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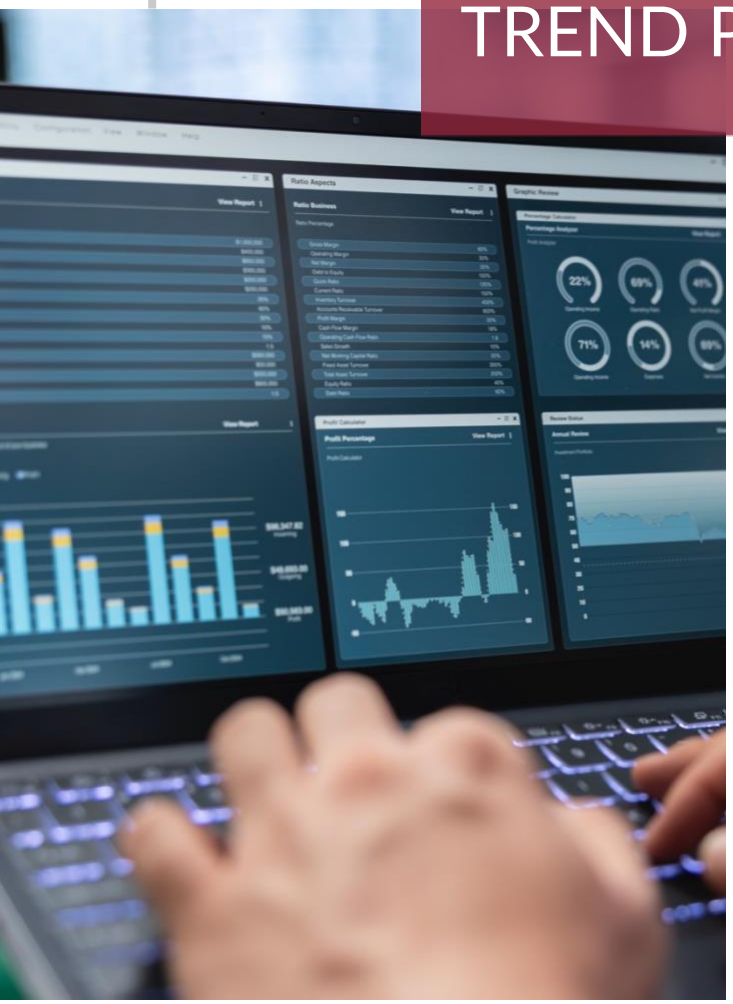
DATA-DRIVEN LEARNING ABOUT TREND PRODUCTIVITY GROWTH



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Data-Driven Learning About Trend Productivity Growth*

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Abstract/Résumé

We investigate the causes of changing productivity growth trend perceptions using a novel state-space framework for statistically efficient estimation of growth trends in the presence of data revision. Uncertainty around contemporary US productivity growth trends has been exacerbated by data revisions that typically occur several years after the initial data release, as well as by publication lags. However, the largest source of revisions in perceived trends comes from future realizations of productivity growth. This underlines the importance of estimation uncertainty in estimates of trend productivity growth.

Nous étudions les causes des changements dans les perceptions relatives aux tendances de croissance de la productivité en utilisant un cadre d'espace d'état novateur, permettant une estimation statistiquement efficiente des tendances de croissance en présence de révisions des données. L'incertitude entourant les tendances contemporaines de la croissance de la productivité aux États-Unis a été amplifiée par les révisions des données, qui interviennent généralement plusieurs années après leur première publication, ainsi que par les délais de diffusion. Toutefois, la principale source de révisions des tendances perçues provient des réalisations futures de la croissance de la productivité. Cela souligne l'importance de l'incertitude d'estimation dans les évaluations de la tendance de la productivité.

Keywords/Mots-clés: productivity, real-time data, news, trend-cycle decomposition / productivité, données en temps réel, nouvelles économiques, décomposition tendance-cycle

JEL Codes/Codes JEL: C32, C51, E6, E24, O47

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Productivity isn't everything, but in the long run, it's almost everything.

– Paul Krugman (1997, p11)

1 Introduction

Perceptions of productivity growth play a key role in many macroeconomic decisions, including households' intertemporal consumption smoothing, firms' investment, the sustainability of government fiscal policy, monetary policy and the solvency of public pension plans, among others.[1] In addition to the interest in productivity growth, the level of trend productivity is commonly used in the calculation of potential output, which in turn is used in guiding monetary and fiscal policy decisions.[2] The past several decades have seen a nearly continuous stream of applied studies estimating recent trend rates of productivity growth, as well as testing for evidence of changes in those rates.[3] This has accompanied important refinements in the measurement of productivity and its sources of growth.[4]

One feature of this applied work that is often overlooked is the extent to which estimates of trend productivity growth may be revised over time. As a simple example, consider the estimates of U.S. Total Factor Productivity (TFP) growth published by the Congressional Budget Office (CBO).[5] Figure 1 shows how CBO estimates of productivity growth trends for specific calendar years (2000, 2002, 2008 and 2015) have been revised over time; the horizontal axis shows when the estimates for each year were published.[6] The red line shows

[1]For example, see Ghirono et al. (2008)) on consumption smoothing, (De Long (1992)) on private investment, Duarte Lledo et al. (2018) and European Commission (2017) on the sustainability of fiscal policy, (Laubach and Williams (2003) on monetary policy and Neumark 2006) on the solvency of public pension plans.

[2]See European Commission (2017) or Carstensen et al. (2024) for examples.

[3]Kahn and Rich (2007) in particular noted some of the challenges inherent in timely detection of persistent changes in growth rates.

[4]For overviews, see Coelli et al. (2005) (particularly Chapters 3, 4, 6 and 9), Syverson (2011), Grifell-Tatjé et al. (2018) or Zelenyuk (2023)).

[5]The terms Multi-Factor Productivity (MFP) and Total Factor Productivity (TFP) have both been used to refer to productivity in a two-factor model of aggregate production using inputs of labor and capital services. We use the terms interchangeably.

[6]The data for the Figure are from the CBO's Budget and Economic Data page, specifically the *Historical Data and Economic Projections* and the *Potential GDP and Underlying Inputs* sections. We used all available releases for TFP in the Non-Farm Business sector at Potential, which covered the period from August 2002 through January 2025. Although those sources also include forecasts and nowcasts, Figure 1 contains only "historical" estimates. For example, the first estimate shown for TFP growth in 2008 is that from the January 2009 estimate (which was before the first BLS release for TFP in that year.)

that growth in 2000 is estimated to be 1.3% in early 2002, this is revised up to 1.5% by 2004, followed by a series of downward revisions to 1.2% by 2010. The minor revisions continued until Jan 2017, when the estimate of growth in 2000 is revised from 1.5 to 2.1% (more than 50% higher than the lowest previous estimate.) Other years have shown greater revisions: the blue line shows that estimates of growth in 2002, while initially slightly lower than those of 2000, were soon revised upwards and by 2010 were thought to be double those of 2000. This was abruptly reversed in January 2017 when estimates for 2002 were revised to below those of 2000. Results for 2008 and 2015 show that more recent estimates may also undergo economically important revisions many years after the fact.[7] The point here is not to single out the CBO's estimates for criticism, but to show the extent to which the perception of *historical* productivity growth may continue to evolve through time.

[Figure 1 about here.]

The fact that perceptions of productivity growth evolve over time has several important implications. First, it raises the question of what drives these changes. Second, since *expected* rates of productivity growth may differ substantially from historical estimates, care must be taken when empirical work requires a proxy for expectations of productivity growth. Third, decision makers may care both about point estimates or forecasts of productivity growth as well as the uncertainty around them.

In this paper, we offer an original linear modeling framework that provides time-varying estimates of productivity growth trends and explains how such estimates may evolve as time passes. It also provides econometrically efficient forecasts and historical estimates of growth trends as well as estimates of their varying estimation uncertainty. It does so by allowing for two key sources of changes in perception: data revision and hindsight.

As Jacobs and van Norden (2016) pointed out, productivity revisions are large relative to those in series used to calculate productivity (such as output or hours worked). This stems in part from the fact that productivity is measured as a residual: it is the variation

[7]The latter results also hint at a change in policy at the CBO. Potential TFP growth was assumed to be constant over many calendar years, although the constant rate would be revised slightly from one release to the next. As of January 2025, the CBO's estimated growth rate of TFP at potential is 1.0% for every year from 2009 to 2024.

in output that is not explained by variations in inputs.[8] The modeling of data revisions has made important advances in recent years, which we draw upon below.[9]

To illustrate the extent of data revision, Figure 2 shows growth rates of one of the two productivity growth measures we analyze in this paper: U.S. MultiFactor Productivity (MFP). As in Figure 1, each series shows the estimate for a particular year and traces how the estimate for that year varies over time. Substantial variations are visible in the graphs for some years, while those for others are more modest. [10]

[Figure 2 about here.]

It is also interesting to compare the CBO estimates in Figure 1 with the BLS estimates in Figure 2. The former estimate MFP growth at Potential Output whereas the latter are official estimates of actual MFP growth. Comparing the estimates for 2000 and 2002 in Figure 2, we see that MFP growth in 2002 has been consistently reported to be above that in 2000, and that the difference between them has been relatively constant. This suggests that the highly variable difference between the CBO estimates for these years shown in 1 are due to changing estimates of potential output, including the major change in 2017 (15 years after the event.) Revisions to MFP growth estimates for 2008 and 2015 have been relatively larger but are not directly reflected in the CBO's estimates of MFP growth at Potential for those years. The BLS sharply revised their growth estimate for 2008 downwards in 2011, holding it roughly constant for the next decade; the CBO initially made little change to their estimate, but gradually lowered it over the following eight years. The BLS estimates for 2015 are consistently much higher than those for 2008 and were initially revised upwards substantially, but the CBO estimates for the two years were the same from early 2017 through the end of 2023. Thereafter, as the BLS revised growth in 2008 upwards and 2015 downwards, the CBO increased its estimate of 2015 over that of 2008.

[8] The correlation between revisions in the input and output series also plays an essential role. See Jacobs and van Norden (2016, Section 2).

[9] A key insight from this literature is that revisions are typically "noisy" and somewhat predictable. One implication is that there are potential gains from taking weighted averages of individual "noisy" releases to produce more efficient estimates. This contrasts with the conventional practice of using only the most recently published estimates, which are assumed to be the best available.

[10] Revision analyses of U.S. productivity measures are provided by Aruoba (2008), Jacobs and van Norden (2016), Bognanni and Zito (2016), Asher et al. (2021, 2022) and Glaser et al. (2024). Below we provide further analysis of the revisions shown here, particularly in Table 2.

To understand what else could cause changes in the CBO's perceptions of productivity growth, we turn to consider the role of hindsight in revising estimates. To illustrate the role of hindsight, Figure 3 shows estimates of the trend growth rate of MFP. [11] Because the trend and cycle components are not directly observed, their estimates (as in any linear state-space model) are based on weighted averages of the available observations. However, the weights vary depending on the point from which the estimates are made. In Figure 3, the red line shows estimates made at the end of the sample period (i.e. using the full sample) of the trend for each point in time. The blue line shows estimates of the trend in year t based only on the sample up to and including year t. The former are referred to as "Smoothed" estimates while the latter are known as "Filtered" estimates. The difference between the two sets of estimates shows the extent to which observations *after* year t (i.e. "hindsight") caused us to revise our estimates for year t.[12] This source of revision has been studied in other macroeconomic contexts. For example, Orphanides and van Norden (2002) examine revisions in estimates of U.S. output gaps and find that the difference between filtered and smoothed estimates is the dominant source of revisions.[13] Cyclically-adjusted or Capacity-Utilization-adjusted series (Fernald (2014)) may also undergo additional revision as assessments of potential output or capacity utilization are adjusted with the benefit of hindsight (Kurmann and Sims (2021)).

[Figure 3 about here.]

In this study, we show how taking into account data revision and filtering methods (as well as publication lags and parameter instability) affects the estimation of trend productivity growth, leading to changing perceptions of historical multi-factor and labor productivity growth trends. We provide an original framework for efficient trend-cycle decompositions

[11] Specifically, the model assumes that (1) productivity growth is the sum of a trend and cycle, (2) the cyclic component follows a stationary AR(2) process, (3) the trend follows a random walk, and (4) shocks to the trend and the cycle are uncorrelated and i.i.d. Gaussian. The model is fit to the "final" release of MFP (described below), and therefore is unaffected by data revision.

