prices are quotes as reported by NYMEX. During the 1990s (top panel), the spot price tended to gyrate around fairly stable oil-price futures, suggesting that market participants viewed economic developments affecting the oil market as mainly temporary. Underlying shocks would tend to move the spot price of oil, at times substantially, but futures prices would indicate an expected return to roughly $18 per barrel, about the average spot price during that period. Clearly, whatever the disturbances affecting the world economy, market participants did not view them as persistent enough to substantially alter their long-term view of oil prices, and these views were substantially correct during the 1990s.

However, the middle panel of Figure 1 indicates that the relationship between spot and futures prices changed during the early 2000s. Between 2000 and 2008, the spot price of oil rose steadily, from roughly $27 per barrel to more than $135 per barrel. In contrast, oil-price futures remained initially low, consistently fluctuating below $20 per barrel until 2003. Then, oil-price futures started a gradual rise, increasing to roughly $50 per barrel by the mid-2000s. Spot and futures prices then tended to move in lockstep between 2005 and 2008.

One possible interpretation of this pattern is that, between 2000-03, market participants perceived movements in spot prices as likely to be temporary, as indeed was the case throughout the 1990s. However, after being consistently surprised by the persistence of the rise in spot prices, market participants reassessed their views, placing more weight on the possibility that the increase in oil prices was persistent rather than transitory.

According to this interpretation, by the time the oil market reached its peak in the spring of 2008, market participants largely expected the movements in spot prices to be highly persistent, remaining at about $135 per barrel over the next five years, as indicated by the futures curves at that time. The graph also suggests that the global financial crisis during the fall of 2008 led to a reassessment of the long-run equilibrium price of oil. Indeed, the far-dated futures prices declined from roughly $140 per barrel in the spring of 2008 to roughly $60 per barrel by the end of that year.

Lastly, the evolution of spot and futures prices since 2010 is shown in the bottom panel of the figure. Between 2010 and 2014, the fluctuations in prices had more in common with the 1990s. That is, market participants appear to have perceived that most of the fluctuations in the spot price of oil were largely transitory, with the long-run futures price remaining fairly stable despite significant movements in the spot price.

However, the oil market changed dramatically in mid-2014 when the spot price collapsed by roughly 50 percent.[4] As in the early 2000s, it took some time for futures markets to change their views about the persistence of the price change, which was initially perceived as being somewhat temporary, with futures curves rising back to a long-run price of about \$80 per barrel during most of 2014. By the end of 2015, this assessment was substantially changed, as far-dated futures prices only reached about \$55 per barrel.

3 Empirical framework

In this section, we develop a simple unobserved components model to account for the role of permanent and temporary shocks in determining oil-price futures. By design, we adopt a straightforward approach to highlight our main point. Accordingly, we abstract from many features of the oil market. Specifically, we postulate that spot oil prices are the result of movements in permanent and transitory components and that market participants use the Kalman filter to assess the relative importance of these two components over time. In addition, under our baseline model, we allow the model parameters to evolve with the data sample. In Appendix A, we assess the robustness of our baseline results by analyzing alternative models that explicitly allow for time-varying parameters. As reported in the appendix, results are largely unchanged.

3.1 A simple model

Consider the following linear process relating the spot price of oil s_t (expressed in logs) to a permanent component, e_t^P, and a stationary one, e_t^τ,

$$s_t = e_t^P + e_t^\tau. \tag{1}$$

Schwartz and Smith (2000) modelled oil prices using this assumption but, unlike in the current paper, they assumed that the model parameters were constant. The permanent component is modelled as a random walk with drift:

$$e_t^P = \mu + e_{t-1}^P + v_t, \tag{2}$$

[4] Further discussion of this decline can be found in Baumeister and Kilian (2016).

where v_t is an independently and identically normally distributed disturbance with mean zero and constant variance σ_p^2. The temporary component is assumed to follow the AR(1) process

$$e_t^\tau = \phi_\tau e_{t-1}^\tau + \varepsilon_t, \tag{3}$$

where ε_t is an independently and identically normally distributed disturbance with mean zero and constant variance σ_τ^2 and with $|\phi_\tau| < 1$.

Assuming full information at time t about the temporary and permanent components underlying oil prices, the k-period-ahead futures price at time t, $f_{t,k}$, is given by the following expression:

$$f_{t,k} = E_t s_{t+k} = k\mu + e_t^P + \phi_\tau^k e_t^\tau. \tag{4}$$

In contrast, absent full information about the current levels of e_t^P and e_t^τ, the futures price will be based on the best forecasts given past values of s_t:

$$f_{t,k} = E_t \left(s_{t+k} | \left\{ s_{t-i} \right\}_{i=t}^1 \right) = E_t \left(k\mu + e_t^P + \phi_\tau^k e_t^\tau | \left\{ s_{t-i} \right\}_{i=t}^1 \right). \tag{5}$$

To determine the relative importance of permanent shocks, a simple statistic can be derived from the expression for the change in the spot price of oil:

$$\triangle s_t = \mu + v_t + (\phi_\tau - 1) e_{t-1}^\tau + \varepsilon_t,$$

which implies that the variance of $\triangle s_t$ can be expressed as

$$\sigma_{\triangle s}^2 = \sigma_p^2 + \frac{2}{(1+\phi_\tau)} \sigma_\tau^2,$$

and the fraction of $\sigma_{\triangle s}^2$ due to permanent shocks is captured by the following expression:

$$\frac{\sigma_p^2}{\sigma_p^2 + \frac{2}{(1+\phi_\tau)}\sigma_\tau^2}.$$

3.2 Learning

We assume that market participants use the Kalman filter to form expectations of future oil prices. Our treatment of the Kalman filter is a standard textbook treatment (Hamilton, 1994). In particular, define ξ_t as the unobserved state vector of the model above, comprising the trend, μ, as well as the permanent and temporary components: $\xi_t = \begin{pmatrix} \mu & e_t^P & e_t^\tau & \mu & e_{t-1}^P & e_{t-1}^\tau \end{pmatrix}'$. Given values of the model's parameters, $\Gamma = [\ \sigma_p^2,$

σ_τ^2, μ, ϕ_τ], the Kalman filtering equation relates how the observed variables (here the change in the spot oil price $\triangle s_t$) respond to the changes in the unobserved state vector ξ_t.

The equations for the dynamics of the observed variables $\triangle s_t$ are given by the following system:

$$\begin{aligned} \triangle s_t &= H\xi_t \\ \xi_t &= F\xi_{t-1} + \eta_t, \end{aligned} \tag{6}$$

where F and H are matrices of parameters and $\eta_t = \begin{pmatrix} 0 & v_t & \varepsilon_t & 0 & 0 & 0 \end{pmatrix}$ are the shocks. In turn, the unobserved state vector evolves according to the following standard equation:

$$\xi_{t|t} = \xi_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H)^{-1}\left(\triangle s_t - H'\xi_{t|t-1}\right) \tag{7}$$

given initial estimates of $\xi_{t|t-1}$ and $P_{t|t-1}$, where the forecast error $\left(\triangle s_t - H'\xi_{t|t-1}\right)$ is used to update the estimates of the size of the permanent and transitory components via the term $P_{t|t-1}H(H'P_{t|t-1}H)^{-1}\left(\triangle s_t - H'\xi_{t|t-1}\right)$. Thus, the value of $P_{t|t-1}$ governs whether a given surprise is assumed to be part of the permanent or the transitory component, which is influenced by the values of σ_p^2 and σ_τ^2.

Whether movements in the spot price of oil are perceived to be permanent or temporary will affect futures prices as well. If a 1 percent increase in the spot price of oil is interpreted as purely transitory, then the futures price k-period ahead, $E_t\left(s_{t+k}|\left\{s_{t-i}\right\}_{i=t}^{1}\right)$, only increases by ϕ_τ^k, as indicated by equation (5) above. In contrast, if the same increase is interpreted as purely permanent, then the value of $E_t\left(s_{t+k}|\left\{s_{t-i}\right\}_{i=t}^{1}\right)$ increases by the value of the permanent component, e_t^P. If a shock is actually permanent but mistaken to be temporary, then the value of the futures price will include this error.

So far our discussion has assumed that the parameter values were known with certainty, but in practice we will need to estimate the model parameters, $\Gamma = [\ \sigma_p^2,\ \sigma_\tau^2,\ \mu,\ \phi_\tau]$, to derive forecasts of future oil prices. We assume that market participants estimate the model's parameters using the standard likelihood function:

$$LL\left(\Gamma\right) = -\sum_{t=1}^{T}\left(\frac{1}{2}\ln 2\pi + 0.5\log\left\|V_t\right\| + \left(\triangle s_t - E\triangle s_t\right)V_t^{-1}\left(\triangle s_t - E\triangle s_t\right)\right), \tag{8}$$

where $V_t = H'P_{t|t-1}H$ is the variance of the prediction errors. In Appendix A, we provide an alternative likelihood function that puts more weight on recent observations.

4 Baseline results

In this section, we present our model's predictions for futures prices assuming that market participants form expectations about the permanent and temporary components of oil prices through the Kalman filter. We estimate our model assuming that market participants form their beliefs using univariate methods, i.e., using only data on the spot price of oil. Although extracting information about the components of oil prices solely from previous spot prices may be suboptimal, our emphasis on univariate methods has the benefit of simplicity and shares similarities with the learning algorithm used in the monetary policy literature (see, for instance, Orphanides and van Norden, 2005, and Primiceri, 2006). Using the price of West Texas Intermediate from 1980Q1 to 2015Q4, we construct the model-implied estimates of the two-year-ahead futures prices and compare them with the actual futures price data.[5]

To compute the k-period ahead futures prices we apply the following three-step procedure. First, we use spot oil prices observed up to time $t-1$ and estimate the model parameters $\Gamma = [\ \sigma_p^2,\ \sigma_\tau^2,\ \mu,\ \phi_\tau^k]$ using the standard likelihood function. Second, we apply the Kalman filter using the estimated model parameters and observed prices through time t to get estimates of the unobserved permanent and temporary components e_t^P and e_t^τ. In the third step, we use the estimated e_t^P and e_t^τ and Γ to construct $f_{t,k}$.

We are particularly interested in the behavior of futures prices since the late 1990s. As such, we first estimate the model from 1980Q1 to 1998Q4 and start calculating futures prices from this period on, using an expanding window of data. Thus, the futures prices at the beginning of 2000Q1 are calculated using the model estimated over the period from 1980Q1 to 1999Q4. Similarly, the sample 1980Q1–2004Q3 would be used to estimate the model and compute the forecast of futures prices in 2004Q4. Our modelled two-year ahead futures price is compared against the actual two-year ahead futures price, measured using the closing quote from NYMEX for WTI crude at the end of the first week of the following quarter (typically near the 7th of the month). For instance, the futures price computed using the spot price through the fourth quarter of 2015 is compared against the closing futures price for January 7, 2016.

[5]We begin the sample in 1980 when U.S. oil production was deregulated. From 1986 onward, we use the WTI prices reported by the Energy Information Administration (EIA), while for earlier observations we use those reported in Alquist et al. (2013). Our measure of the quarterly price is the average price during the last month of the quarter.

One concern is whether there is sufficient activity in these far-dated contracts to provide useful information. One relevant metric to assess the market's liquidity is the fraction of total open interest that has a duration greater than 18 months. Before 2003, these further-dated futures contracts accounted for roughly 10 percent of open interest. However, their importance rose substantially between 2004 and 2013, averaging just under 20 percent of total open interest. With the decline in oil prices in mid-2014, far-dated futures contracts went back down to about 10 percent of total open interest. Although subject to fluctuations, the liquidity of the far-dated futures market appears sufficient to provide relevant price information.

We first present the behavior of the model's parameter estimates over the expanding estimation window in Figure 2 and Figure 3. In both figures, solid black lines report the estimated coefficients while the grey intervals are two-standard-deviation confidence intervals. The results are broadly in line with the narrative of Figure 1. First, the left panel of Figure 2 reports the estimated value of μ. Using only the pre-2000 part of the sample, the point estimate of μ is slightly below zero, implying a negative trend for the nominal oil price. However, as the estimation sample includes more of the post-2000 data, the estimated trend first turns positive and then begins to increase, although the uncertainty around the estimated value is large. The maximum value of the trend occurs for the sample ending in the second quarter of 2008, before starting to decline and stabilizing around 1.3 percent per quarter, or at an annual rate of just over 5 percent. In contrast to μ, the value of ϕ_τ, the autoregressive coefficient of the temporary component, is more stable, varying only slightly around a value of 0.7 (right panel of Figure 2).