[12] The figure ignores the effects of publication lags. As we explain below, these increase the importance of ex post revisions by causing decision makers to rely on forecasts or "nowcasts." Publication lags for MFP have been variable and particularly long.

[13] The output gap is similar to productivity growth in the sense that it too is constructed as a residual (the difference between observed and potential or trend output). Their study differs from ours in several important respects however; it examines the cyclical component rather than the trend, the level of output rather than productivity growth, and they do not model data revision.

using advances in state-space modeling that allow for data revisions that may be a combination of both news and noise.[14] Standard tools for linear state-space models can then produce updated estimates and forecasts of trend growth, as well as calculate their confidence intervals, while allowing for missing observations due to publication lags or other sources.

Using original-vintage data on both multi-factor and labor productivity, our model prefers to estimate trends using a weighted average of many releases that puts relatively little weight on initial estimates compared to revisions released one or more years later. This results in uncertainty around the estimate of trend growth that dissipates substantially as years pass. In addition to showing how uncertainty and the optimal revision weighting change as time passes, we also document the extent to which "real-time" trend estimates tend to lag retrospective estimates in detecting changes in trends.

The applied productivity literature typically employs the most recent vintage to understand the sources of changes in trend productivity.[15] Models analogous to ours which only use the most fully-revised release (Figure 3) produce trend productivity growth estimates that are much smoother than most commonly accepted estimates. Compared to several vintages of trend productivity estimates from the Congressional Budget Office (CBO), our real-time estimates capture similar broad trends but with periods of significant deviations.

In the next section we detail the data used for our analysis, which are among the most widely studied U.S. productivity series. We also show the extent to which the series are revised over time, and distinguish regular and "benchmark" revisions. Thereafter we lay out the statistical model we use to estimate trend productivity growth for each series. The model is a conventional linear state space model with uncorrelated Gaussian errors that may be estimated and manipulated with conventional tools. Readers less interested in econometric details may pass lightly over this section. We then look in detail at the

[14] The concepts of "news" and "noise" in the data revision literature differ from the ones adopted in the "news"-driven business cycle literature. As will become clear in section 3, in the data revision literature "news " are "noise" are measurement errors, whereas in the news-driven business cycle literature, a "news" shock refers to signals that economic agents receive about future productivity growth (e.g. Beaudry and Portier (2004) and Fujiwara et al. (2011)). For a reconciliation of these concepts see Goodwin and Tian (2017).

[15] For example, Feenstra et al. (2015) present a new generation of the Penn World Table with new estimates of total factor productivity figures across countries. Pinkovskiy and Sala-i-Martin (2016) comment, however, that "newer need not be better".

historical estimates of trend growth that the model produces. In addition to comparing them to CBO estimates, we compare them to the degree of uncertainty surrounding the estimates and discuss the weights that they place on various releases and revisions of the MFP and OPHA series. The subsequent section contrasts these historical estimates with those made under conditions resembling those faced by decision makers, who effectively face a forecasting problem. We show how this affects the weights assigned to various releases and revisions, and how the estimates of trend growth and their uncertainty evolve as time passes and more information arrives. Finally, we consider the role that instability in the estimated parameters of our model may influence our results.

2 Data

Our analysis uses 32 vintages of the MultiFactor Productivity (MFP) series for the U.S. Non-Farm Business Sector covering the years 1949 to 2021 and published between February 1995 and November 2022.[16] In addition, 408 vintages from May 1968 to June 2023 of Output per Hour (OPH) series are used, covering the quarters from 1947Q2 to 2023Q1. Both series are from ALFRED; historical releases for MFP were provided by the Bureau of Labor Statistics.[17] We use log differences of the level of both series to mitigate effects of benchmark revisions (see Croushore 2011).

We initially compared revisions in annual estimates of MFP to those in quarterly estimates of OPH and found strikingly dissimilar results.[18] To eliminate artifacts due to differences in their reporting frequencies we instead compare MFP to annualized OPH data (OPHA), which is calculated as the annual averages of OPH.[19] Figure 4 shows the MFP, OPH and OPHA series for the last vintage in our sample. The patterns in MFP and OPHA are similar, but the quarterly OPH series is much more volatile.

[16]The November 2022 release of preliminary estimates for 2022Q3 incorporated the 2022 update of the National Income and Product Accounts (NIPA) and included a new methodology for estimating hours worked. Accordingly, we ended our sample with that benchmark revision. For details on the methodological changes, see Eldridge et al. (2022).

[17]The authors thank Corby Garner at the BLS for help in accessing early vintages.

[18]Results for quarterly OPH are available on request.

[19]OPHA retains the quarterly releases of OPH, but the use of annual averages means that our last observation is for 2022.

[Figure 4 about here.]

While both series are subject to a fairly regular revision cycle (e.g. the publication of preliminary and then "revised" estimates, and perhaps annual revisions of seasonal factors) they are also subject to irregular "benchmark" revisions. These are due to methodological or definitional changes and typically revise several decades of previously published estimates. Table 1 displays the release dates for benchmark revisions.

Due to their irregular nature, we treat them differently from regularly occurring revisions. The value of the benchmark release for period t is defined as the value for period t in the first benchmark revision published for that period. If the initial release for period t coincides with a benchmark revision, the value of the next benchmark revision for period t is used. For example, in the case of MFP, the value of the benchmark release for 1996 corresponds to the year 2000 vintage, the value of the benchmark release for 1997 also corresponds to the year 2000 vintage, but the value of the benchmark release for 1998 corresponds to the year 2004 vintage. We also define the pre-benchmark release, which is simply the last release published before a benchmark release, as well as a "Final" release, which is simply the last release in our data set.[20]

[Table 1 about here.]

Table 2 shows descriptive statistics for our productivity growth measures and their revisions. Regarding the variation in the series, we see similar results using Std. Dev. and MA. Dev. The temporal decomposition of revisions shows that initial revisions (2nd - 1st) are relatively much more important for the MFP series than for the OPHA series. They also show that revisions may continue long after the initial release; variation more than 5 years after the initial release (Final - 5Yr) is roughly 60% or more of the size of the observed aggregate variation. Similarly, revisions after the initial benchmark revision (Final - BM) also appear to be substantial.

The extent to which these revisions simply reflect "noise" or may lead to initial misperceptions of longer-term productivity growth trends requires careful modeling, to which we turn in the next section.

[20]This usage of the term "Final" has become endemic in the data revision literature; see Orphanides and van Norden (2002) or Croushore (2011) for examples.

[Table 2 about here.]

3 Methodology

The model introduced here builds on work by Jacobs and van Norden (2011), Jacobs et al. (2022) and Goto et al. (2023) on modeling time series that are subject to periodic revision.[21] All use a linear state-space framework to model multiple series that are assumed to be estimates of the same (unobserved) "true" series $\tilde{y}_t$. The jth estimate of $\tilde{y}_t$, $y_{j,t}$, is assumed to be equal to the target series $\tilde{y}_t$ but for the presence of two kinds of measurement errors:

Noise errors $(\xi_{j,t})$ which are mean zero and independent of $\tilde{y}_t$.

News errors $(\nu_{j,t})$ which are mean zero and are independent of all information available at the time of their publication.

While noise errors follow classical assumptions for the behavior of measurement errors, news errors mirror the behavior of rational forecast errors. Among other things, this implies that $\text{cov}(\xi_{j,t}, \tilde{y}_t) = 0$, but $\text{cov}(\nu_{j,t}, \tilde{y}_t) > 0$.[22] The presence of noise errors also implies that some positive fraction of the variance of data revisions is predictable, while revisions due to news errors are unpredictable (by definition).

The framework that we present below further distinguishes between cycles (c_t) and trends (τ_t) so that

$$\tilde{y}_t \equiv c_t + \tau_t = y_{j,t} + \nu^c_{j,t} + \nu^\tau_{j,t} + \xi_{j,t},$$

[21] Other advances in estimation and forecasting with data subject to revision include Koenig et al. (2003), Garratt et al. (2008), Kishor and Koenig (2012), Cunningham et al. (2012), Aruoba et al. (2013, 2016), Jacobs, et al. (2022), Anesti et al. (2022), Goto et al. (2023), Koop et al. (2023), and Almuzara et al. (2024). Jacobs and van Norden (2011) introduce the basic state-space framework for news and noise revisions. Although they discuss how to incorporate trend-cycle decompositions in their framework, this is not examined in their application and our modeling of noise shocks is slightly different. Anderes et al. (2024) also consider the problem of trend-cycle decomposition when data are subject to revision, but focus on the estimation of the cyclical component (the output gap) rather than the trend. Jacobs et al. (2022) and Goto et al. (2023) consider how to reconcile estimates subject to revision, using bivariate models where two series provide alternative estimates of the same underlying quantity, such as US GDP (Jacobs et al. (2022)) or US employment (Goto et al. (2023)).

[22] This also implies that $\text{cov}(\nu_{j,t}, \nu_{i,t}) \geq 0$, and if $y_{j,t}$ is published after $y_{i,t}$, then $\text{var}(\nu_{j,t}) \leq \text{var}(\nu_{i,t})$.

for the jth release, so that news errors have distinct cycle and trend components, while noise errors are assumed to be independently distributed. Therefore a key challenge for decision makers will be to decide whether surprises in $y_{j,t}$ are due to measurement noise, news about cyclical variation in productivity, or news about productivity growth trends.

3.1 A State-Space Model

For simplicity, our model assumes that the (true, unobserved) cycle c_t follows a stationary AR(2) process while the trend is assumed to be a random walk. While we use five distinct data releases in our application, for compactness we specify the model below for the case where we have only two releases $y_{1,t}$ and $y_{2,t}$.

State vector

The state vector $\boldsymbol{\alpha}_t \equiv [c_t, c_{t-1}, \tau_t, (\boldsymbol{\nu}_t^c)', (\boldsymbol{\nu}_t^\tau)']'$, where c_t, c_{t-1}, τ_t are all scalars and $\boldsymbol{\nu}_t^c$ and $\boldsymbol{\nu}_t^\tau$ are 2×1 vectors. Note that the news errors, but not the noise errors, form part of the state vector.

Measurement Equation

$$\boldsymbol{Y}_t \equiv \begin{bmatrix} 1 & 0 & 1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 1 & 0 & -1 & 0 & -1 \end{bmatrix} \cdot \boldsymbol{\alpha}_t + \begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix}$$

where $\boldsymbol{Y}_t \equiv [y_{1,t}, y_{2,t}]'$ and $[\xi_{1,t}, \xi_{2,t}]' \sim i.i.d.\, N(\mathbf{0}, \boldsymbol{H})$ and $\boldsymbol{H}$ is a diagonal matrix with elements $[\sigma_1^\xi, \sigma_2^\xi]$. We see that noise errors appear exclusively as random errors in the measurement equation.

State Equation

$$\boldsymbol{\alpha}_t = \boldsymbol{T} \cdot \boldsymbol{\alpha}_{t-1} + \boldsymbol{R} \cdot \begin{bmatrix} \nu_{1,t}^c \\ \nu_{2,t}^c \\ \nu_{1,t}^\tau \\ \nu_{2,t}^\tau \end{bmatrix},$$

$$\text{where } \boldsymbol{T} \equiv \begin{bmatrix} \rho_1 & \rho_2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{R} \equiv \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{and } \begin{bmatrix} \nu^c_{1,t} \\ \nu^c_{2,t} \\ \nu^\tau_{1,t} \\ \nu^\tau_{2,t} \end{bmatrix} \sim i.i.d.\ N\left(\mathbf{0}, \begin{bmatrix} \sigma_1^{c,\nu} & 0 & 0 & 0 \\ 0 & \sigma_2^{c,\nu} & 0 & 0 \\ 0 & 0 & \sigma_1^{\tau,\nu} & 0 \\ 0 & 0 & 0 & \sigma_2^{\tau,\nu} \end{bmatrix} \right).$$

The first two rows of the $\boldsymbol{T}$ matrix determine the dynamics of the cyclical component c_t, while the third row is responsible for the random walk trend τ_t. The remaining rows of $\boldsymbol{T}$ are filled with zeros as the properties of the news errors are determined via $\boldsymbol{R}$ and the measurement equation.

3.2 Estimation

Our estimated model includes five releases in the measurement vector: [1st, 2nd, 3rd, benchmark, final] vintages for MFP, and [1st, 2nd, 1yr, benchmark, final] vintages for OPHA.[23] Our earliest release of MFP ends in 1993, while that for OPHA ends at 1968Q2. Consequently, any releases prior to 1993 for MFP and prior to 1968Q1 for OPHA are treated as missing values. Recent periods also contain missing values for some releases. For example, for MFP in 2021, our sample only includes the 1st and 2nd releases, so the 3rd and benchmark releases are treated as missing.

[23] We also experimented with four releases (excluding final vintage.) The Bayes factor preferred five releases.

Estimation uses a random-walk Metropolis-Hastings algorithm with a diffuse prior. The prior information is detailed in Table 3.[24] We generate 100,000 draws, discarding the initial 80,000. For each parameter draw, latent variables are estimated.

[Table 3 about here.]

We report the median and 50% credible bands for the parameters based on our full data sample in Table 4. The AR coefficients for the cyclical component are similar for the two series and they display the "hump-shape" (opposite signs) often associated with simple business cycle models. The first and final releases for MFP are the noisiest, while the fourth and final release noises are the noisiest for OPHA. News about the trend is evenly distributed across the various releases for MFP while that for OPHA is largest in the 1 Yr release while the initial release contains relatively little information. News about the cycle is dominated by the last release included in the model.

[Table 4 about here.]

To better understand what the above full-sample estimates imply for changing perceptions of productivity growth trends, we consider a variety of measures in the next section, before turning to consider the effects of publication lags and parameter instability thereafter. The former gives a better sense of the model's properties and its historical assessments of productivity growth trends, while the latter shows how the model behaves when used to inform decision making and current analysis.

4 Full Sample Results

4.1 Productivity Growth Trends

Figure 5 shows the model's estimates of the trend growth rate of productivity. These are based on the full-sample parameter estimates shown in Table 4 and are smoothed estimates. These should be interpreted as the model's best "historical" perspective on trend

[24] We use the same prior for both measures of productivity growth. Given the small sample size for MFP, we also investigated an inverse gamma distribution as the prior for the standard deviation parameters. The alternative prior is specified in Table 7 and results are presented in Appendix A.

productivity growth, a mostly retrospective view that takes into account the full sample for all five releases of each variable. The red bands around the estimates indicate the 68% credible interval for the trend. These intervals are slightly wider near the beginning and end of the sample, but are generally narrow enough to imply very statistically significant variation in productivity growth trends, particularly for OPHA.

The trends for MFP and OPHA are broadly similar until the 1980s, with growth not far from 2% through to around 1970 before declining to near 0% by 1980. Although both series show a gradual recovery over the next 20 years, the recovery in OPHA is slightly more than double that in MFP and exceeds its previous peak in the early 1960s. After 2000, MFP stagnates somewhat while OPHA shows a much steeper decline. By 2022, both are close to 1%.

[Figure 5 about here.]

This pattern (decline in the 1970s, recovery in the 1980s & 90s, decline thereafter) is unremarkable and similar results may be produced by much simpler techniques. However, note that these variations in trend growth are much larger than those we saw in Figure 3, which were produced by an analogous model with the same dynamics for trend and cycle, but which used only the "final" data release and ignored data revision. Figure 6 directly compares the two models' filtered estimates of trend (black for our multi-release model, green for the No Revision model). While filtered estimates from the No Revision model are more volatile than that model's smoothed estimates, they largely miss the decline of growth in both series to near zero around 1980 and much of the recovery in growth rates thereafter.

[Figure 6 about here.]

Figure 6 also compares both of these model's filtered estimates to several releases of the CBO's estimates of growth at potential for MFP and OPHA. While our model largely tracks the CBO estimates for MFP from the 1960s through 1980, it provides smaller estimates of the rebound in MFP growth through to 2000. For OPHA, the multi-revision model tracks the CBO estimates more closely, with the exception of the 2000–2015 period where our model produces substantially higher estimates of trend productivity growth.

Weighing Revisions

To understand how our model arrives at its filtered estimates of trend growth and why they differ from other estimates, we follow Koopman and Harvey (2003) and examine the weights it places on the initial release and on each subsequent revision.[25] Figure 7 shows these weights, with results for MFP in Figure 7a and those for OPHA below in Figure 7b. Filter-implied weights for both series decay smoothly, with those for OPHA decaying slightly faster than those for MFP, producing slightly more volatile growth trends for OPHA. In both cases, several different releases receive substantial weight in estimating the trend growth rate.[26] The benchmark revision receives substantial weight. For OPHA, it is the most important component of the estimated trend and receives a weight that is slightly larger than that of the next two components combined, or just under half of the total weight assigned to all five components. For MFP however, the revision from the 2nd to the 3rd release receives a still greater weight. Together with the benchmark revision, these two components account for about two-thirds of the total weight assigned to all five components. Interestingly, revisions in the final available vintage receive relatively little weight for either series, which contrasts with the conventional practice of using only the most recent available vintage to estimate the trend. This suggests that filtered estimates may continue to undergo important revisions for several years after the initial release but should change little after the first benchmark revision, something we investigate in greater detail below.

[Figure 7 about here.]

[25] In the following discussions of Kalman Gains and Weights, all calculations are based on our median parameter estimates and assume the absence of missing observations. The formulas in Koopman and Harvey (2003) provide a set of five linear weights $[\omega_1, \ldots, \omega_5]$ for the five releases $[y_1, \ldots, y_5]$ in our model. We convert these into the implied weights on the first release and subsequent revisions using the fact that

$$\omega_1 y_1 + \omega_2 y_2 + \omega_3 y_3 + \omega_4 y_4 + \omega_5 y_5 = \left(\sum_{j=1}^{5} \omega_j\right) y_1 + \left(\sum_{j=2}^{5} \omega_j\right)(y_2 - y_1) + \left(\sum_{j=3}^{5} \omega_j\right)(y_3 - y_2) + (\omega_4 + \omega_5)(y_4 - y_3) + \omega_5 (y_5 - y_4)$$

[26] We found analogous results for estimates of the cyclical component of productivity growth c_{t+1}, since these are simply $-1\times$ the weights for τ_{t+1}.

5 The View at the Leading Edge

The filtered estimates presented in the previous section for a given period t may differ from what an analyst using our model in period t would have calculated in two important ways. First, all the analysis presented above used the full-sample parameter estimates presented in Table 4. The filtered estimates may therefore vary to the extent that the parameter estimates are unstable and differ over shorter samples. Second, and more importantly, the estimates presented above ignored publication lags. When estimating growth trends in year t, the latest data available is that for year $t-1$ at best; MFP estimates were often published with multi-year delays. Analysts making a "nowcast" are therefore forecasting one (or more) years into the future. The fact of missing observations due to publication lags is commonly referred to as "the ragged edge".

The ragged edge problem becomes much more serious when data revisions are used in a model. Although the first estimate for year t may be published at $t+1$, the second estimate may not be available until $t+2$, and the benchmark revision may be delayed further still (see Table 1).[27] The "final" estimate used in our model is also unavailable for current analysis, although the results in Figure 7 show that it receives modest weight.

In this section we address the ragged edge and the parameter instability issues separately to understand how they may affect changing perceptions of productivity growth. We begin by revisiting the filter weights shown above in Figure 7 to consider how they vary when one or more series are unavailable. We then compare the filtered and smoothed estimates from the previous section to filtered estimates with varying amounts of missing information. We also examine how quickly the precision of the estimated trends improves as more information becomes available. Thereafter we repeat our analysis using rolling estimates of our model parameters to understand the degree to which estimates of productivity growth trends are affected.

[27] Recalling that OPH estimates are published at a higher frequency and with shorter lags than those for MFP, this problem is most serious for MFP. Note also that the BLS has increased the publication and revision frequency for both series in recent years, which should mitigate this problem to some degree. See the discussion, below.

5.1 At the Ragged Edge

To understand how the estimation of the trend productivity growth rate τ_{t+1} changes at the ragged edge, we now consider how the weights on the observations at t change when one or more releases are unavailable. Table 5 presents these results.[28]

The first point to note is that the weights assigned by the filter change only slightly as more components are released. This implies that revisions to the estimated trend growth rate largely reflect the incorporation of new revisions rather than a re-weighting of available releases. Taken together with the weights assigned to the various revisions shown above in Figure 7, this implies that much of the information needed to estimate the trend productivity growth rate is not available until several years have passed. In the case of MFP, the largest weight is on the revision from the 2nd to the 3rd release. When revisions are released annually, this will typically imply a delay of 3 years before even half of the weight in filtered series can be allocated.[29] Even then, substantial weight (43% of the total) for the filtered estimate must wait until the benchmark and then the final revisions are available. For OPHA, while revisions are released more frequently (quarterly), just over half of the total weight (54%) is put on the benchmark and final revisions. This again implies that several years may need to pass before much of the information needed for reliable estimation of the trend growth rate is available.[30]

[Table 5 about here.]

The above analysis of the filter weights has an important shortcoming, however. Table 2 showed that some revisions tended to be much larger than others, so that the apparent importance of a larger weight might be (partially or totally) offset by a lower variability of the associated revision. Furthermore, as shown in Figure 7, estimates of the current trend growth rate depend on a long distributed lag of past observations. The most recent filter weights alone could therefore give a distorted view of how quickly or slowly statistical

[28] When all 5 releases are available, as shown in the first line of the table for MFP and for OPHA, the weights correspond to those shown in Figure 7 at a lag of one period.

[29] Historically, MFP revisions have not always been released at regular intervals. There were no releases in 1997 or 2005. In 2020, the BLS began releasing estimates in the spring and revising them in the fall of the same year.

[30] As shown in Table 1, benchmark revisions for OPHA were released in 1971, 1981, 1990, 1996, 2010, 2013 and 2018.

uncertainty about productivity growth trends dissipates over time as more information becomes available.

An alternative approach is to compare estimates of the error variance of the trend estimation. Specifically, consider $\text{var}(\hat{\tau}_{t+1|\boldsymbol{\Omega}} - \tau_{t+1})$, where τ_{t+1} is now the unknown true value of the growth rate and $\hat{\tau}_{t+1|\boldsymbol{\Omega}}$ is the estimated growth rate from our model conditional on the information in $\boldsymbol{\Omega}$ which contains model parameters Θ as well as data $\boldsymbol{Y}$.[31] Table 6 compares results for MFP and OPHA as more information is used to estimate τ_{t+1}. Results in the table are shown relative to the 1-period ahead overall estimation uncertainty $\text{var}(\tau_{t+1}|\boldsymbol{\Omega} = \boldsymbol{Y}_t)$, where $\boldsymbol{Y}_t$ contains all releases for all years up to and including year t. Note that in this case $\boldsymbol{\Omega}$ does not include Θ; instead the distribution of Θ is inferred from $\boldsymbol{Y}_T$ and the conditional variance $\text{var}(\tau_{t+1}|\boldsymbol{\Omega} = \boldsymbol{Y}_t, \Theta)$ is integrated over the marginal distribution of Θ. The first line of Table 6 shows how the uncertainty about the trend growth rate changes when we condition on the median values of the estimated parameters shown in Table 4. We see that the conditional variance drops just less than 8% for the MFP trend and less than 3.5% for the trend in OPHA; this suggests that model parameter uncertainty is not a major contributor to overall trend uncertainty (something that we return to below.)

[Table 6 about here.]

The subsequent lines in Table 6 show how the error variance of our estimated trend at $t+1$ is further reduced as we take account of preliminary productivity estimates for $t+1$ and their successive revisions. When all series are available for $t+1$ ($\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_{t+1}\}$), the change in the relative uncertainty gives us a measure of the overall impact of data revision at $t+1$. We see that the relative uncertainty for trend growth in MFP has decreased from 92.3% to 78.0%; just over three quarters of the uncertainty about trend growth from year t remains. The situation is marginally better for OPHA, where the relative uncertainty falls from 96.7% to 67.1%, implying that just over two thirds of uncertainty remains. While these reductions show that productivity data revisions are economically significant, they are not the only source of changes in productivity growth perceptions. The final line shows that

[31] This differs slightly from standard notation (such as that of Durbin and Koopman (2012, Section 4.3.1), where $\boldsymbol{P}_{t+1} \equiv \text{var}(\boldsymbol{\alpha}_{t+1}|\boldsymbol{Y}_t)$ and $\boldsymbol{\alpha}_{t+1}$ is the state vector and it is assumed that the parameters $\boldsymbol{\Theta}$ of the system matrices are known rather than estimated.

after 12 additional years of data there remains just over one-third (36.1%) of the original uncertainty for MFP and just over one-quarter (26.1%) for OPHA. This shows that the future evolution of productivity (i.e. "hindsight") provides somewhat greater information about current growth trends than data revisions will.