Next, Figure 3 reports the estimation results for the standard deviation of the permanent and temporary components, σ_p and σ_τ, respectively. These estimated coefficients do vary considerably as the estimation sample period expands and are also more precisely estimated. In particular, the estimated value of σ_p is notably zero for the initial sample ending in 1998Q4, implying that market participants perceived oil prices to be solely driven by temporary factors. As more data from the 2000s are included in the estimation sample, the perceived contribution of the permanent component steadily increases, peaking in the second quarter of 2008, before the financial meltdown and global recession. In contrast, the estimated value of σ_τ broadly follows the opposite pattern.

The evolution of these model parameters can also be viewed by considering the role of permanent shocks in affecting the variance of $\triangle s_t$, which is reported in Figure 4. The figure shows that the estimated contribution of permanent shocks only slowly increases over time. In the early part of the sample, because the standard deviation of innovations to the permanent component is extremely small, the permanent component's contribution to the variance of $\triangle s_t$ is negligible, so that the temporary component is the main driver of changes in the spot price. These results are very much in line with our assessment of Figure 1, where the futures curves show transitory deviations from a long-term price during the 1990s and early 2000s. However, the estimated contribution of permanent shocks rises roughly steadily between 2002 and the first half of 2008, when it accounted for more than 60 percent of the variance of $\triangle s_t$. Thereafter, the sharp fall in oil prices in the last half of 2008 resulted in lower estimates of the role of permanent shocks, which remained fairly stable thereafter. In turn, our estimates suggest that when the price of oil collapsed in June 2014, market participants perceived oil-price movements as more likely to be transitory. This perception has continued through 2014, but has risen slightly in early 2015.

Given the estimates of the model's parameters, we now construct forecasts of the one- and two-year ahead futures prices, using the Kalman filtering formula (7) for each quarter from 1999Q1 onward. Figure 5 illustrates the evolution of the estimated and actual two-year-ahead futures prices, as well as the spot price of oil. Comparing the actual futures prices with our model-implied futures price, the figure shows that our simple framework model does reasonably well in matching what happened.[6] As are the actual futures prices, our estimated futures prices are well below the observed spot prices in the early 2000s, suggesting again that market participants viewed the underlying factors pushing spot prices up to be mostly transitory. By the mid-2000s, our estimated futures prices move closely together with the spot price. In line with the rising estimate of the contribution of the permanent component to the variance in oil-price changes in Figure 5, changes in the spot price of oil by the mid-2000s are perceived as being mostly permanent and are thus being reflected rapidly in futures prices. As

[6]The grey bands indicate the confidence interval, which is defined as the following set:

$$\left\{ E_t \left(s_{t+8} | \left\{ s_{t-i} \right\}_{i=t}^{1}, \tilde{\Gamma}_{t-1} \right) | \left(\Gamma_{t-1} - \tilde{\Gamma}_{t-1} \right)' W \left(\Gamma_{t-1} \right)^{-1} \left(\Gamma_{t-1} - \tilde{\Gamma}_{t-1} \right) \leq 4.5 \right\},$$

where Γ_{t-1} is the maximum likelihood estimate and $W(\Gamma_{t-1})$ is the corresponding estimated variance-covariance matrix. The critical value of 4.5 is chosen as the 66 percentile of the chi-squared distribution with four degrees of freedom.

the financial crisis intensified in mid-2008, the spot price of oil fell rapidly, but this decline was much more pronounced than the fall in the actual futures price, which is well captured by our estimated value. Our model tracked well the actual futures price between 2010 and the end of 2013. A gap then developed as our predicted futures kept rising, while the actual futures price was relatively stable. This gap was largely erased by mid-2014.

To assess the model fit, Figure 6 reports the cumulative mean-squared error (MSE) of fitting the futures price using the implied value from our Kalman Filtering approach relative to using the previous quarter's spot price. As is clear from the figure, the Kalman filtering approach is much better at fitting the futures prices than the spot price. As noted from Figure 5, the model does do somewhat worse in matching the futures price after 2012. However, the overall performance is still considerably better than using the spot price. Furthermore, as described in Appendix A, alternative models, such as the particle filter, can improve the fit along that dimension.

Overall, our results indicate that learning about the persistence of underlying shocks helps capture movements in oil-price futures. Although our model is simple and only uses information from past movements in the spot price of oil, it accounts reasonably well for the fluctuations in oil-price futures over the past 15 years. In the appendix, we also consider extensions of our approach to assess robustness. Section A.1 considers the possibility that market participants may be concerned with structural breaks and thus examines a form of learning that places relatively more weight on recent observations. Section A.2 extends our framework to one with time-varying volatility, which allows for time variation in our model parameters. Overall, we find our baseline results to be robust to these relevant extensions.

5 A DSGE model with learning

In the previous sections, we showed that investors' perception of the persistence of oil prices clearly changed over the past 15 years and that these changes are captured reasonably well by a learning process about the role of transitory and persistent factors in the economy. We now assess the importance of this learning process for the impact of oil shocks on economic activity, using a DSGE model in which agents learn about the persistence of oil-price movements via the Kalman filter.[7] Throughout our theoretical analysis, we assume that economic agents are subject to information constraints similar to those that investors face in the futures markets, as documented above. The model consists of households that supply labor and rent capital to firms and save over time by holding one-period, pure discount bonds and by accumulating capital. One novel aspect of our approach is the use of a storage model. This framework is a priori appealing for our purpose, since storage directly links oil prices to expectations of future oil prices. We follow Arseneau and Leduc (2013) and assume that households hold oil inventories. As is typical in the rational expectations storage literature, speculation in inventory holdings allows the household to smooth temporary volatility in the oil market. The production side of the model is composed of firms producing a consumption good using labor, capital, and oil.

We analyze three specific scenarios, which are model parameterizations of the model that are meant to capture salient features of our empirical results at important points during the past 15 years. Our first scenario is the *2003 scenario*, which assumes that agents in our economy perceive shocks affecting the outlook for oil prices as being mostly transitory, as was the case in the early 2000s, according to our empirical findings. For our second scenario, the *2007 scenario*, we calibrate the economy such that the shocks underlying oil-price movements are perceived to be mostly permanent, in line with investors' perceptions on the eve of the Great Recession. Finally, we calibrate a *2014 scenario* that assesses the role of learning during the recent oil-price collapse. For each scenario, we simulate the impact of an oil-demand shock on economic activity. We then contrast the responses of the economy to those under full information to capture the effect of agents' perceptions. We focus on an oil-demand shock since it played an important role during the run-up in oil prices during the past decade (Kilian, 2009). However, the gist of our results does not depend on the source of oil price fluctuations and generalizes to oil-supply shocks or to environments in which oil prices are exogenous and subject to random fluctuations as in Leduc and Sill (2004).

[7] Erceg and Levin (2003) and Andolfatto et al. (2008) develop related frameworks, in which economic agents assess persistent and transitory shocks to monetary policy via a filtering mechanism.

5.1 Households

The representative household's utility function is defined over the consumption of a composite good, c_t, and hours worked, n_t

$$U_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \eta \frac{n_t^{1+\varphi}}{1+\varphi} \right] \tag{9}$$

where β denotes the subjective discount factor, σ represents the coefficient of relative risk aversion, and φ controls the Hicksian labour supply elasticity. In turn, the composite consumption good is itself the following CES combination of final good consumption, c_t^g, and oil consumption, c_t^o:

$$c_t = \left[(1-\omega_c)^{1/\xi_c} c_t^{g\,\frac{\xi_c-1}{\xi_c}} + \omega_c^{1/\xi_c} c_t^{o\,\frac{\xi_c-1}{\xi_c}} \right]^{\frac{\xi_c}{\xi_c-1}}, \tag{10}$$

with ξ_c denoting the elasticity of substitution between the final good's consumption and oil consumption and where ω_c represents the share of oil consumption in the composite good.

The price of the composite good, p_t^c, is given by the standard expression:

$$p_t^c = \left[(1-\omega_c) + \omega_c p_t^{o\,1-\xi_c} \right]^{\frac{1}{1-\xi_c}}, \tag{11}$$

where the price of the final good is taken to be the numéraire and p_t^o is the relative price of oil.

Households supply labor and capital services to firms producing the final good, and the associated income from these two activities are $w_t n_t$ and $r_t k_t$, with the real wage and rental rate of capital denoted by w_t and r_t, respectively.

Households hold three types of assets: bonds, capital, and oil inventories. First, we assume they can purchase a one-period real discount bond, denoted b_{t+1}, at price $p_{b,t+1}$; the current holding b_t thus constitutes an additional source of revenue. Next, capital, k_t, accumulates according to the usual law of motion

$$k_{t+1} = i_t + (1-\delta) k_t, \tag{12}$$

where i_t denotes investment and δ represents the depreciation rate of capital.

Finally, we assume that households can purchase s_{t+1} units of oil, to hold as storage until the next period. Because households cannot borrow oil from the future, inventories must be non-negative. However, to simplify the numerical analysis below, we assume that the economy fluctuates around a steady state characterized by inventory holdings sufficiently large that the non-negativity constraint is never binding.[8] We verify that this condition is met in our simulations below. Holding inventories entails a per-unit cost $\phi(s_{t+1})$, with $\phi'(s_{t+1}) > 0$ in terms of oil. Finally, households receive a fixed endowment of oil each period, $\overline{o}$, which they can sell to firms on the spot market for oil.

In this context, the household optimization problem is to choose sequences of c_t, n_t, $k_{t+1,}$ s_{t+1}, and b_{t+1} to maximize (9) subject to (12) and an infinite sequence of flow budget constraints given by:

$$p_t^c c_t + p_{b,t+1} b_{t+1} + i_t + p_t^o s_{t+1} \left[1 + \phi(s_{t+1})\right] = w_t n_t + r_t k_t + b_t + p_t^o s_t + p_t^o \overline{o}_t, \qquad (13)$$

The efficiency conditions being standard, we concentrate on the optimal demand for oil inventories by households, which is characterized by the following condition:

$$p_t^o(1 + \phi(s_{t+1}) + \phi'(s_{t+1}) s_{t+1}) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} p_{t+1}^o\right], \qquad (14)$$

where λ_t denotes the marginal utility of wealth. This expression states that the household will accumulate oil inventories up until the marginal cost of holding one additional unit, inclusive of the cost of storage, is equal to the expected gain from holding the commodity for one period and then selling it at next period's spot price.

Finally, we define the current futures price of oil for delivery at time $t+1$ as $f_{t+1|t} \equiv E_t\left[p_{t+1}^o\right]$, i.e., the expected future spot price of oil. The model solution then allows us to also compute $f_{t+k|t}$, the futures price for delivery at a future date $t+k$.

5.2 Production

Final Goods

Firms combine capital, labor, and oil inputs to produce a final good, using the following two-stage production process. First, production of the final good y_t is given by

$$y_t = \left[(1-\omega_y)^{1/\xi_y} va_t^{\frac{\xi_y - 1}{\xi_y}} + \omega_y^{1/\xi_y} z_t^o\, o_t^{\frac{\xi_y - 1}{\xi_y}}\right]^{\frac{\xi_y}{\xi_y - 1}}, \qquad (15)$$

[8] See Williams and Wright (1991) and Arseneau and Leduc (2013) for partial and general equilibrium analyses directly tackling the non-negativity constraints on inventories.

where o_t represents the oil input, ω_y is the share of oil in final output, and ξ_y is the elasticity of substitution between energy and value-added va_t, which itself is the following CES combination of capital and labor:

$$va_t = \left[(1 - \omega_{va})^{1/\xi_{va}} k_t^{\frac{\xi_{va}-1}{\xi_{va}}} + \omega_{va}^{1/\xi_{va}} n_t^{\frac{\xi_{va}-1}{\xi_{va}}} \right]^{\frac{\xi_{va}}{\xi_{va}-1}}, \tag{16}$$

where ω_{va} is the share of labor in value added. In (15), the shock z_t^o affects the relative weight of oil in the production of a given level of final goods; we interpret this shock as a demand shock for oil, in the spirit of Bodenstein and Guerrieri (2011).