A way to visualize the effect of ragged edges on estimated productivity growth trends is to compare the filtered and smoothed trend estimates shown above (which ignored ragged edges) with their ragged-edged counterparts. Figure 8 presents the results. It shows the same smoothed estimates shown in Figure 5 (red solid line with credible bands) alongside the filtered estimates shown in Figure 6 (brown solid line) and adds several sets of ragged-edge filtered estimates for comparison. For example, when the sample ends in 2000, we include the first release of the 2000 value, the first and second releases of the 1999 value, the first, second, and third/one-year releases of the 1998 value, and so forth. We then retain the estimate for 2000 as the ragged-edge series with one release, the estimate for 1999 as the ragged-edge series with two releases, the estimate for 1998 as the ragged-edge series with three releases, and so on. We then proceed to the next year where the sample ends in 2001. All are calculated using the same median parameter estimates from Table 4.

[Figure 8 about here.]

For MFP, we see that the ragged edge estimates lie close to the filtered estimates which ignore ragged edges, implying that the overall effects of publication lags are modest. This is consistent with the results of Table 6 which suggested that smoothing over several years is more important to trend estimation than the replacement of missing observations. It is also interesting to note that nearly all the filtered estimates lie within the 68% credible interval around the smoothed estimates. Finally, all the filtered estimates appear to lag the smoothing estimates in the detection of turning points (seen most clearly around 2001). There is also some suggestion that estimates with more missing observations have slightly longer lags. The latter effect is seen most clearly around the 2001 turning point in OPHA where the lag at the peak decreases steadily as we receive the 1st, 2nd, 1yr and BM releases in turn. The ragged edge estimates now follow the filtered estimates that ignore ragged edges a bit less closely, and they more frequently depart from the 68% credible

interval around the smoothed estimates. This is consistent with the more modest degree of smoothing of the OPHA data that we saw in Figure 7.

5.2 Real-Time Behavior

As noted at the outset of this section, parameter instability may also contribute to changing (model-based) estimates of trend productivity growth. This question is of potentially great concern to decision makers, so we now investigate the extent to which this may contribute to variability in our model's trend growth estimates. We do so via a (pseudo) real-time simulation.

To this point our results have been based on full-sample estimates of our model parameters $\hat{\Theta}$ as shown in Table 4. We now compare the resulting estimates of trend growth rates with those produced using rolling estimates of Θ. Estimation uses an expanding window whose end-point starts in 1999 and ends in 2021 for MFP and 2022 for OPHA, and includes the ragged edge effects considered in the previous section to make the simulation as realistic as possible. As an example, when estimating the trend component of MFP growth for a given year, say 2000, the available data includes values from the 1st release for 1998, the 1st, 2nd, and benchmark releases for 1997, the 1st, 2nd, 3rd, and benchmark releases for 1996, and so on.

Figure 9 compares the full-sample smoothed estimates of trend growth shown above in Figure 5 with the results of our real-time simulation. The real-time filtered estimates it shows for each year t are based solely on data and parameter estimates that were available to agents in that year. Because the preliminary data for year t are not yet published at t, the filtered estimates shown reflect a real-time "nowcast" of trend productivity growth. We see that, like their full-sample counterparts, the real-time filtered estimates are somewhat more volatile than the smoothed estimates and appear to lag turning points somewhat, but again mostly lie within the 68% credible intervals for the full-sample smoothed estimates.

[Figure 9 about here.]

However, there is no clear analogue to the smoothed estimate in a real-time simulation: at the end of the sample, "smoothed" estimates are identically equal to the filtered estimate. As an approximate analogue therefore, the figure compares the full-sample smoothed

estimates for year $t-5$ with our real-time smoothed estimates for the same year based on information as of year t. Figure 9 shows that the two series are tightly bound together, as we would expect from stable parameter estimates.[32]

Figure 10 provides more evidence on the convergence of real-time filtered estimates to the full-sample smoothed estimates. It includes the full-sample smoothed series previously shown in Figures 5 and 8 (red solid line with credible bands), as well as the real-time filtered estimates ($Y_{t|t}$, blue solid line) together with their 68% credible bands. The green line (denoted $Y_{t-2|t}$) shows real-time estimates for year $t-2$ conditional on the information available in year t. At that point at least one release for period $t-2$ is typically available. Again, the corresponding 68% credible bands are shown.

For both measures of productivity, we again see that the real-time filtered estimates lag the smoothed series, with $Y_{t|t}$ exhibiting the greatest lag, followed by $Y_{t-2|t}$. A similar pattern appears in the ragged-edge results of Figure 8, where filtered releases lag the smoothed estimate, and additional releases gradually bring the latent series closer to the smoothed series. The credible bands for the filtered estimates are wider than those of the smoothed estimates, with $Y_{t|t}$ being the widest, followed by $Y_{t-2|t}$, showing how additional data releases and revisions help reduce estimation uncertainty.

[Figure 10 about here.]

As expected, the full-sample smoothed estimates of trend productivity growth are not particularly volatile around NBER recessions. In contrast, the real-time filtered estimates often (though not always) exhibit considerable volatility around recessions, leading to economically significant differences between the two approaches. For MFP, the three recessions for which we have real-time estimates (2001, 2008–2009, 2020) show notable declines in the trend productivity growth rate before and after recessions, without corresponding movements in the full-sample smoothed estimates. For OPHA, aside from a sharp but transitory rebound following the 2008 recession, this pattern does not appear in other recessions.

[32] This is also consistent with the results in the first line of Table 6, which showed that conditioning on the median parameter estimates rather than their full distribution did little to reduce estimation uncertainty, as one would expect if parameters were estimated with good precision. This also gives indirect evidence that smoothed estimates of trend growth have largely converged after 5 years.

6 Conclusion

As we showed in Figure 1, the extent to which perceptions of productivity growth trends are revised over time is sometimes puzzling. To help understand the importance of various factors that may shape the changing perceptions of productivity growth trends, we presented a simple linear model that produces statistically efficient estimates of those trends even after taking account of data revisions, publication lags, and the uncertainty stemming from trend/cycle decompositions. Unlike analogous models that ignore data revision, the historical trends for the U.S. that it estimates match the widely-accepted narrative of high-growth through the mid-1960s, a slowdown through the 1970s to near zero, a period of recovery peaking around 2000 and a renewed slowing of growth thereafter. It finds that the variations in trend growth are much more pronounced for annual labor productivity (OPHA) than for multi-factor productivity (MFP).

We then use the model to understand how we should expect perceptions of productivity growth trends to be revised over time as more data become available and existing series are revised. We compare contemporaneous and historical estimates, estimates from models with and without data revision, from models with and without publication lags, and from models with and without rolling parameter estimates. We also examine how the standard errors of the growth trend estimates vary as we increase the information available to the model.

Because of the noisy character of productivity series, our model prefers to use a weighted average of several different releases, with relatively higher weights on those associated with benchmark revisions and only modest weights on subsequent changes. With relatively low weights for initial releases, this increases the imprecision of early trend estimates. Publication lags, particularly for MFP, tended to be long and variable, adding further uncertainty to trend growth estimates used for decision-making. However, the most important reductions in uncertainty around growth trends came not from the elimination of publication lags, nor the use of fully revised data, but from the ability to observe the future evolution of the series.

This result has some implications for how decision makers should interpret recent productivity growth trends. First, without minimizing the importance of providing timely and

accurate data, and the improvements that have been made in this regard in recent decades, it suggests that there are diminishing returns to further improvements and that even rapid and precise measurement may leave agents with considerable economic and statistical uncertainty about the rate of trend productivity growth. Second, we found that preliminary estimates of trend growth tended to detect shifts in trends with a few years delay, and sometimes detected false changes in trends around recessions. However, these changes lay well within the uncertainty bands surrounding even historical estimates, and so should not be misleading to those mindful of the limited precision of preliminary estimates. Third, our model's growth trends estimates typically undergo little revision after five years, by which time the most influential revisions have typically taken place. It therefore offers little insight into major changes in perceptions that may occur a decade or more after the fact.

We chose to keep the model used here as simple as possible while addressing the multiple sources of uncertainty that we highlighted above. No doubt it could be usefully extended in a variety of ways. For example, one might prefer a mixed-frequency model that would jointly model labor and multi-factor productivity, as they no doubt are subject to some common cyclical and trend shocks. Alternatively, a three-factor model could be used to jointly model output, labor, and capital services. To the extent that theory or applied work have suggested variables that are predictors of future productivity growth, the extent to which they can improve the precision of current trend estimates may produce useful insights.[33] Of course, there are a plethora of alternative trend/cycle decompositions or selections of data vintages that might also be explored.

[33]For an example of a multivariate approach, see Zaman (2025).

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A Alternative prior: Inverse gamma distribution

Due to concerns about the small sample size of MFP, an inverse gamma distribution prior has been imposed on the standard deviation parameters in addition to the benchmark diffuse prior presented in Table 3 (see Table 7). This alternative prior is introduced to mitigate potential issues associated with a diffuse prior, which is analogous to MLE. Results are shown in Tables 8 and 9 and Figure 11.

[Table 7 about here.]

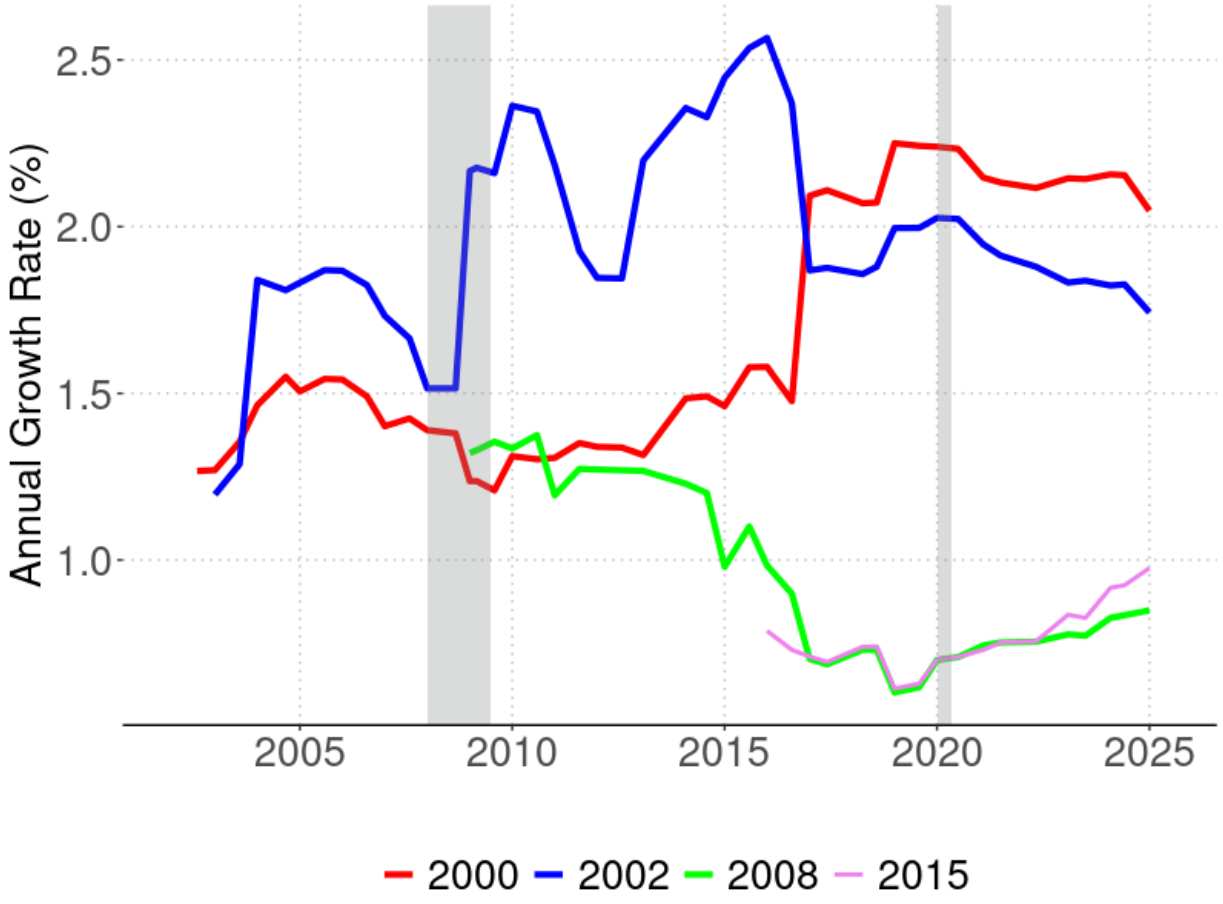
[Table 8 about here.]

[Table 9 about here.]

[Figure 11 about here.]

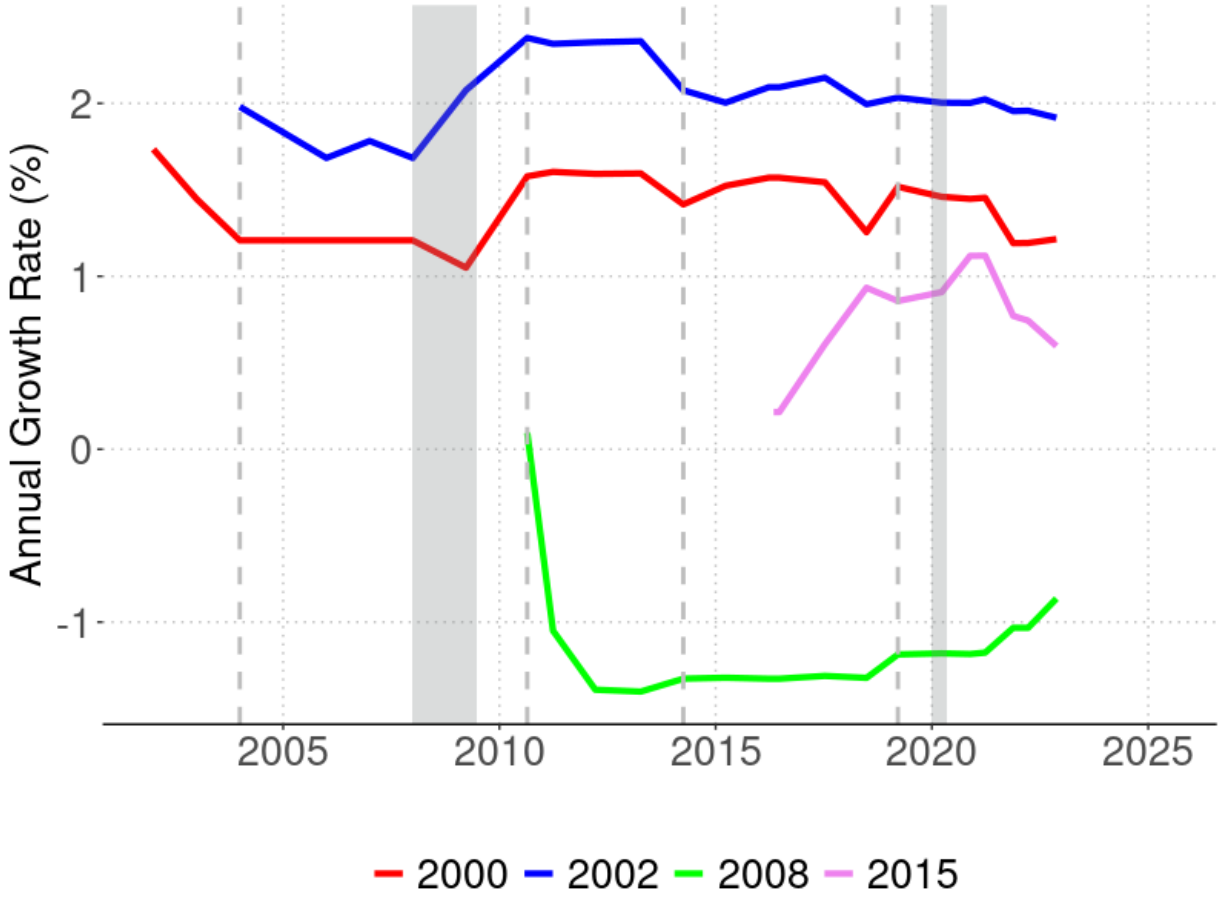
List of Figures

Figure 1: CBO's Changing Estimates of U.S. Productivity Growth Trends



The horizontal axis shows when the estimates for each year were published. Shaded areas show NBER recessions.

Figure 2: Revisions of U.S. MFP Growth Rates



Dashed vertical lines indicate changes in base year. For details, please refer to Section 2.

Figure 3: MFP Growth Trend Estimates from a Simple Model

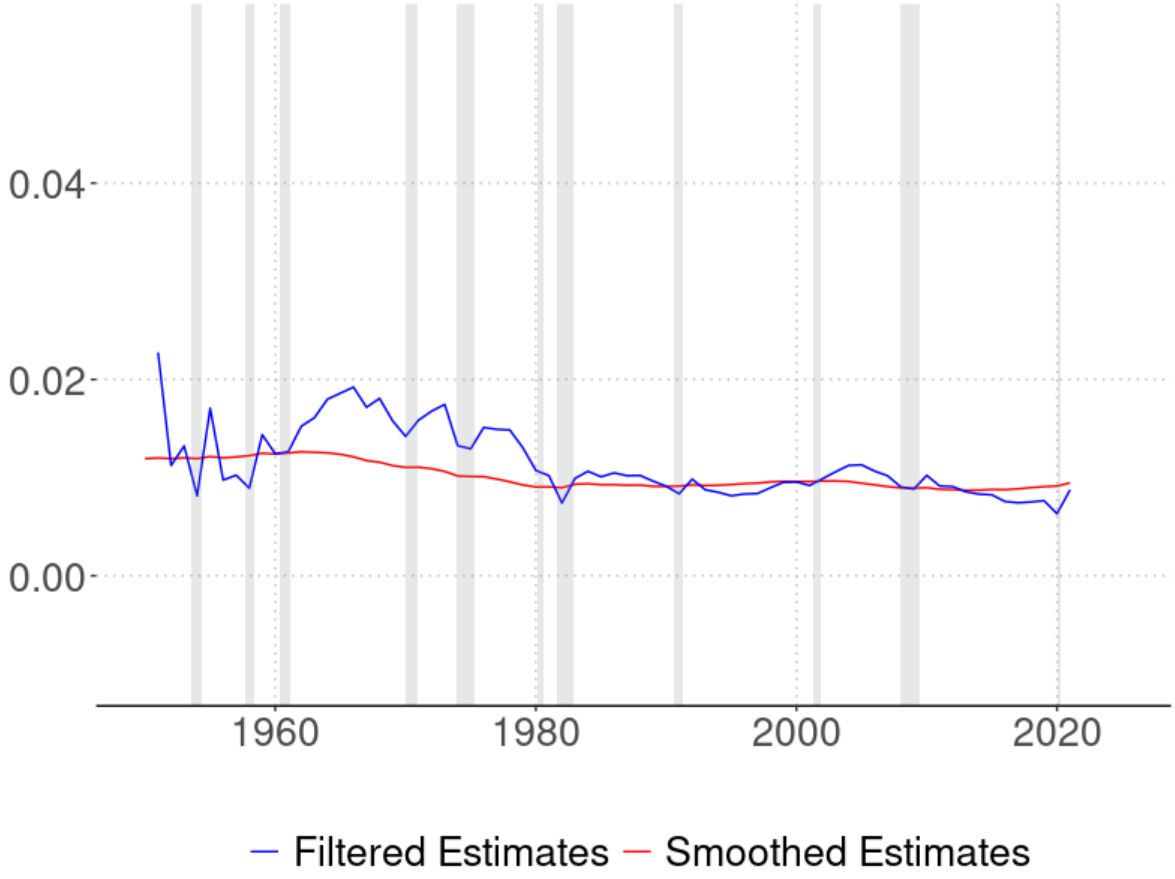


Figure 4: MFP, OPH, and OPHA

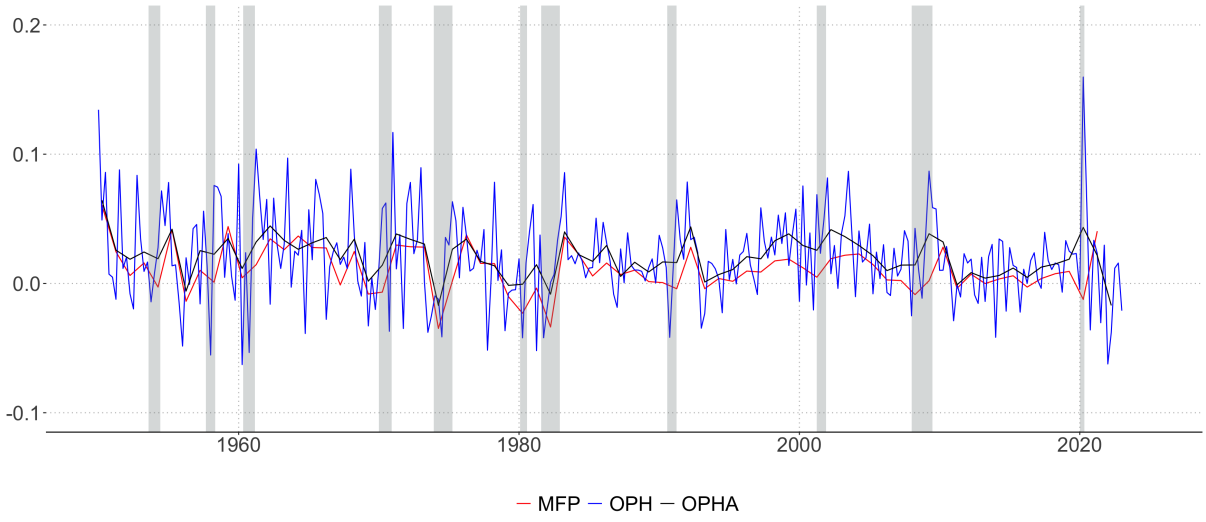
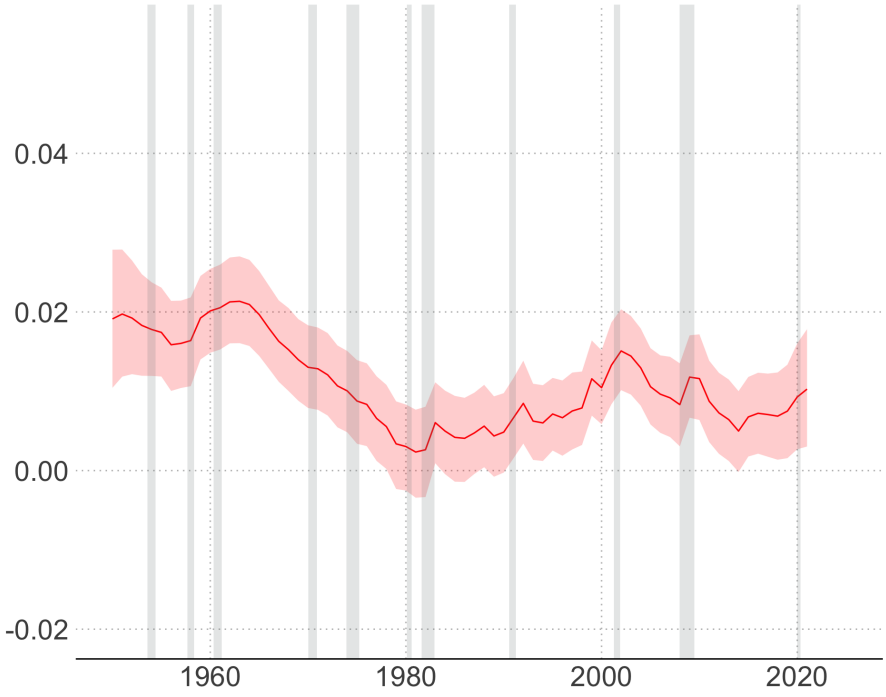
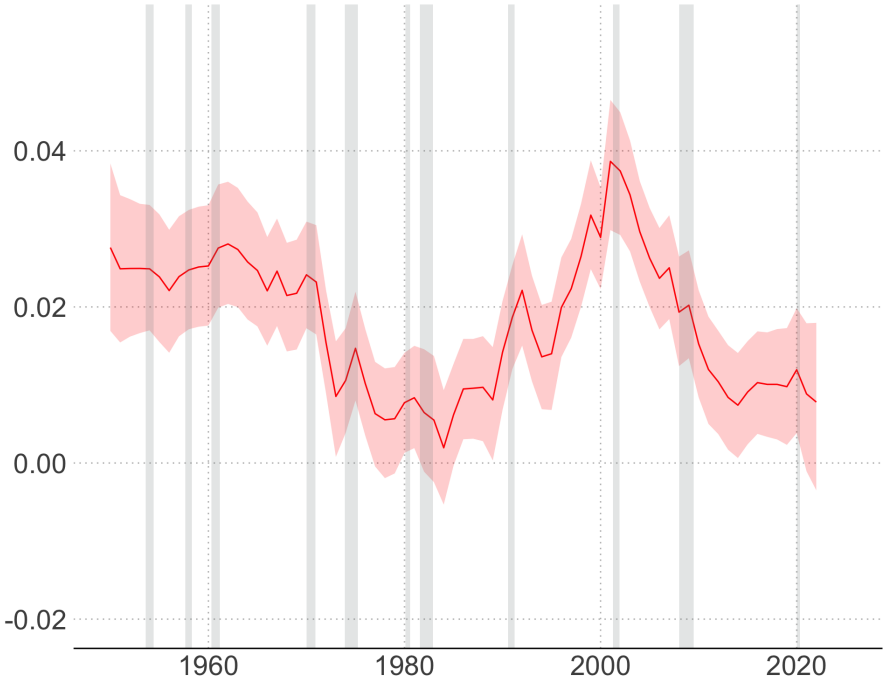