Shocks

The shock z_t^o to the demand for oil is affected by both persistent and transitory components, as captured by the following expression:

$$z_t^o = \overline{z^o}\, \varepsilon_{z,t}^P\, \varepsilon_{z,t}^\tau, \tag{17}$$

where the persistent and transitory components $\varepsilon_{z,t}^P$ and $\varepsilon_{z,t}^\tau$ follow AR(1) processes:

$$log(\varepsilon_{z,t}^P) = \rho_z^P log(\varepsilon_{z,t-1}^P) + u_{z,t}^P, \tag{18}$$

$$log(\varepsilon_{z,t}^\tau) = \rho_z^\tau log(\varepsilon_{z,t-1}^\tau) + u_{z,t}^\tau. \tag{19}$$

Below, we calibrate the AR coefficient ρ_z^P to be 0.999, as a near-random walk and $u_{z,t}^P$ as a zero-mean Gaussian innovation with standard deviation σ_z^P. The AR coefficient for the transitory component, ρ_z^τ, is assumed to be strictly less than one, and u_t^τ is similarly a Gaussian disturbance with mean zero and standard deviation σ_z^τ. Importantly, we assume two different information structures: in the first, agents have full information and can directly observe the two components affecting the shock. In the second, agents know all the model parameters, but do not separately observe the permanent and transitory shocks to the demand for oil. Consistent with our empirical results above, we assume instead that they use the Kalman filter to learn about the permanent and transitory shocks.

5.3 Equilibrium

Taking as given the exogenous shocks z_t^o, the equilibrium of the model is a sequence of $\{y_t,\ va_t,\ c_t,\ c_t^g,\ c_t^o,\ n_t,\ k_t,\ o_t,\ s_t,\ w_t,\ p_t,\ r_t\}$ that satisfy the household optimality conditions; the optimality conditions for firms producing final and consumption goods; the bond market clearing condition, as well as the oil-market clearing conditions $(1+\phi(s_{t+1}))s_{t+1} - s_t + c^o_t + o_t = \overline{o}$;[9] and the resource constraint $c^g_t + i_t = y_t$.

5.4 Calibration

We calibrate the structural parameters to be consistent with the literature and broadly match some characteristics about the importance of oil in the economy. First, parameters common to most business cycle models are calibrated. The model is calibrated to a quarterly frequency, and we set the discount factor β and the depreciation rate δ to 0.99 and 0.025, respectively. The parameter σ, controlling the extent of intertemporal substitution in consumption, is 2, and the Hicksian elasticity of labor supply φ is 1. Finally, the scaling parameter η is set in order for the labor input in the economy's steady state to be 1.

We next calibrate the parameters related to the production and storage of oil. Following Unalmis et al. (2012), we adopt the following storage cost function, $\phi(s) = \kappa + \frac{\phi}{2}s$, where the constant κ can be thought of as a convenience yield that we set to 5 percent and where ϕ is set such that the steady-state stock of oil stored as a ratio of total (quarterly) output is 50 percent, also as in Unalmis et al. (2012).

Next, the three pairs of production parameters, ω_{va} and ξ_{va}, ω_y and ξ_y, as well as ω_c and ξ_c, are assigned values. First, we set $\omega_{va} = 0.66$ and $\xi_{va} = 1.0$, which matches the values used in standard business cycle models where substitution between capital and labor in production is unity and the capital share in final output is around one-thrid. The remaining parameters are assigned the values $\omega_y = \omega_c = 0.0585$ and $\xi_y = \xi_c = 0.1$, which ensures that the elasticity of substitution between oil and other inputs is small, at 0.1, and that the steady-state value of oil used in production and in composite consumption is about 4 percent, the values also used in Bodenstein, Erceg, and Guerrieri (2011).

The evolution of the permanent and transitory shocks to oil demand is governed by the parameters ρ^P_z, ρ^τ_z, σ^P_z, σ^τ_z. Their values are key for determining the agents' inference about the persistence of oil-price movements. First, we set ρ^P_z arbitrarily close to 1, so

[9] The aggregate endowment of oil, $\overline{o}$, could also be subjected to shocks; as mentioned above, simulation results analyzing such *supply* shocks are broadly similar to those discussed below, which result from the demand shocks z^o_t.

that these shocks have a near unit root. In assigning values for the remaining three parameters, we consider three scenarios, labelled *2003*, *2007*, and *2014*, respectively, that are meant to replicate the spirit of our empirical findings at three key moments.

First, the *2003 scenario* captures an environment where agents consider most of the shocks affecting the oil market to be transitory. Under this scenario, permanent shocks will only gradually be recognized and integrated in agents' expectations. To this end, we first use our estimate of the autocorrelation coefficient of transitory shocks of roughly 0.75 in 2003 to calibrate ρ_z^τ (see right panel of Figure 2). Next, we calibrate σ_z^P and σ_z^τ to match the perceived relative importance of permanent and transitory shocks in 2003. As shown in Figure 4, less than 10 percent of the variance in spot price changes was attributed to permanent shocks around that time. In practice, this strategy only allows us to pin down the relative magnitudes of σ_z^P and σ_z^τ. We thus set $\sigma_z^\tau = 0.01$ as a benchmark, and the procedure yields $\sigma_z^P = 0.003$.

By contrast, the *2007 scenario* captures the view that the perceived relative importance of permanent shocks had risen substantially by 2007. As documented above, roughly 50 percent of shocks affecting the spot price of oil were perceived to be permanent on the eve of the Great Recession. In addition, our estimate of the autocorrelation of transitory shocks indicates a slight decrease, to around 0.65. To reflect this environment, we set $\rho_z^\tau = 0.65$ and $\sigma_z^P = 0.005$, while keeping the benchmarked value of σ_z^τ unchanged at 0.01. All told, the *2007 scenario* represents an environment where agents consider the occurrence of permanent shocks to be very likely, so that their consequences for the oil market and the macroeconomy will be rapidly internalized when they occur.

Finally, the *2014 scenario* is based on our empirical results on the eve of the steep decline in oil prices in the middle of 2014. Our findings indicate that at that time investors perceived that about 30 percent of oil-price shocks were permanent. This scenario is thus midway between the previous two, and so we set $\sigma_z^P = 0.013$.

In all our simulations below, the consequences of learning for oil prices and for economic activity are assessed by integrating Kalman filtering about the two types of shocks in the King and Watson (2002) first-order solution method (see Appendix B for details).

6 Macro effects of learning

We now examine the response of the economy to near-permanent shocks to oil efficiency (i.e., a shock to the demand for oil captured by the variable $\varepsilon^P_{z,t}$ above), comparing the effect of such shocks in the case where agents can fully observe them to the case when they must instead infer their persistence via learning. We linearize the model's equilibrium conditions around the economy's steady state and study a shock whose amplitude entails a 10 percent increase in oil prices under full information. Agents observe the increase in the price of oil and its associated economic impact, but must infer its persistence based on their perception of the relative importance of permanent and transitory shocks. Over time, as the economy evolves, agents reassess their views regarding the persistence of the shock to oil efficiency using the Kalman filter, which in turn informs their forward-looking decisions.

To demonstrate the model mechanism, we first contrast the transmission of a permanent shock to that of a transitory one, assuming that agents have complete information about the source of the shock underlying the rise in the price of oil. Figure 7 compares these responses, with the solid line denoting the economy's response following a permanent shock and the dotted line representing those stemming from a transitory one.

The figure shows that the transitory shock has a substantially more muted impact on output relative to its response following a permanent shock, partly reflecting both the large decrease in oil inventories and the temporary nature of the shock. With the price of oil temporarily higher, inventory demand declines sharply, which helps mitigate the reduction in the effective oil supply and the associated reduction in output. In contrast, inventories decline much less following a permanent increase in oil prices, so the rise in the price of oil and the associated decline in oil supply leads to a larger decline in output. The figure also shows that the more muted response of economic activity following the transitory shock is also shared by consumption and investment.

Next, we examine the effect of the perceptions of oil-shock persistence and their interaction with storage. In the following exercises, we look at the response of the economy to the same permanent oil-demand shock, but under two alternative scenarios about available information. We first reproduce the effects of the shock when full information about its persistence is available, but then contrast those to responses when the shock is misperceived to be temporary, as parameterized under our *2003 scenario.*

We then contrast the results to those under the *2007 scenario* in which agents expect the shock to be nearly permanent.

Figure 8 reports results under the *2003 scenario*. It shows that when agents misperceived the shock as being largely transitory ,the decline in GDP is substantially smaller, about one-half, than when agents have complete information. This partly reflects the much greater decline in inventories and thus the associated smaller rise in the price of oil. With incomplete information, the spot price of oil rises persistently above the one-year-ahead futures price, a feature observed empirically during that period. In addition, the muted response of output is once again shared by investment and consumption.

Even when agents expect the shock to be fairly persistent, as under the *2007 scenario*, the near-term decline in activity continues to be substantially smaller relative to an economy with complete information, as shown in Figure 9. However, agents learn more rapidly about the persistence of the shock, so the impact on the economy quickly resembles that of an economy with complete information. Under the *2007 scenario*, spot prices do not rise as much above futures prices, and the quantitative differences between the full-information and learning scenarios are relatively modest.

The macroeconomic responses following this adverse shock stem from the combination of expectation effects, which describe agents' views about the perceived durability of the shock, and the storage capabilities of the economy, which allow transitory shocks in oil's efficiency to be smoothed over. Figure 10 shows that the ability to store oil interacts importantly with expectations to create the macroeconomic responses described above. The figure contrasts full-information and learning responses in the economy with storage (charts in the left column) with those arising when agents are unable to store oil (on the right).

The figure shows that this storage ability is important in dampening the fall in output during the quarters immediately following the shock: the left-hand panels show that GDP decreases significantly less when inventories are available to smooth the shocks' effects, and the rise in oil prices is concurrently much more modest. When storage is not available to ease the transition following the shock (right-hand panels), the decreases in GDP and oil-price rises are broadly similar under the two information regimes, showing that storage ability is an important factor in our framework for learning to have significant effects on the macroeconomy.

Overall, our model findings indicate that the learning process in the futures market documented in this paper can have a meaningful impact on the transmission of oil shocks on the economy. In particular, our empirical evidence implies that the negative effect of oil-demand shocks on output are only about one-half of its effect in an economy with full information. Agents expecting most movements in oil prices to be transitory may therefore account for the muted effects of oil shocks on the macroeconomy during the 2000s.

In that context, it is also instructive to bring the model to bear on the more recent oil-price collapse: between June 2014 and the beginning of 2015, oil prices fell by roughly 50 percent. Using our *2014 scenario*, we examine the effect of such a decline. Figure 11 reports the results of a permanent *negative* shock to oil efficiency (oil demand) calibrated to result in a 50 percent fall in oil prices under full information. The figure shows that the boost in economic activity following this positive shock strongly depends on its perceived persistence: when agents correctly perceive that the shock will be highly persistent, output, investment, and consumption rise substantially more than when agents have incorrect perceptions. According to our estimates, this learning process leads to an output response that is about 25 percent more muted in the first few quarters following the shock than that under full information. This weaker response partly reflects a rise in inventories that helps mitigate the effect on oil prices. Note that in this case the spot price falls substantially more than the futures price compared with the economy with full information. This analysis highlights the possible role of learning for the relatively weak response of economic activity following the recent oil-price collapse.

7 Conclusion

We provide an analysis of movements in oil-price futures since 2000 based on learning and assess the macroeconomic implications brought about by this learning process. We show that a simple unobserved component model in which investors must form beliefs about the persistence of changes in oil prices accounts well for the fluctuations in oil-price futures. Our simple framework captures the relatively slow increase in futures prices at the beginning of the past decade and their unprecedented run-up between 2004 and 2008. Even during the first half of 2008, a period during which oil prices reached historic highs, the model predicts a level of futures prices that is broadly in line with the data. Our estimates suggest that through learning investors revised up the contribution of permanent shocks to the variance of oil prices over this period. Similarly, our results suggest that throughout 2014 investors perceived the dramatic decline in oil prices as largely temporary.

Using a DSGE model in which oil is storable and used in production, we show that agents' learning in the form suggested by our empirical results can substantially alter the impact of oil shocks. Consistent with our empirical results, we calibrate three scenarios that capture market participants' perceptions regarding the persistence of oil prices in 2003, when changes in oil prices were largely thought to be transitory; in 2007, when oil-price changes were expected to be much more persistent, and more recently, during the oil-price collapse in 2014, midway between the first two scenarios. We show that compared with a framework with full information, the recessionary effects of oil-price increases are significantly more muted under learning than with full information, particularly in our 2003 scenario, where the negative impact of that shock on output is roughly halved in the year following the rise in the price of oil. As such, our analysis highlights expectations formation as an additional factor accounting for the smaller effects of oil shocks during the past 15 years, complementing the role of demand shocks, changes in monetary policy, and a smaller dependence on oil than in previous decades.

References

Acharya, Viral V., Lochstoer, Lars A., and Tarun Ramadorai, 2013. "Limits to Arbitrage and Hedging: Evidence from Commodity Markets," *Journal of Financial Economics* 109(2), pp. 441-465.

Alquist, Ron and Lutz Kilian, 2010. "What Do We Learn from the Price of Crude Oil Futures?" *Journal of Applied Econometrics* 25(4), pp. 539-573.

Alquist, Ron, and Olivier Gervais, 2013. "The Role of Financial Speculation in Driving the Price of Crude Oil," *The Energy Journal*, International Association for Energy Economics 34(3), pp. 35-54.

Alquist Ron, Lutz Kilian and Robert J. Vigfusson, 2013. "Forecasting the Price of Oil," in: G. Elliott and A. Timmermann (eds.), Handbook of Economic Forecasting 2, pp. 427-507.

Andolfatto, David, Hendry, Scott, and Kevin Moran, 2008. "Are inflation expectations rational?" *Journal of Monetary Economics* 55(2), pp. 406-422.

Arseneau, David M. and Sylvain Leduc, 2013. "Commodity Price Movements in a General Equilibrium Model of Storage," *IMF Economic Review* 61(1), pp. 199-224.

Baumeister, Christiane and Lutz Kilian, 2015. "A General Approach to Recovering Market Expectations from Futures Prices With An Application to Crude Oil," Manuscript.

Baumeister, Christiane and Lutz Kilian, 2016a. "Understanding the Decline in the Price of Oil since June 2014 " *Journal of the Association of Environmental and Resource Economists*, 3(1), pp. 131-158.

Baumeister, Christiane and Lutz Kilian, 2016b. "Forty Years of Oil Price Fluctuations: Why the Price of Oil May Still Surprise Us", *Journal of Economic Perspectives* 30(1), pp. 139-160.

Blanchard, Olivier J. and Jordi Galì, 2009. "The Macroeconomic Effects of Oil Shocks: Why are the 2000s So Different from the 1970s?" in Galì Jordi and Mark Gertler (eds.) *International Dimensions of Monetary Policy*. Chicago: University of Chicago Press.

Bodenstein, Martin and Luca Guerrieri, 2011."Oil efficiency, demand, and prices: a tale of ups and downs", International Finance Discussion Papers 1031, Board of Governors of the Federal Reserve System.

Bodenstein, Martin, Christopher J. Erceg, and Luca Guerrieri, 2011. "Oil shocks and external adjustment," *Journal of International Economics* 83(2), pp. 168-184.

Buyuksahin, B., Haigh, M., Harris, J., Overdalh, J. and M. Robe 2008. *Fundamentals, Trader Activity and Derivative Pricing,* Manuscript.

Cho, In-Koo., Noah Williams, and Thomas Sargent, 2002. "Escaping Nash inflation," *Review of Economic Studies* 69(1), pp. 1-40.

Creal, Drew 2012. "A Survey of Sequential Monte Carlo Methods for Economics and Finance," *Econometric Reviews* 31(3), pp. 245-296,

Erceg, Christopher and Andrew T. Levin, 2003. "Imperfect credibility and inflation persistence," *Journal of Monetary Economics* 50(4), pp. 915-944.

Fattouh, Bassam, Lutz Kilian, and Lavan Mahadeva, 2013. "The Role of Speculation in Oil Markets: What Have We Learned So Far?" *Energy Journal* 34(3), pp. 7-33.

Fornero, Jorge and Markus Kirchner, 2014. "Learning About Commodity Cycles and Saving- Investment Dynamics in a Commodity-Exporting Economy," Working Papers Central Bank of Chile 727, Central Bank of Chile.

Hamilton, James D., 2009. "Causes and Consequences of the Oil Shock of 2007-08," *Brookings Papers on Economic Activity* Spring 2009, pp. 567-582.

Hamilton, James D., 1994. Time Series Analysis. Princeton University Press, Princeton,

Hamilton, James D. and Jing Cynthia Wu, 2014. "Risk Premia in Crude Oil Futures Prices," *Journal of International Money and Finance* 42(C), pp. 9-37.

Irwin, Scott H. and Dwight R. Sanders, 2012. "Testing the Masters Hypothesis in Commodity Futures Markets" *Energy Economics* 34, pp. 256-269.

Kellog, Ryan, 2014. "The Effect of Uncertainty on Investment: Evidence from Texas Oil Drilling," *American Economic Review* 104(6), pp. 1698-1734.

Kilian, Lutz, 2009. “Not All Oil Prices Are Alike: Disentengling Demand And Supply Shocks in the Crude Oil Market,” *American Economic Review* 99(3), pp. 1053-1069.

Kilian, Lutz, and Daniel P. Murphy, 2014. “The Role of Inventories and Speculative Trading in the Global Market for Crude Oil,” *Journal of Applied Econometrics* 29(3), pp. 454-478.

King, Robert and Mark Watson, 2002. “System Reduction and Solution Algorithms for Singular Linear Difference Systems under Rational Expectations,” *Computational Economics* 20: pp. 57–86.

Leduc Sylvain and Keith Sill, 2004. “A quantitative analysis of oil-price shocks, systematic monetary policy, and economic downturns,” *Journal of Monetary Economics* 51(4), pp. 781-808.

Milani, Fabio, 2009. “Expectations, learning, and the changing relationship between oil prices and the macroeconomy,” *Energy Economics* 31(6), pp. 827-837.

Ormeno, Arturo and Krisztina Molnar, 2015. “Using Survey Data of Inflation Expectations in the Estimation of Learning and Rational Expectations Models,” *Journal of Money, Credit and Banking* 47(4), pp. 673-699.

Orphanides, Athanasios and Simon van Norden, 2005. “The Reliability of Inflation Forecasts Based on Output Gap Estimates in Real Time,” *Journal of Money, Credit and Banking* 37(3), pp. 583-601.

Orphanides, Athanasios and John C. Williams, 2007. “Robust monetary policy with imperfect knowledge,” *Journal of Monetary Economics* 54(5), pp. 1406-143

Primiceri, Giorgio E., 2006. “Why Inflation Rose and Fell: Policy-Makers’ Beliefs and U. S. Postwar Stabilization Policy,” *The Quarterly Journal of Economics* 121(3), pp. 867-901.

Schwartz, Eduardo, and James E. Smith 2000. “Short-Term Variations and Long-Term Dynamics in Commodity Prices,” *Management Science* 46(7), pp. 893-911.

Singleton, Kenneth J. 2014. “Investor Flows and the 2008 Boom/Bust in Oil Prices,” Management Science 60(2), pp. 300-318.

Slobodyan, Sergey and Raf Wouters, 2012. "Learning in a Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models," *American Economic Journal: Macroeconomics* 4(2), pp. 65-101.

Stock, J. H. and Mark Watson, 2007. "Why Has U.S. Inflation Become Harder to Forecast?" *Journal of Money, Credit and Banking* 39(1), pp. 3-33.

Unalmis, Deren, Unalmis, Ibrahim, and Derya Filiz Undal, 2012. "On Oil Price Shocks: The Role of Storage," *IMF Economic Review* 60(4), pp. 505-532.

Williams, J.C., and B. D. Wright, 1991, Storage and Commodity Markets, Cambridge University Press, Cambridge, U.K.

A Alternative Statistical Models

We consider two alternatives to the standard Kalman filtering approach used in the main body of the paper. The first alternative is to modify the likelihood function to allow for greater weight being placed on more recent observations relative to earlier observations, a form of constant-gain learning. The second is to use a particle filtering approach to model the evolution of model parameters in a modified version of Stock and Watson's unobserved components with stochastic volatility model.

A.1 Constant-Gain Learning

Our baseline results highlight the importance of time variations in the model's parameter estimates. This finding suggests that investors may be concerned with structural breaks and may choose to weight recent observations relatively more than distant ones. As a result, we now consider the possibility that market participants use a modified likelihood function, which in the spirit of the recursive least squares algorithm in Cho, Williams and Sargent (2002), we define as follows

$$LL_T = (1-\chi_T)\, LL_{T-1} - \chi_T \left(\frac{1}{2} \ln 2\pi + 0.5 \log \|F_t\| + (s_t - Es_t)\, V_t^{-1} (s_t - Es_t) \right). \tag{20}$$

If $\chi_t = \frac{1}{t}$, then all observations have the same weight, equivalent to the standard likelihood function described above. In contrast, if χ_t is a constant, then recent observations are more important than lagged observations (in the learning literature, this approach is referred to as constant-gain learning). In particular, for a dataset of T observations, the first observation contributes $\prod_{t=1}^{T} (1-\chi_t)\, \chi_1$,whereas the most recent observation (observed at time T) has a much greater weight of χ_T. In conducting this exercise, we consider different constant values of χ_T in the weighted maximum likelihood estimation.

As a starting point, we use information from the literature on learning and monetary policy to parameterize the gain. We first set χ_T to 2 percent based on the value reported by Orphanides and Williams (2007) who estimate the (constant) gain that best fits the inflation forecasts from the Survey of Professional Forecasters. This value implies that an observation eight years in the past gets only half as much weight in the likelihood as the current observation. Figure A1 compares our baseline results with the estimate of the two-year-ahead futures price when $\chi_T = 0.02$. The figure shows that discounting past observations at this rate generally leads to worse forecasts. Relative to our baseline model, it significantly overpredicts oil-price futures from 2004 onward, predicting a peak of \$180 per barrel in the second quarter of 2008, well above the actual peak value.

Is there a value of χ_T such that the weighted maximum likelihood estimation results in a better model fit? The value of χ_T that best matches the two-year-ahead futures path is 1.5 percent. Remarkably, this is also the value used by Primeceri (2006) in a model of U.S. inflation in which policymakers learn about the natural rate of unemployment using a constant-gain algorithm. However, as shown in Figure A2, even with this optimized value, the constant-gain estimation tends to overpredict the run-up in futures prices between 2004 and 2008 relative to our baseline model, partly reflecting the large weight ascribed to permanent shocks as sources of oil-price fluctuations during this period (Figure 8). A similar pattern emerges since the end of the Great Recession.

A.2 Particle Filter

The three-step procedure that we used to derive our baseline results has the benefit of simplicity, but it also presents some potentially important limitations. It assumes that the model's parameters are constant. As a result, while the procedure allows investors to learn about the importance of temporary and permanent shocks, it restricts the evolution of the parameter values by requiring them to fit the entire sample rather than just recent observations.

In this section, we address this limitation by assessing the robustness of our baseline results to a more general learning process. In particular, we consider a variant of Stock and Watson's (2007) unobserved component model with stochastic volatility, in which we introduce an additional temporary shock to the level of oil prices. Although the model is similar to our baseline framework, it differs by allowing for time variation in Γ. Therefore, as before, the model for the log spot price of oil is

$$s_t = e_t^P + e_t^\tau, \tag{21}$$

where we continue to assume that the permanent component follows a random walk with drift:

$$e_t^P = \mu_t + e_{t-1}^P + v_t. \tag{22}$$

However, in contrast to our baseline framework, we allow for time variation in μ_t and assume that the disturbance v_t is Gaussian with time-varying variance $\sigma^2_{p,t}$: $v_t \sim N(0, \sigma^2_{p,t})$. Moreover, we postulate that the drift parameter, μ_t, follows a random walk:

$$\mu_t = \mu_{t-1} + \xi_{\mu,t} \tag{23}$$

and that $\sigma^2_{p,t}$ evolves according to

$$\ln \sigma^2_{p,t} = \ln \sigma^2_{p,t-1} + \xi_{P,t} \tag{24}$$

where $\xi_{\mu,t}$ and $\xi_{P,t}$ are Gaussian disturbances with zero mean and constant variance $\sigma^2_{\xi_\mu}$ and $\sigma^2_{\xi_P}$, respectively.

As before, the temporary component follows an AR(1) process:

$$e^\tau_t = \phi_{\tau,t} e^\tau_{t-1} + \varepsilon^\tau_t, \tag{25}$$

where $\phi_{\tau,t}$ is allowed to vary through time and $\varepsilon^\tau_t \sim N(0, \sigma^2_{\tau,t})$. Again, we assume that

$$\ln \sigma^2_{\tau,t} = \ln \sigma^2_{\tau,t-1} + \xi_{\tau,t}, \tag{26}$$

where $\xi_{\tau,t}$ is a homoskedastic, Gaussian error term with zero mean and constant variance $\sigma^2_{\xi_\tau}$.

In addition, following Schwartz and Smith (2000), we allow for the presence of measurement error in the pricing of oil futures that could capture errors in reporting or deviations between our model's fit and observed prices:

$$f^k_t = E s_{t+k} + \xi_t, \tag{27}$$

where the measurement error term, ξ_t, is assumed to be independently and identically normally distributed with zero mean and time-varying variance $\sigma^2_{\xi,t}$, which evolves according to

$$\ln \sigma^2_{\xi,t} = \ln \sigma^2_{\xi,t-1} + \xi_{\Phi,t}, \tag{28}$$

where $\xi_t \sim N(0, \sigma^2_\xi)$.