Figure 5: Trend Productivity Growth
(Smoothed Estimates, Full Sample)



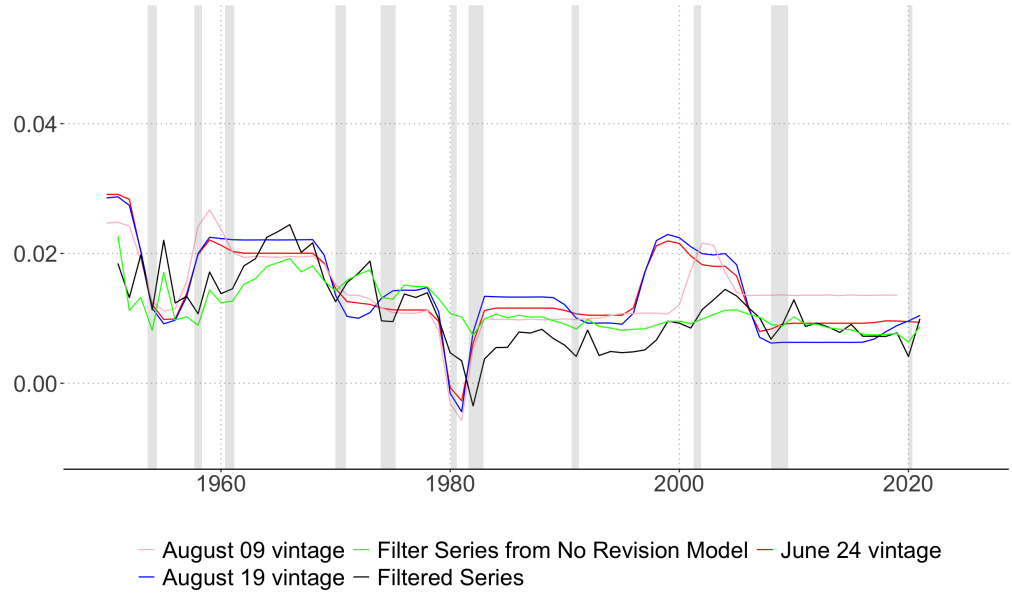
(a) MFP



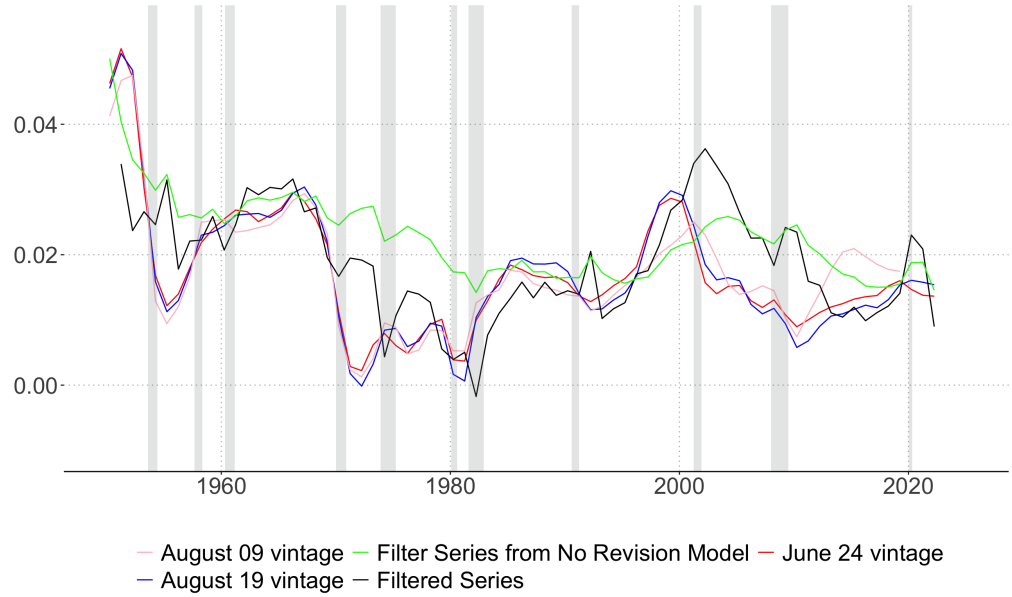
(b) OPHA

Shaded areas indicate the 68% credible interval.

Figure 6: Comparison to No-Revision and CBO Estimates (Full Sample)