To bring our model to the data, we use the following set of equations consisting of the growth rate of the spot price of oil

$$\triangle s_t = \left[\mu_t + \varepsilon^P_t + (\phi_\tau - 1)\, e^t_{t-1}\right] + \varepsilon^\tau_t, \tag{29}$$

the expression for the spread between the k-period-ahead futures price and the spot price

$$f_t^k - s_t = \left[k\mu_t + \rho\Phi_{t-1} + \left(\phi^k - 1\right)\phi e_{t-1}^{\tau}\right] + \phi^k \varepsilon_t^{\tau} + \xi_{\Phi,t}, \tag{30}$$

as well as (22), (25), (23), (24), (26), and (28). To better discipline the particle filter, we complement the use of the price of West Texas Intermediate oil used for estimating our baseline model with the nine-month-ahead futures price. We then use the model's estimates to forecast two-year-ahead futures prices. Because we are limited by the availability of one-year ahead futures contracts, our estimation period begins in 1989Q1. The sample still ends in 2015Q4. The estimation is done using the particle filter as described in Creal (2012).

Figure A1 compares the expected futures prices using the Kalman filter model to those from the particle filter. The figure shows that, overall, both models have very similar predictions, especially in the 2002–05 period. In 2006 and 2007, the particle-filter-implied futures estimates are slightly higher, but the differences are relatively slight. In addition, the particle filter tracks better the decline of the two-year-ahead futures prices since 2013.

B Model Solution

We extend King and Watson's (2002) solution method to allow Kalman filtering of persistent and transitory shocks. Denote dynamic (d_t) and flow (f_t) variables as functions of exogenous expected future variables x_t, as follows (King and Watson, Sec. 4):

$$E_t d_{t+1} = W d_t + E_t \left[\Psi_d(F)\, x_t\right]; \tag{31}$$

$$f_t = -K d_t - E_t \left[\Psi_f(F)\, x_t\right]; \tag{32}$$

with $\Psi_d(F)$ and $\Psi_f(F)$ matrix polynomials in the forward operator (i.e, $F x_t = x_{t+1}$). Dynamic variables in d_t can be further separated into non-predetermined and predetermined variables labelled λ_t and k_t, respectively, so that $d_t = [\lambda_t\, k_t]'$.
King and Watson use the following decomposition of the matrix W in (31):

$$V_u W = \mu V_u, \tag{33}$$

where μ is a lower-triangular matrix with the unstable eigenvalues of W on its diagonal. Next, they define $u_t \equiv V_u d_t$, or since $d_t = [\lambda_t\ k_t]'$,

$$u_t = V_{u\lambda}\lambda_t + V_{uk} k_t. \tag{34}$$

Finally, they apply V_u to (31) and use the definition of u_t and (31) to obtain:

$$E_t u_{t+1} = \mu u_t + V_u \Psi_d(F) E_t(x_t). \tag{35}$$

Meanwhile, the exogenous variables x_t evolve as

$$x_t = \Theta \xi_t, \tag{36}$$

$$\xi_t = \rho \xi_{t-1} + \theta \eta_t,\ \eta_t \sim (0, Q); \tag{37}$$

where η_t is a martingale difference sequence.

B.1 Solution when shocks in ξ_t are observed

For reference, we first illustrate how to solve the model when all of the shocks ξ_t are observed. The system (36)–(37) is used to evaluate expressions that depends on expected future values of x_t. Notably, the last part of expression (35) becomes

$$\begin{aligned} V_u \Psi_d(F) E_t(x_t) &= V_u \Psi_{d,0} x_t + V_u \Psi_{d,1} E_t(x_{t+1}) + V_u \Psi_{d,2} E_t(x_{t+2}) + \ldots \\ &= V_u \left[\Psi_{d,0}\Theta + \Psi_{d,1}\Theta\rho + \Psi_{d,2}\Theta\rho^2 + \ldots\right] \xi_t \\ &\equiv \varphi_u \quad \xi_t; \end{aligned} \tag{38}$$

while the last part of (32) becomes

$$\begin{aligned}\Psi_f(F)E_t(x_t) &= \Psi_{f,0}x_t + \Psi_{f,1}E_t(x_{t+1}) + \Psi_{f,2}E_t(x_{t+2}) + \dots \\ &= \left[\Psi_{f,0}\Theta + \Psi_{f,1}\Theta\rho + \Psi_{f,2}\Theta\rho^2 + \dots\right]\xi_t \\ &\equiv \varphi_f \quad \xi_t. \end{aligned} \tag{39}$$

Eigenvalues of μ are unstable so (35) can be solved forward, equation-by-equation and yield

$$u_t = \nu\xi_t, \tag{40}$$

where ν is a function of the coefficients in φ_u, in μ, and in ρ. Once u_t is solved as a function of ξ_t, the remainder is straightforward: using (34) allows us to compute λ_t as a function of the predetermined variables k_t and the exogenous shocks ξ_t:

$$\lambda_t = -V_{u\lambda}^{-1}V_{uk}k_t + V_{u\lambda}^{-1}\nu\xi_t, \tag{41}$$

and the stable part of (31) allows us to compute the dynamic evolution of predetermined variables k_t. Finally, (32) is used to solve for f_t, and the complete solution reads as

$$\begin{bmatrix} f_t \\ \lambda_t \\ k_t \\ x_t \end{bmatrix} = \begin{bmatrix} \Pi_{fk} & \Pi_{f\xi} \\ \Pi_{\lambda k} & \Pi_{\lambda\xi} \\ I & 0 \\ 0 & \Theta \end{bmatrix} \begin{bmatrix} k_t \\ \xi_t \end{bmatrix}$$

$$\begin{bmatrix} k_{t+1} \\ \xi_{t+1} \end{bmatrix} = \begin{bmatrix} M_{kk} & M_{k\xi} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} k_t \\ \xi_t \end{bmatrix} + \begin{bmatrix} 0 \\ \theta \end{bmatrix} \eta_{t+1}.$$

B.2 Solution when x_t is observed but not ξ_t

Consider now that only the composite shock x_t in (36) is observable and that its decomposition into components of ξ_t is not. Knowledge about (36)–(37) can still be used to infer probable values for ξ_t through the application of the Kalman filter on the observed values of x_t. In turn, these inferences can be used to compute $E_t[x_{t+h}]$ for any h. To this end, denote the best estimate of ξ_t based on information up to time t as $\hat{\xi}_t|_t$ and the best linear forecast for ξ_{t+1} as $\hat{\xi}_{t+1}|_t$. Further, $P_{t+1}|_t$ is the mean-squared error of that forecast. The following recursions obtain for these quantities:

$$\begin{aligned}
\widehat{\xi}_t|_t &= \widehat{\xi}_t|_{t-1} + K_t\left(x_t - \Theta\widehat{\xi}_t|_{t-1}\right); &(42)\\
\widehat{\xi}_{t+1}|_t = \rho\widehat{\xi}_t|_t &= \rho\widehat{\xi}_t|_{t-1} + \rho K_t\left(x_t - \Theta\widehat{\xi}_t|_{t-1}\right); &(43)\\
K_t &= P_t|_{t-1}Q'\left(QP_t|_{t-1}Q'\right)^{-1}; &(44)\\
P_{t+1}|_t &= (\rho - K_t\Theta)P_t|_{t-1}(\rho' - \Theta' K_t') + Q; &(45)
\end{aligned}$$

Before proceeding, notice that (42) can be rewritten

$$\begin{aligned}
\widehat{\xi}_t|_t &= K_t\Theta\widehat{\xi}_t + \left[I - K_t\Theta\right]\widehat{\xi}_t|_{t-1};\\
&= A\widehat{\xi}_t + B\widehat{\xi}_t|_{t-1} &(46)
\end{aligned}$$

We can now use (42)–(45) in computations involving expectations of future values of exogenous shocks above: (38) and (39) become

$$\begin{aligned}
V_u\Psi_d(F)E_t(x_t) &= V_u\Psi_{d,0}x_t + V_u\Psi_{d,1}E_t(x_{t+1}) + V_u\Psi_{d,2}E_t(x_{t+2}) + \ldots\\
&= V_u\Psi_{d,0}\Theta\xi_t + V_u\Psi_{d,1}\Theta\widehat{\xi}_{t+1}|_t + V_u\Psi_{d,2}\Theta\widehat{\xi}_{t+2}|_t + \ldots\\
&= V_u\Psi_{d,0}\Theta\xi_t + V_u\Psi_{d,1}\Theta\rho\widehat{\xi}_t|_t + V_u\Psi_{d,2}\Theta\rho^2\widehat{\xi}_t|_t + \ldots\\
&\equiv \varphi_{d,\xi}\xi_t + \varphi_{d,\hat{\xi}}\widehat{\xi}_t|_t; &(47)
\end{aligned}$$

$$\begin{aligned}
\Psi_f(F)E_t(x_t) &= \Psi_{f,0}x_t + \Psi_{f,1}E_t(x_{t+1}) + \Psi_{f,2}E_t(x_{t+2}) + \ldots\\
&= \Psi_{f,0}\Theta\xi_t + \Psi_{f,1}\Theta\widehat{\xi}_{t+1}|_t + \Psi_{f,2}\Theta\widehat{\xi}_{t+2}|_t + \ldots\\
&= \Psi_{f,0}\Theta\xi_t + \Psi_{f,1}\Theta\rho\widehat{\xi}_t|_t + \Psi_{f,2}\Theta\rho^2\widehat{\xi}_t|_t + \ldots\\
&\equiv \varphi_{f,\xi}\xi_t + \varphi_{f,\hat{\xi}}\widehat{\xi}_t|_t. &(48)
\end{aligned}$$

Again, use (46) and (47) to help solve (35) forward, equation-by-equation to get

$$u_t = \nu_\xi\xi_t + \nu_{\hat{\xi}}\widehat{\xi}_t|_{t-1}, \tag{49}$$

The solution for u_t allows us to express λ_t as a function of predetermined variables k_t and exogenous shocks ξ_t, as follows:

$$\lambda_t = -V_{u\lambda}^{-1}V_{uk}k_t + V_{u\lambda}^{-1}\nu_\xi\xi_t + V_{u\lambda}^{-1}\nu_{\hat{\xi}}\widehat{\xi}_t|_{t-1}. \tag{50}$$

Further the "stable" part of (31) is again used to find a dynamic solution for the evolution of the predetermined variables k_t; and (32) can then be used to find the solution for f_t. In the end, the complete solution has the following form:

$$\begin{bmatrix} f_t \\ \lambda_t \\ k_t \\ x_t \end{bmatrix} = \begin{bmatrix} \Pi_{fk} & \Pi_{f\xi} & \Pi_{f\widehat{\xi}} \\ \Pi_{\lambda k} & \Pi_{\lambda\xi} & \Pi_{\lambda\widehat{\xi}} \\ I & 0 & 0 \\ 0 & \Theta & 0 \end{bmatrix} \begin{bmatrix} k_t \\ \xi_t \\ \widehat{\xi}_t \end{bmatrix}$$

$$\begin{bmatrix} k_{t+1} \\ \xi_{t+1} \\ \widehat{\xi}_{t+1} \end{bmatrix} = \begin{bmatrix} M_{kk} & M_{k\xi} & M_{k\widehat{\xi}} \\ 0 & \rho & 0 \\ 0 & \rho A & \rho B \end{bmatrix} \begin{bmatrix} k_t \\ \xi_t \\ \widehat{\xi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \theta \\ 0 \end{bmatrix} \eta_{t+1}.$$

Note that, in effect, we have added $\widehat{\xi}_t|_t$ as an additional vector of dynamic variables to the original solution and taken into account its impact on the other variables through the matrices $\Pi_{f\widehat{\xi}}$, $\Pi_{\lambda\widehat{\xi}}$ and $M_{k\widehat{\xi}}$. Further details about the solution method can be obtained from the authors.

Figure 1: Oil Spot and Futures Prices

Figure 2: Estimates of Oil-Price Trend and Persistence of Transitory Shocks

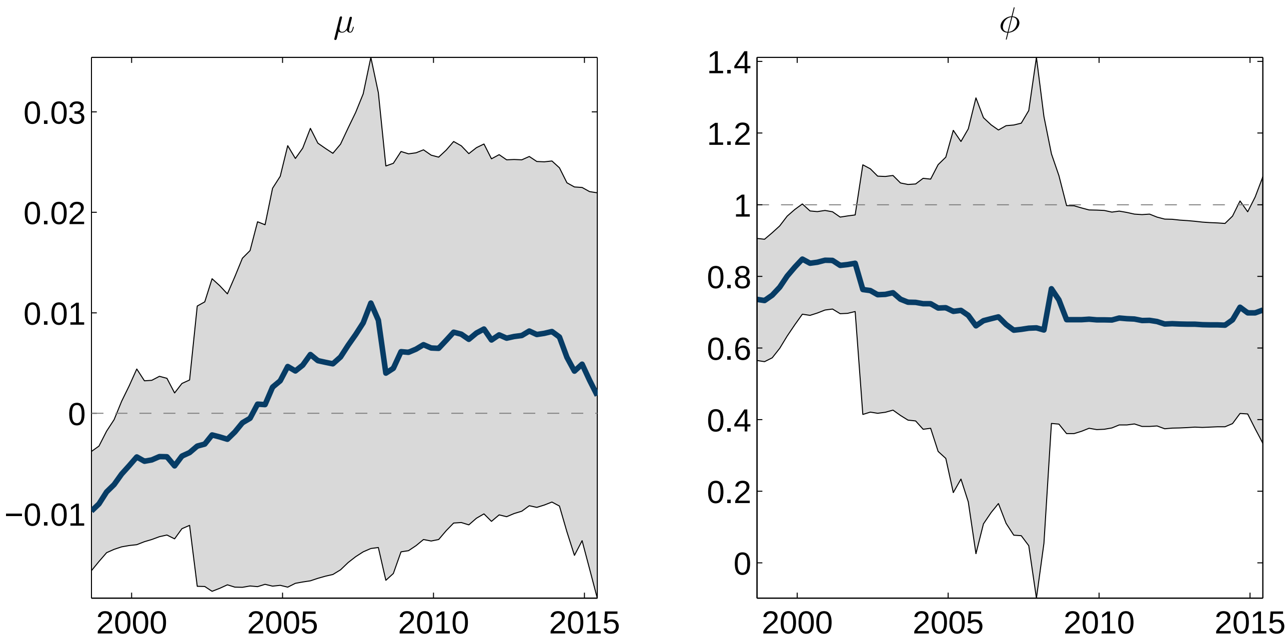


Figure 3: Estimated Standard Deviations of Permanent and Transitory Shocks

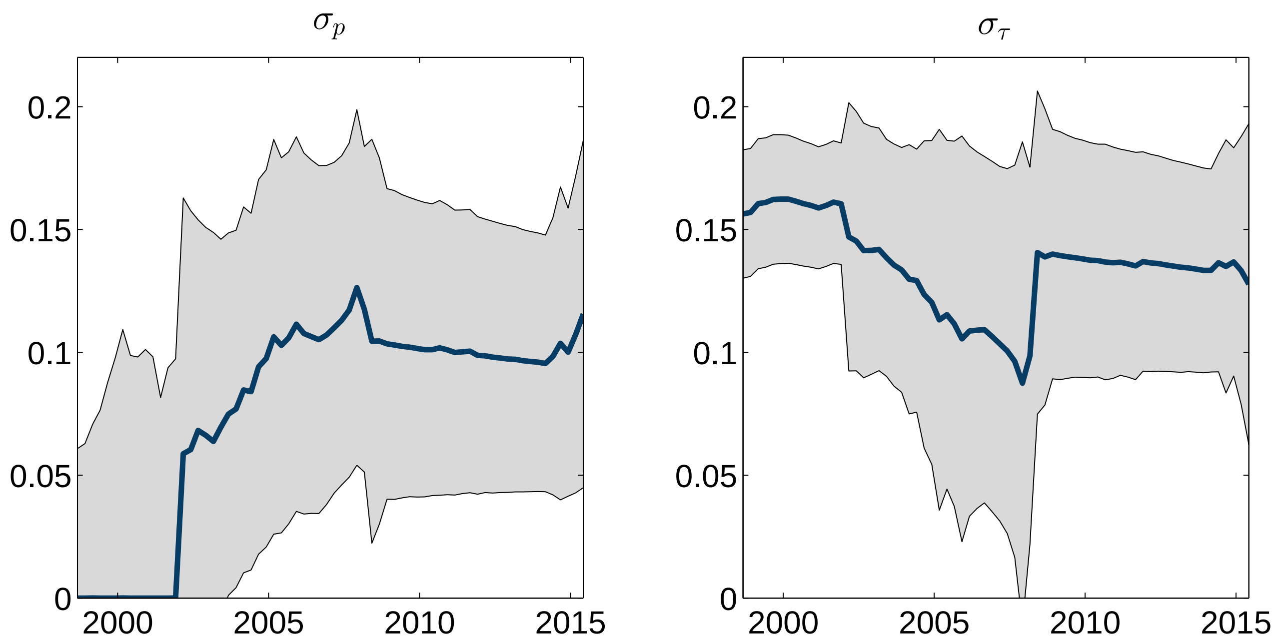


Figure 4: The Relative Importance of Permanent Shocks

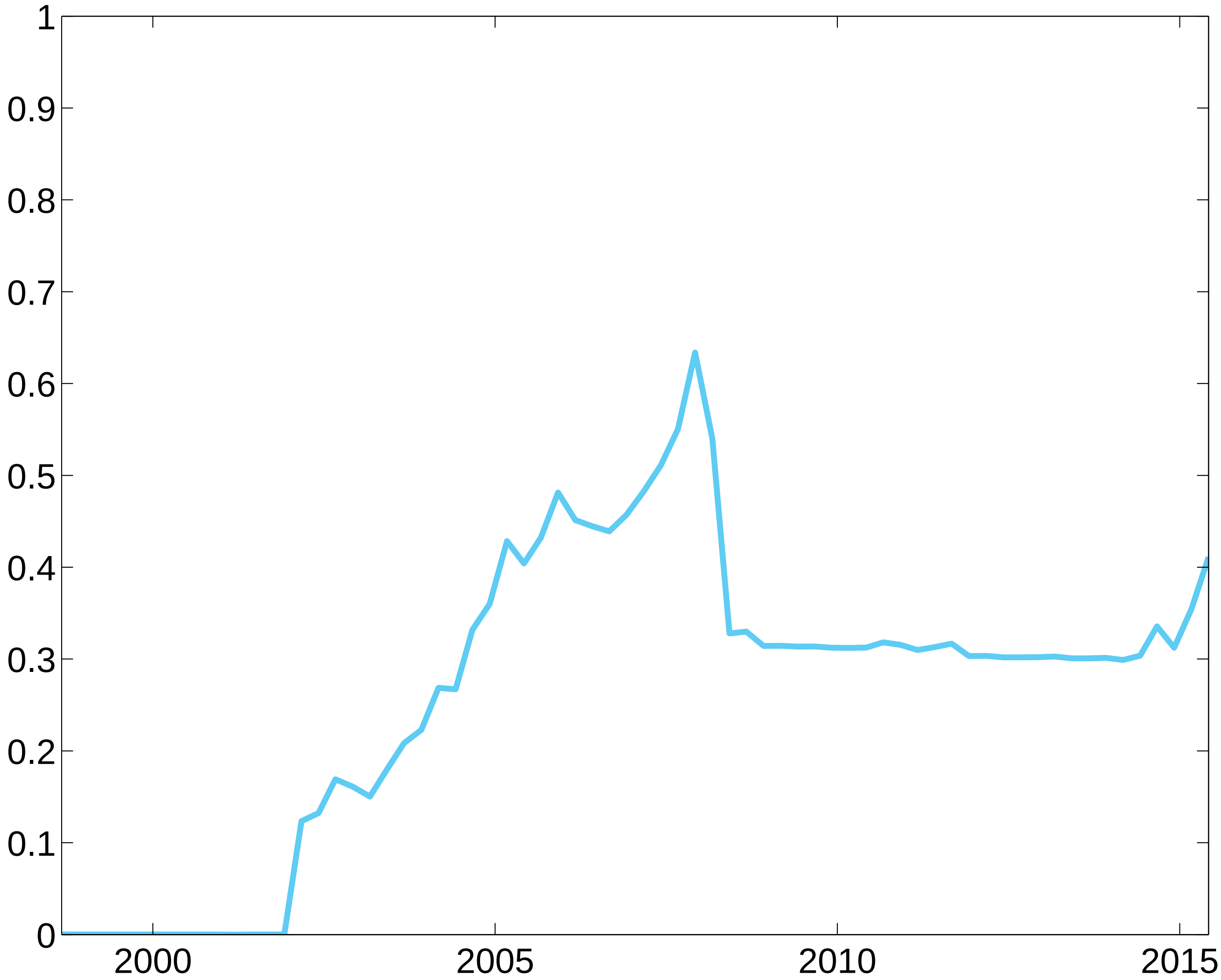


Figure 5: Predicting Futures Prices