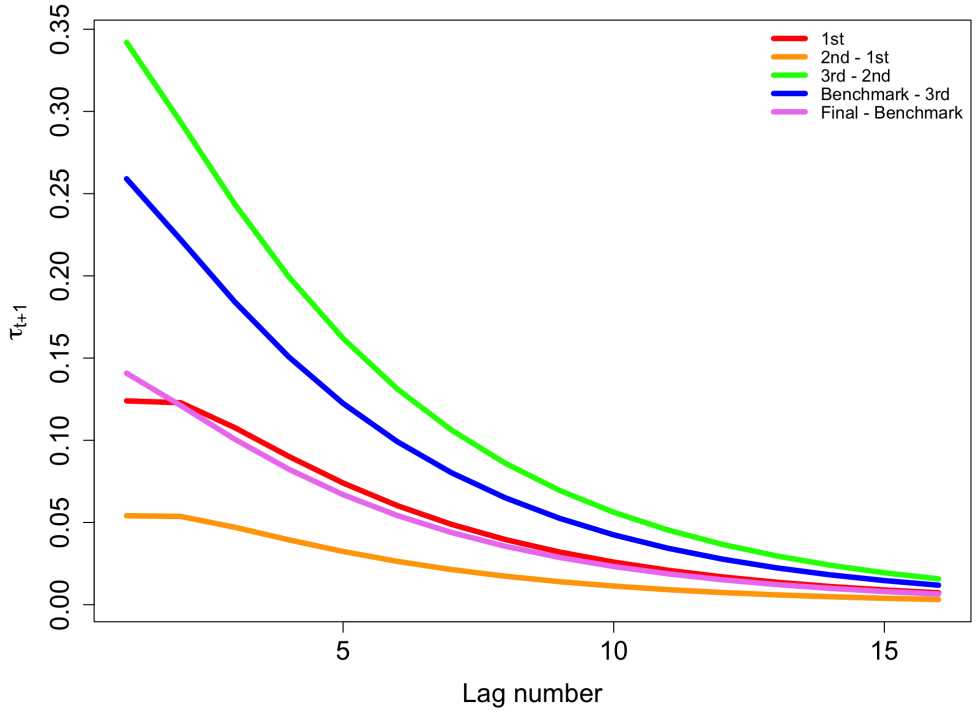


(a) MFP

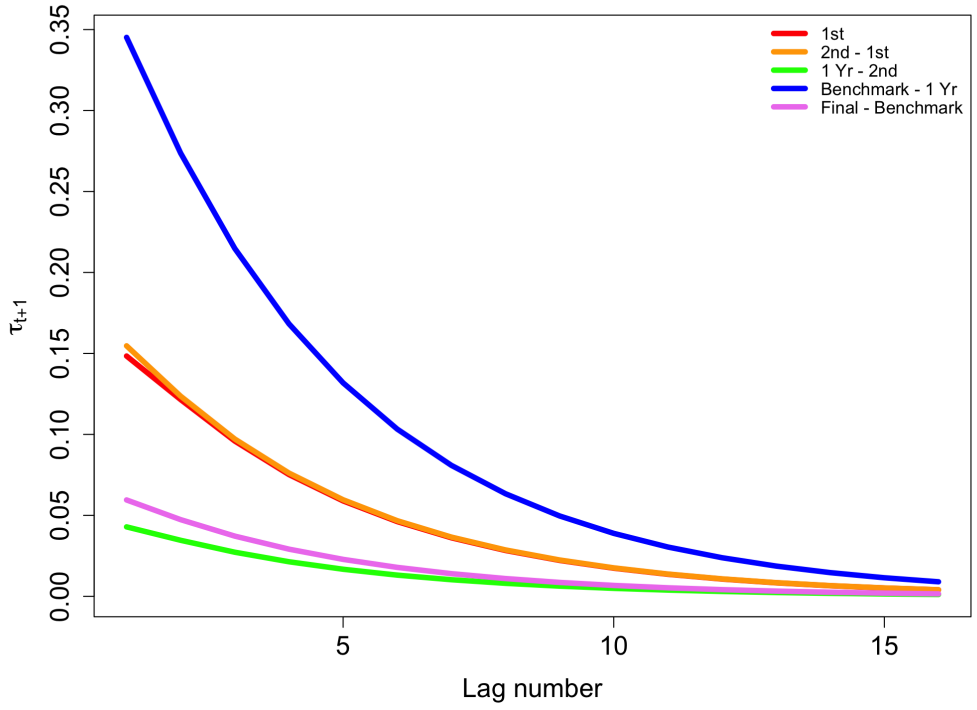


(b) OPHA

Figure 7: Kalman Filter Weights for τ_{t+1}

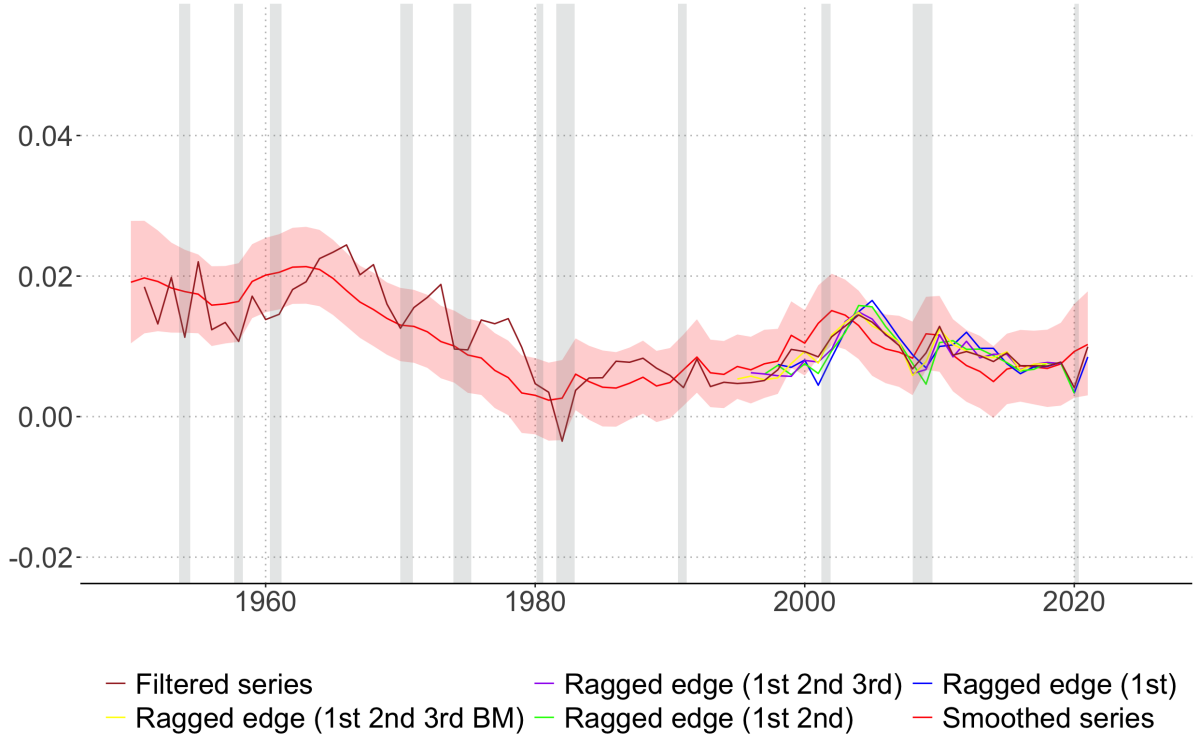


(a) MFP

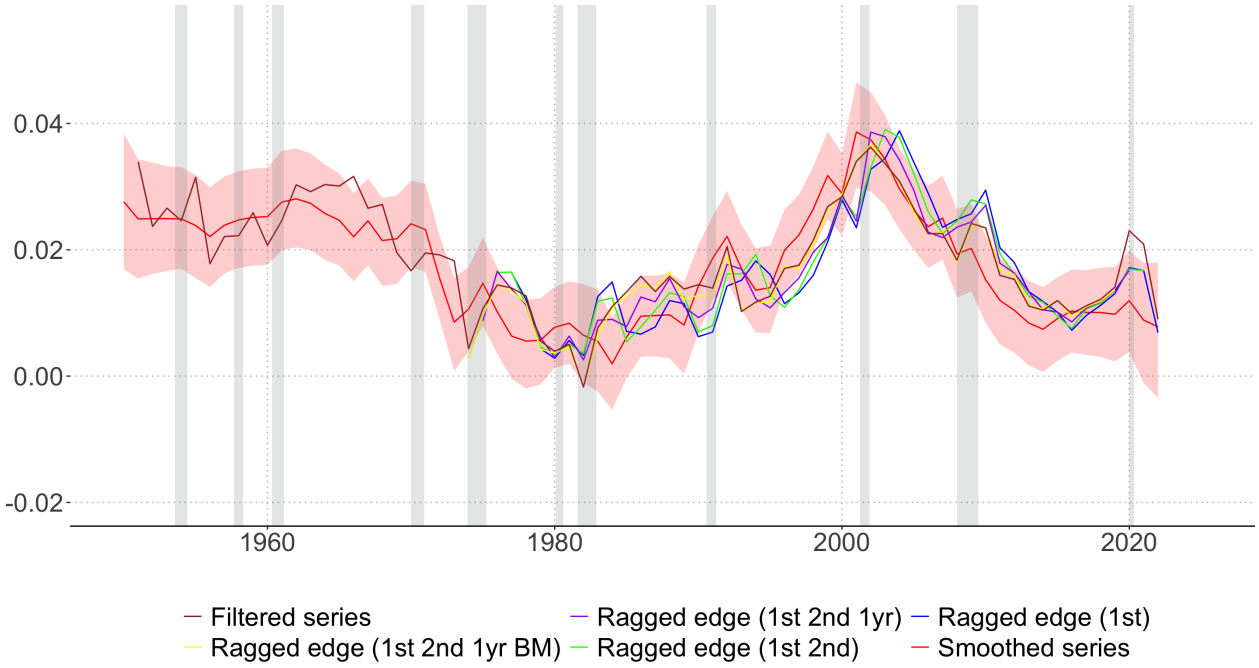


(b) OPHA

Figure 8: Trend Productivity Growth (Ragged Edge)

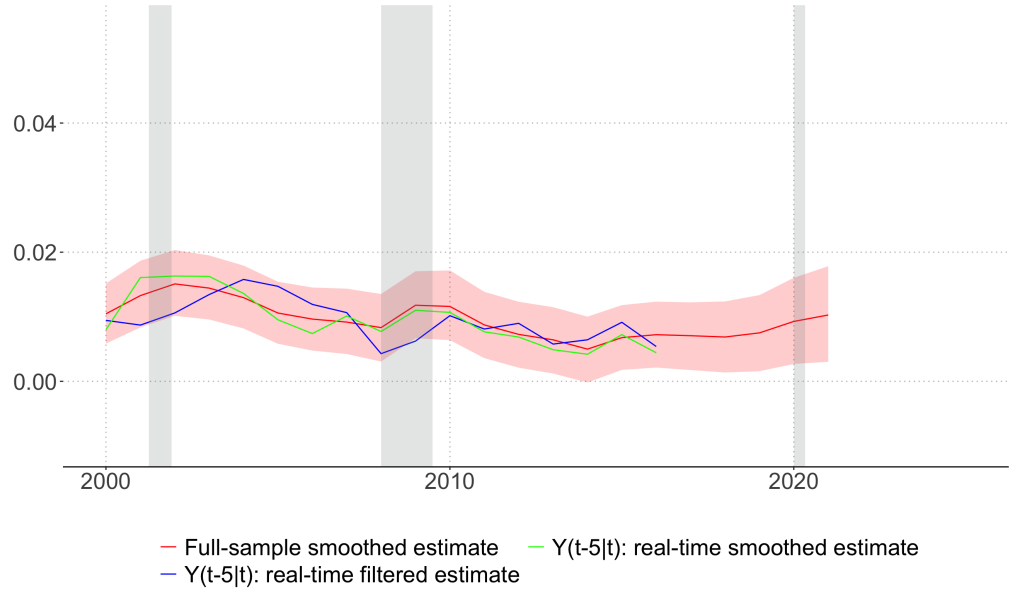


(a) MFP

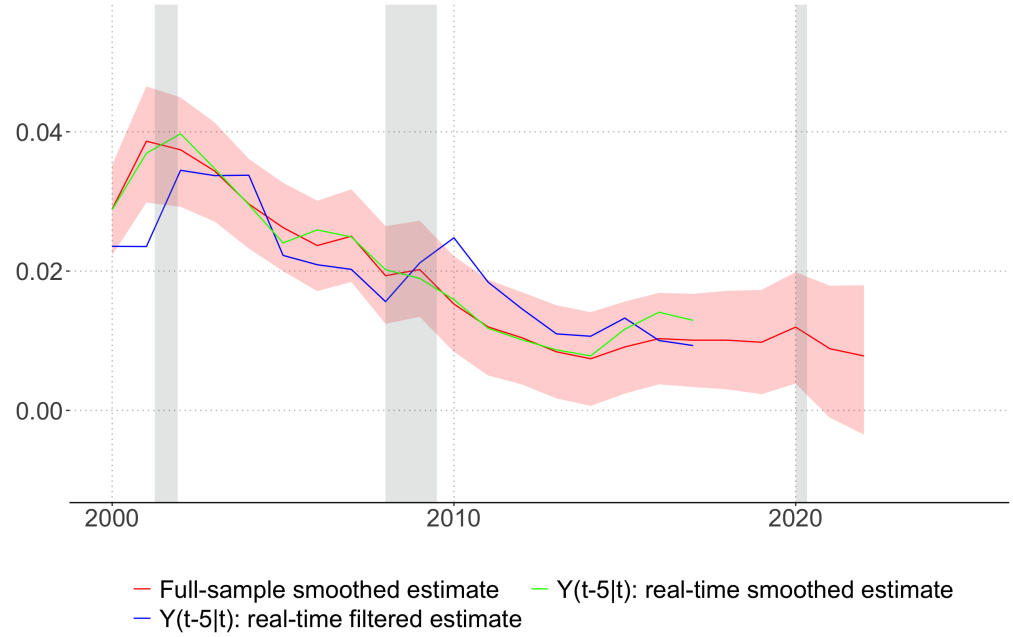


(b) OPHA

Figure 9: Real-Time Filter vs Full-Sample Smoother

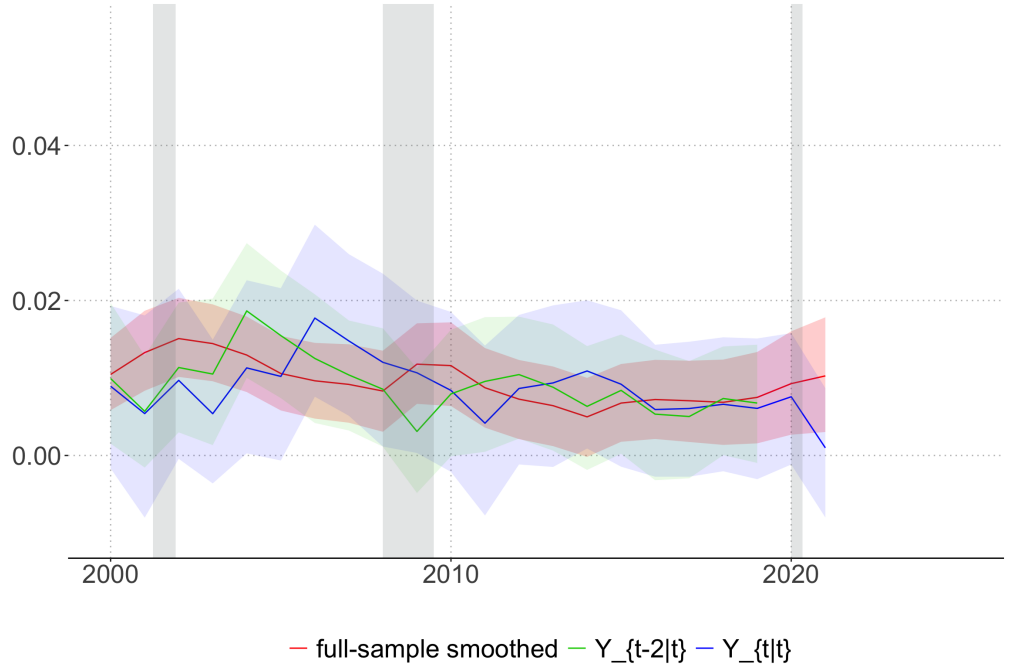


(a) MFP

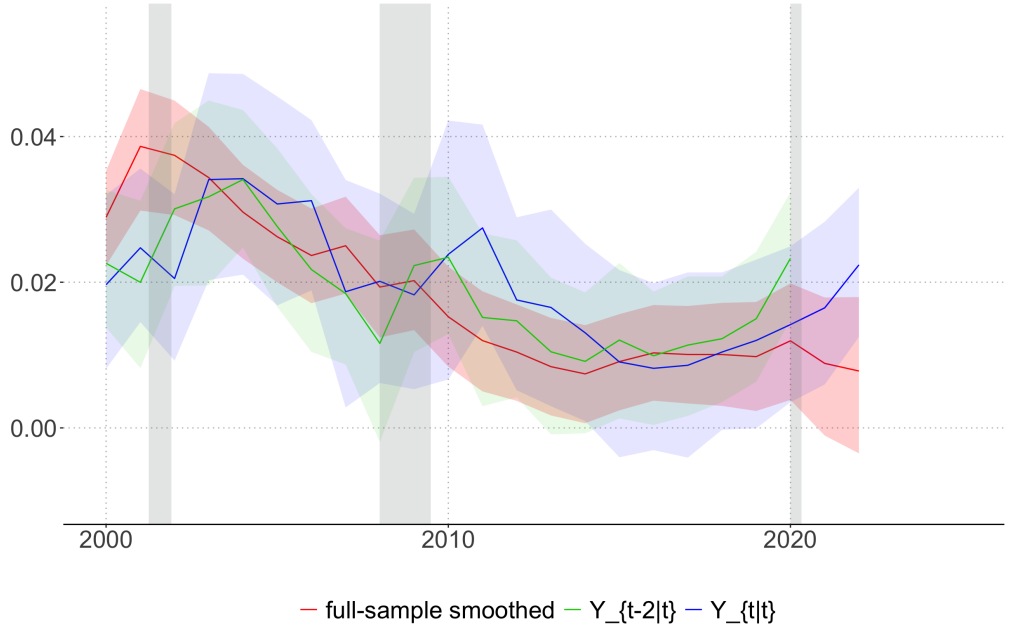


(b) OPHA

Figure 10: Trend Productivity Growth (Real-Time)