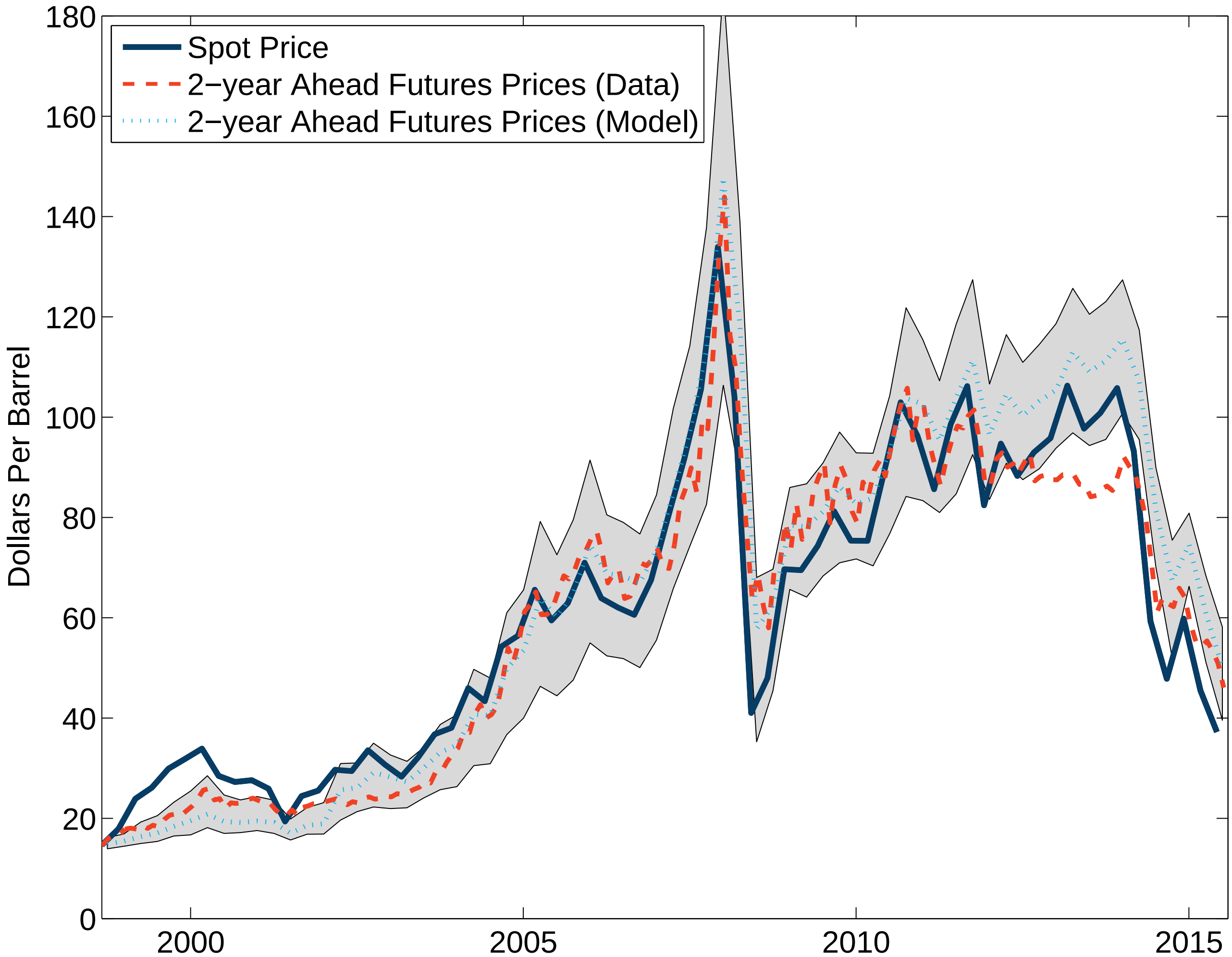


Figure 6: Assessment of Model Performance

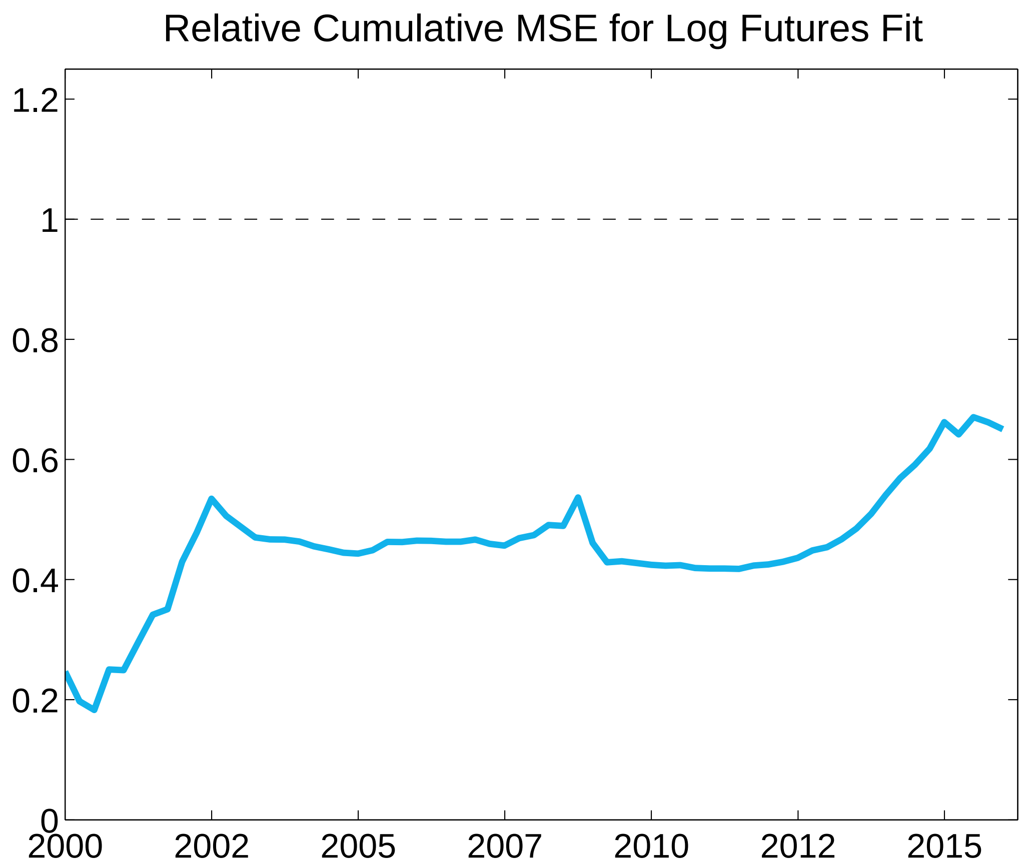


Figure 7: Impact of a Permanent Oil-Demand Shock: Complete Information

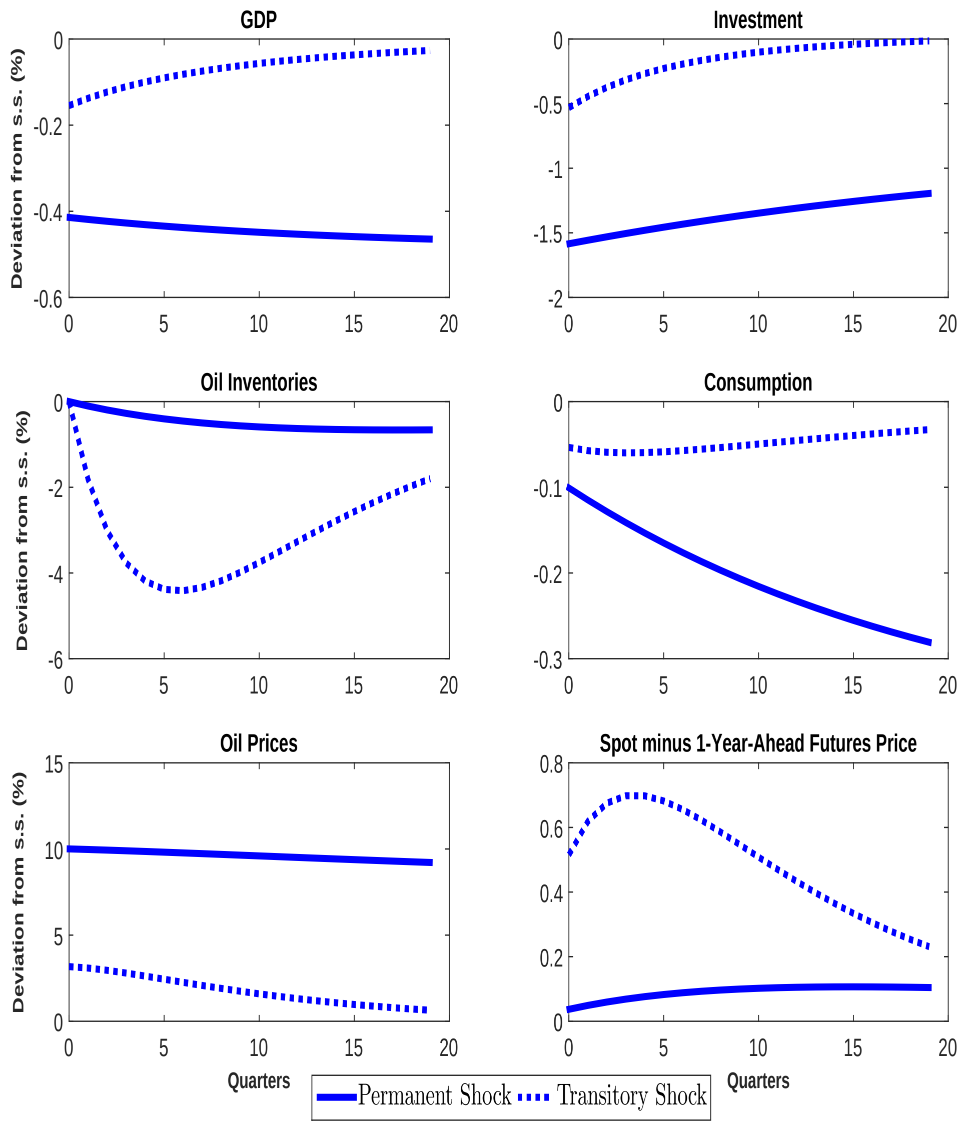


Figure 8: Impact of a Permanent Oil- Demand Shock: 2003 Scenario

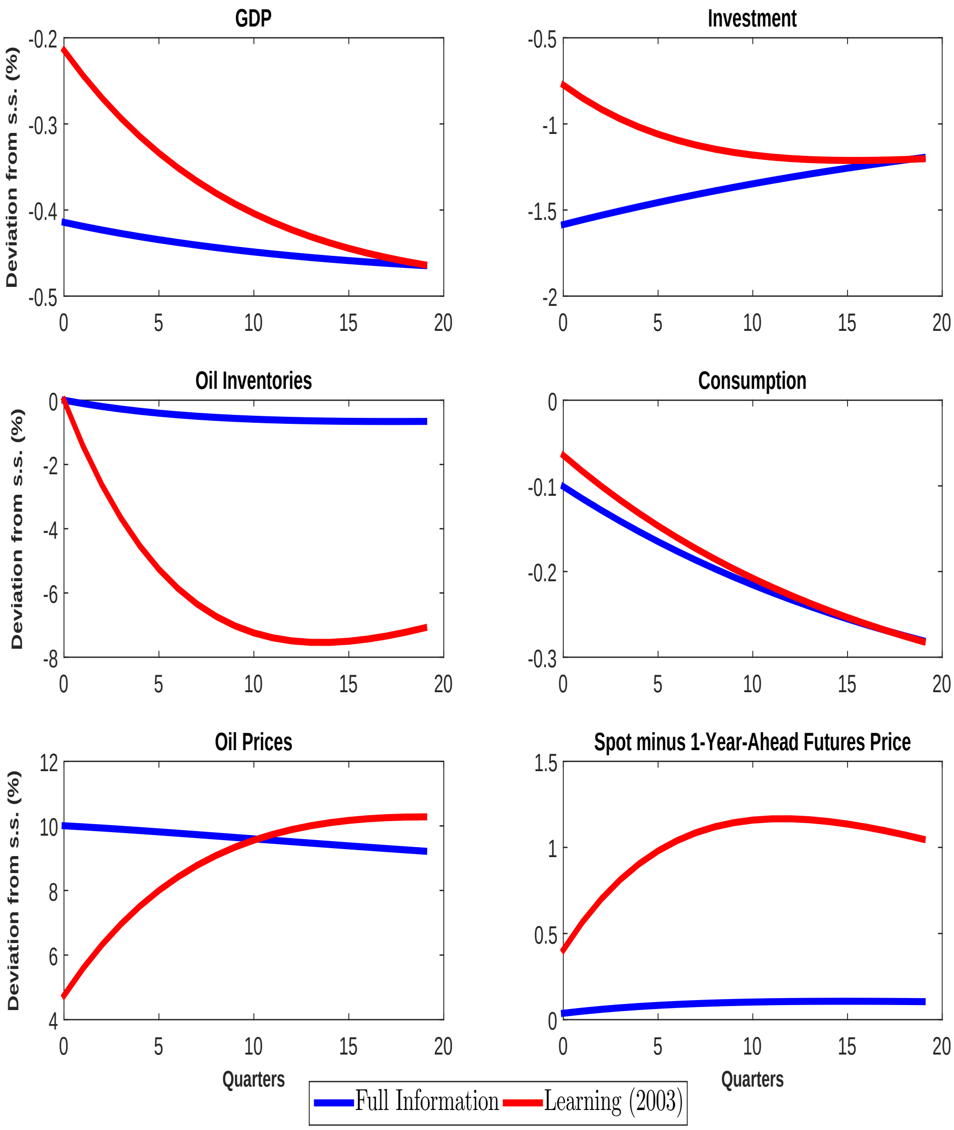


Figure 9: Impact of a Permanent Oil-Demand Shock: 2007 Scenario

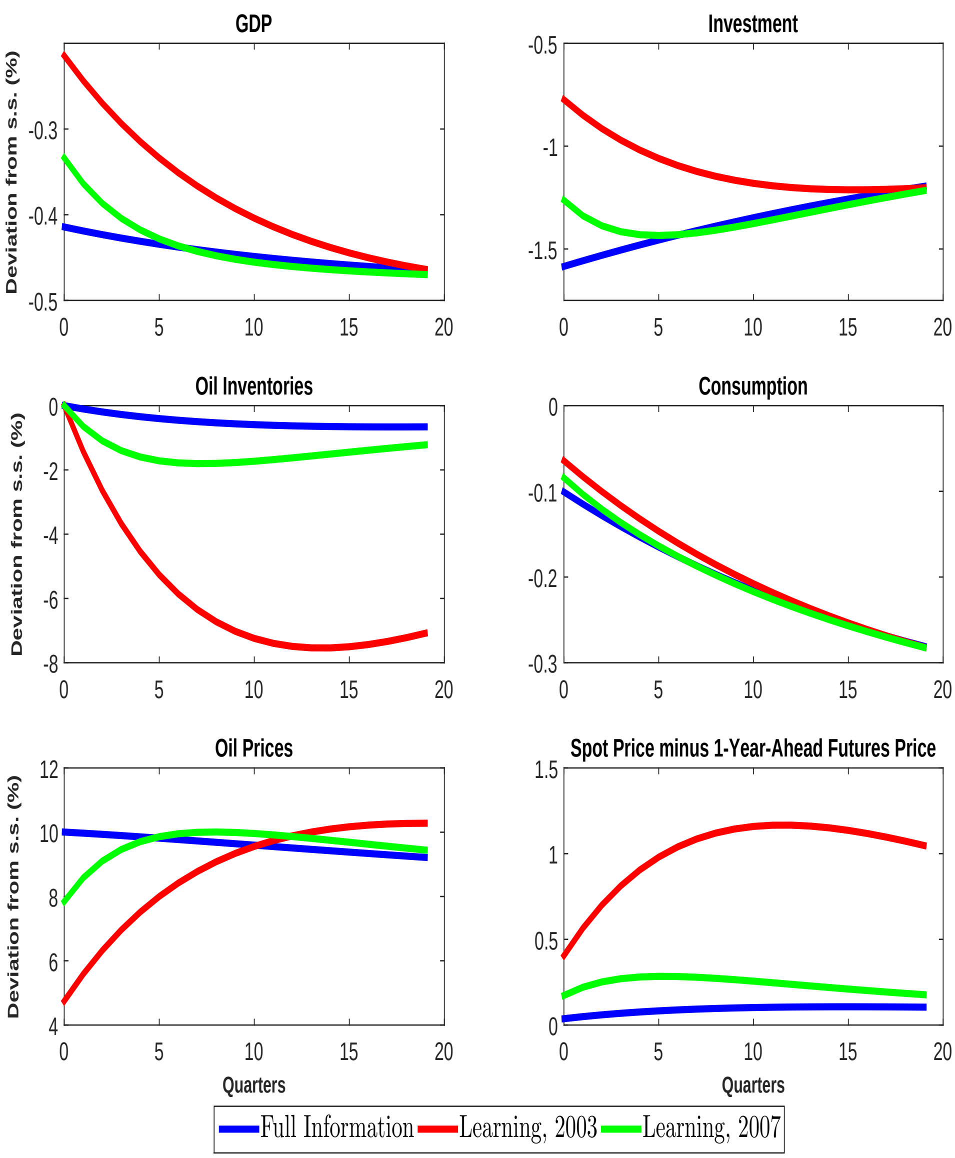


Figure 10: Interaction of Learning with Storage

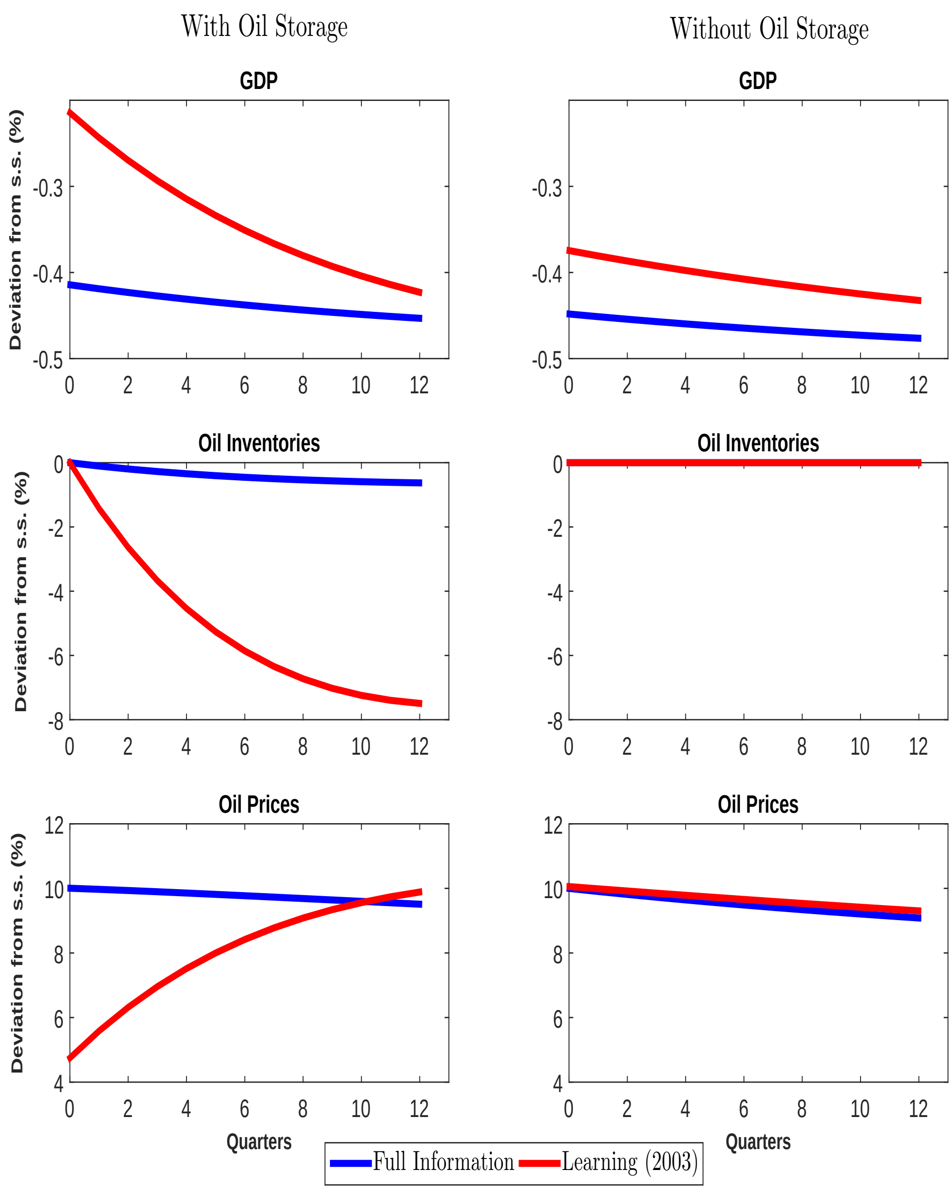


Figure A.1: Predicted Futures Prices: Constant-Gain Learning

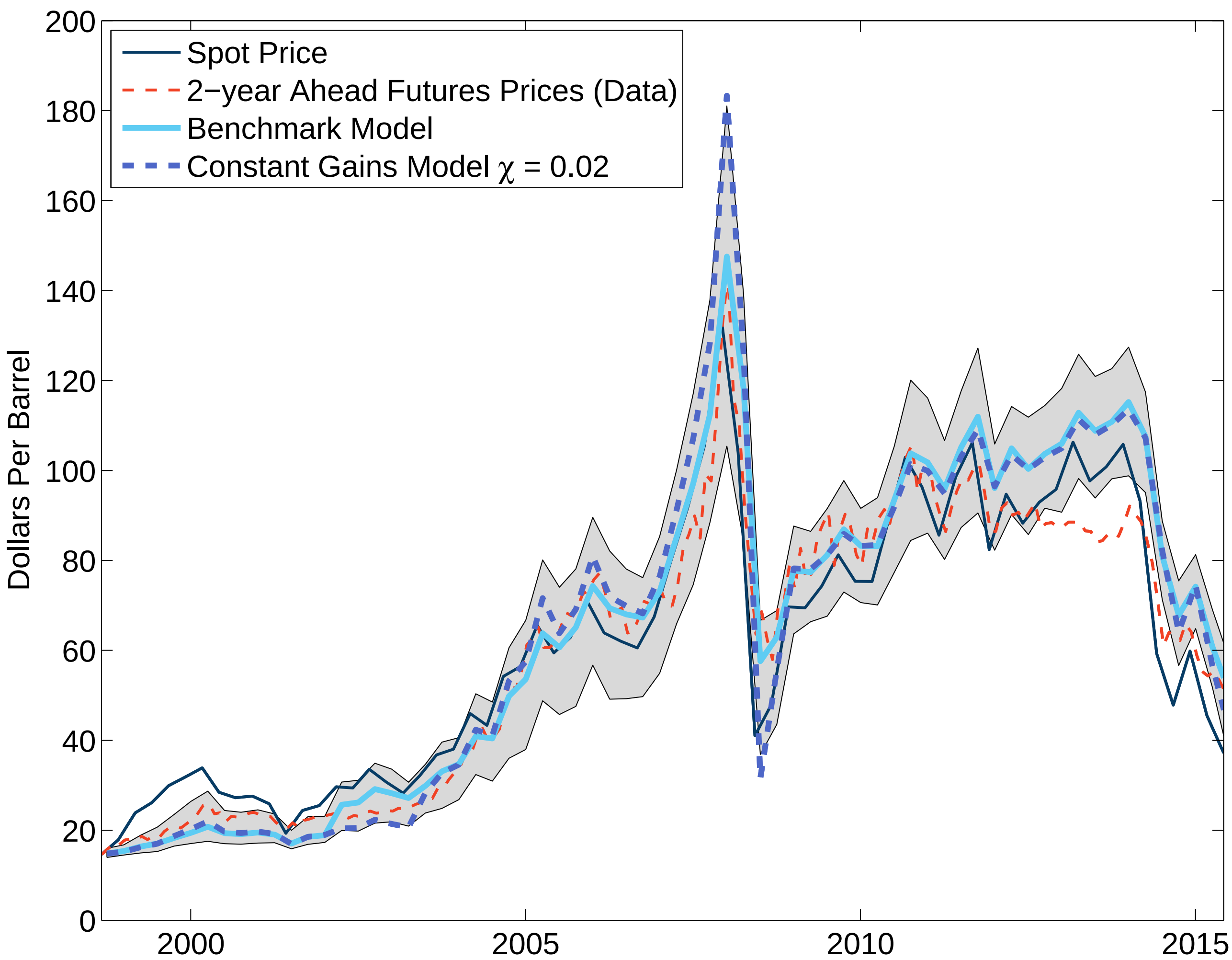
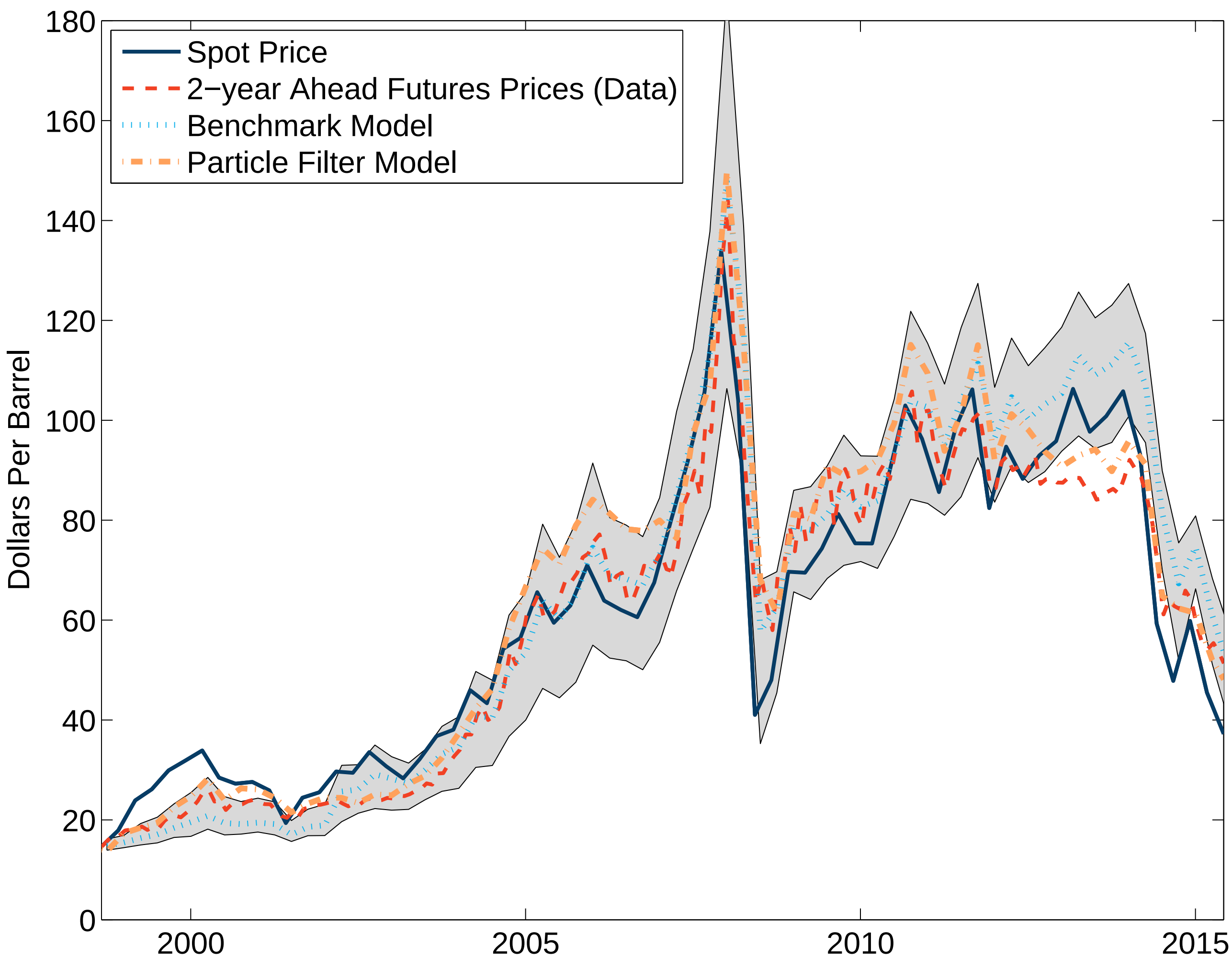


Figure A.2: Predicted Futures Prices: Particle Filter



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