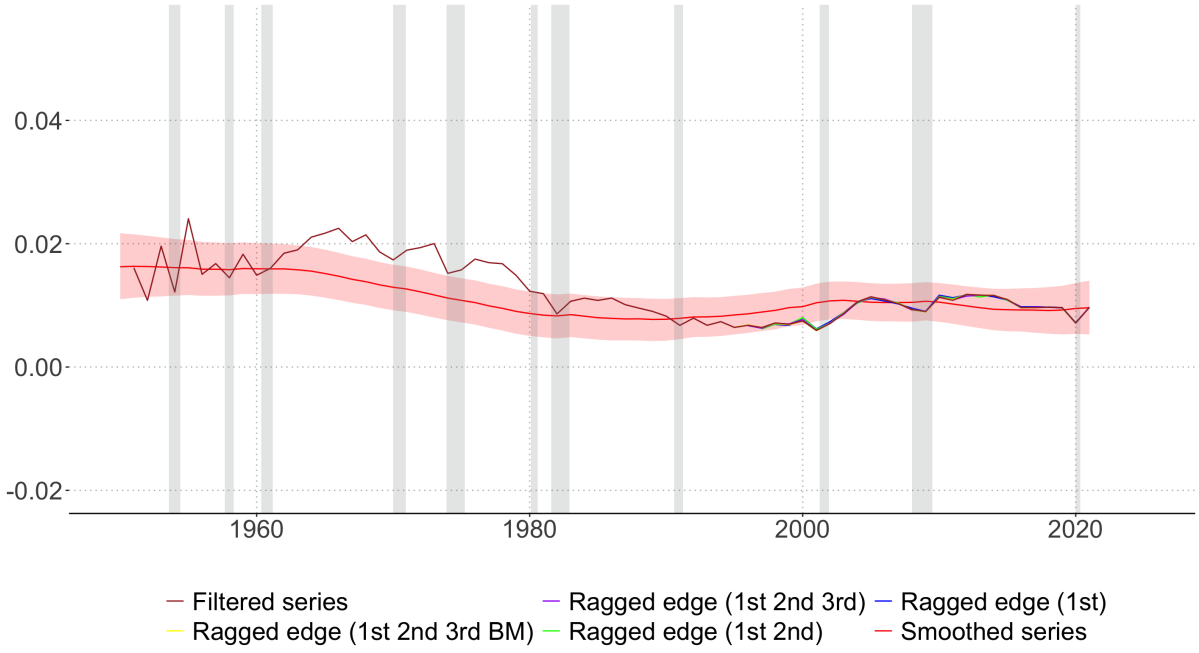


(a) MFP

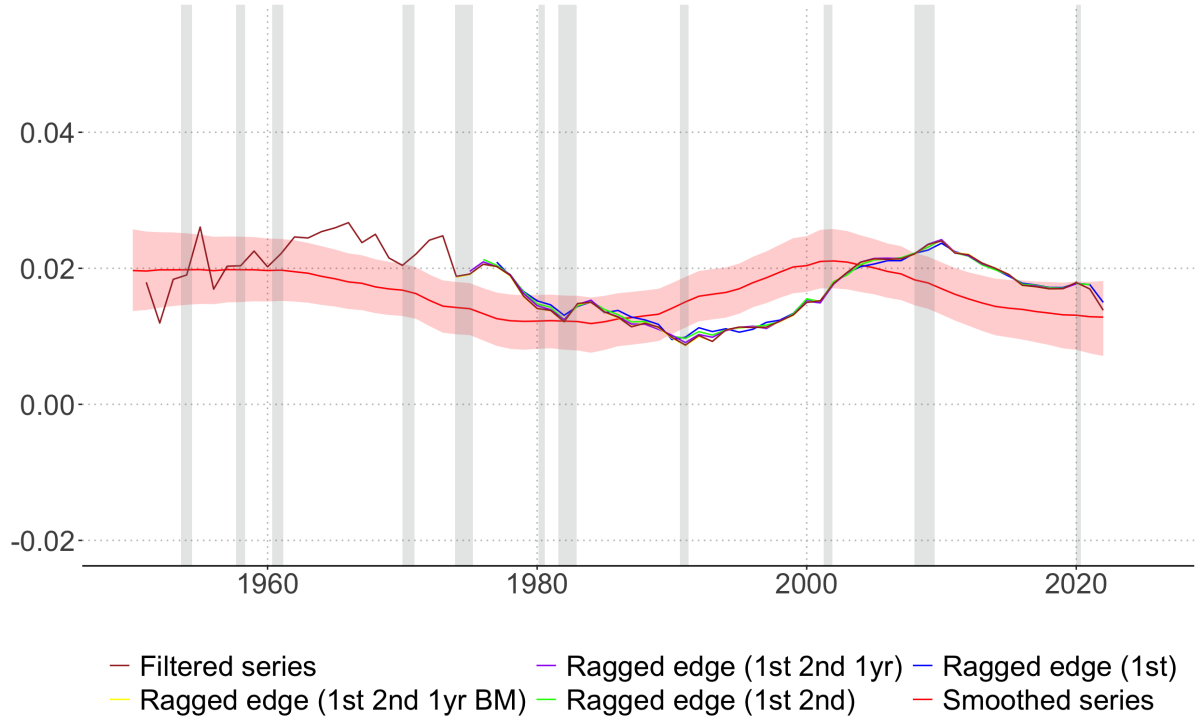


(b) OPHA

Figure 11: Trend Productivity Growth, Inverse Gamma Prior (Ragged Edge)



(a) MFP



(b) OPHA

List of Tables

Table 1: Benchmark Revisions

Variable	Base Year	Start	End
MFP	1995&96 = 100	1987	
	1998&99 = 100	1992	
	2000-2003 = 100	1996	
	2004-2009M3 = 100	2000	
	2010M8-2013M4 =100	2005	
	2014M4-2018M6	2009	
	2019M3-present (2022M11)	2012	
OPH	1957-1959=100	1968-05-27	1971-02-01
	Index 1967=100	1971-02-02	1981-01-29
	Index 1977=100	1981-01-30	1990-08-05
	Index 1982=100	1990-08-06	1996-02-07
	Index 1992=100	1996-02-08	2010-08-09
	Index 2005=100	2010-08-10	2013-08-15
	Index 2009=100	2013-08-16	2018-08-14
	Index 2012=100	2018-08-15	Current

Table 2: Revision in Productivity Growth Rates

	MFP	**OPHA**	**MFP**	**OPHA**
[-.1in] **Release**	**Std. Deviation**		**Mean Absolute Deviation**	
1st	0.012	0.016	0.010	0.016
Final	0.017	0.015	0.015	0.016
Revision	**RMS Revision**		**Mean Absolute Revision**	
Final - 1st	0.57	0.63	0.56	0.49
2nd - 1st	0.47	0.12	0.44	0.07
1 yr - 2nd	0.18	0.38	0.10	0.28
5 yr - 1 yr	0.21	0.40	0.20	0.32
Final - 5yr	0.33	0.41	0.31	0.30
RBM - 1st	0.43	0.48	0.37	0.37
BM - PBM	0.24	0.21	0.23	0.09
Final - BM	0.43	0.38	0.41	0.29

Revision statistics are expressed as a fraction of the variability (Std. Deviation or Mean Absolute Deviation) of the 1st release.

PBM indicates Pre-Benchmark Release (the last release prior to 1st Benchmark revision).

BM indicates Benchmark Release.

Final indicates the last release in our sample.

Table 3: Prior Specification

Parameter	**Density**	**Parameter 1**	**Parameter 2**
ρ_1	Normal	0.5	1
ρ_2	Normal	0	1
σ_i^{ξ}	Uniform	0	1
$\sigma_i^{c,\nu}$	Uniform	0	1
$\sigma_i^{\tau,\nu}$	Uniform	0	1

Parameter 1 is the mean of the normal distribution and the minimum value of the uniform distribution.
Parameter 2 is the standard deviation of the normal distribution and the maximum value of the uniform distribution.

Table 4: Model Parameter Estimates

Parameter	**MFP**	**OPHA**
ρ_1	0.428	0.404
	(0.420,0.439)	(0.398,0.415)
ρ_2	-0.158	-0.055
	(-0.177,-0.142)	(-0.065,-0.041)
σ_1^{ξ}	3.206	0.661
	(1.499,4.600)	(0.337,1.068)
σ_2^{ξ}	0.887	0.942
	(0.441,1.444)	(0.550,1.313)
σ_3^{ξ}	0.965	1.285
	(0.534,1.410)	(0.632,2.022)
σ_4^{ξ}	1.742	2.231
	(1.264,2.148)	(1.453,2.917)
σ_5^{ξ}	2.795	4.514
	(1.469,3.993)	(2.747,5.379)
$\sigma_1^{c,\nu}$	4.272	0.895
	(2.392,5.203)	(0.464,1.322)
$\sigma_2^{c,\nu}$	1.358	5.029
	(0.727,2.003)	(4.364,5.602)
$\sigma_3^{c,\nu}$	0.841	2.902
	(0.388,1.387)	(1.730,4.106)
$\sigma_4^{c,\nu}$	2.584	2.817
	(1.218,3.893)	(1.387,4.539)
$\sigma_5^{c,\nu}$	34.982	32.363
	(33.553,36.740)	(28.798,34.464)
$\sigma_1^{\tau,\nu}$	1.306	0.756
	(0.607,2.147)	(0.377,1.166)
$\sigma_2^{\tau,\nu}$	1.470	2.260
	(0.810,2.007)	(1.051,3.467)
$\sigma_3^{\tau,\nu}$	0.945	3.974
	(0.511,1.400)	(3.011,4.720)
$\sigma_4^{\tau,\nu}$	1.761	1.347
	(0.949,2.604)	(0.653,2.090)
$\sigma_5^{\tau,\nu}$	1.343	2.199
	(0.586,2.449)	(1.101,4.268)
log-likelihood	689.957	1080.306

Values in ( )'s are 25th% and 75th% quantiles; σ's are shown multiplied by 10^3.
Subscripts [1,2,3,4,5] refer to releases [1st, 2nd, 3rd, BM, Final] for MFP, and [1st, 2nd, 1 Yr, BM, Final] for OPHA.

Table 5: Kalman Filter Weights for τ_{t+1} at the Ragged Edge

	1st	2nd - 1st	3rd/1yr - 2nd	benchmark - 3rd/1yr	Final - benchmark
MFP	0.124	0.054	0.342	0.259	0.141
	0.124	0.054	0.329	0.180	
	0.126	0.053	0.298		
	0.127	0.045			
	0.129				
OPHA	0.148	0.155	0.043	0.345	0.060
	0.148	0.155	0.042	0.336	
	0.149	0.148	0.024		
	0.157	0.132			
	0.159				

Table 6: Resolution of Estimated Trend Uncertainty

Conditioning Information	Relative Uncertainty MFP	OPHA
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_t\}$	0.923	0.967
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_t, y_{1,t+1}\}$	0.867	0.812
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_t, y_{1,t+1}, y_{2,t+1}\}$	0.836	0.808
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_t, y_{1,t+1}, \ldots, y_{3,t+1}\}$	0.813	0.789
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_t, y_{1,t+1}, \ldots, y_{4,t+1}\}$	0.800	0.686
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_{t+1}\}$	0.780	0.671
$\boldsymbol{\Omega} \equiv \{\boldsymbol{\Theta}_T, \boldsymbol{Y}_{t+12}\}$	0.361	0.261

Relative Uncertainty is defined as $\text{var}(\tau_{t+1}|\boldsymbol{\Omega})$ / $\text{var}(\tau_{t+1}|\boldsymbol{Y}_t)$.
$\boldsymbol{\Theta}_T$ are the median full-sample estimates of model parameters.
$\boldsymbol{Y}_{t+j}$ contains all elements of $\boldsymbol{Y}$ for periods 1 to $t+j$.
$y_{j,t+1}$ is the value at $t+1$ of the jth series in $\boldsymbol{Y}$.

Table 7: Inverse Gamma Prior

Parameter	**Density**	**Parameter 1**	**Parameter 2**
ρ_1	Normal	0.5	1
ρ_2	Normal	0	1
$100\sigma_i^{\xi}, i = 1, \ldots, 4$	Inverse Gamma	0.2	4
$100\sigma_i^{c,\nu}, i = 1, \ldots, 4$	Inverse Gamma	1	4
$100\sigma_i^{\tau,\nu}, i = 1, \ldots, 4$	Inverse Gamma	0.05	4

For the normal distribution, Parameter 1 is the mean and Parameter 2 is the standard deviation.
For the inverse gamma distribution, parameter 1 is s and parameter 2 is ν where $p_{IG}(\sigma|s,\nu) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

Table 8: Five Release Parameter Estimates, Inverse Gamma Prior

Parameter	**MFP**	**OPHA**
ρ_1	0.428	0.503
	(0.425,0.433)	(0.474,0.510)
ρ_2	-0.122	-0.102
	(-0.129,-0.115)	(-0.108,-0.098)
σ_1^{ξ}	1.885	1.321
	(1.578,2.346)	(1.176,1.489)
σ_2^{ξ}	1.586	1.348
	(1.372,1.833)	(1.179,1.540)
σ_3^{ξ}	1.540	1.726
	(1.320,1.832)	(1.469,2.035)
σ_4^{ξ}	1.585	1.885
	(1.373,1.885)	(1.610,2.221)
σ_5^{ξ}	1.844	2.090
	(1.527,2.242)	(1.715,2.570)
$\sigma_1^{c,\nu}$	6.399	3.354
	(5.787,7.060)	(3.134,3.588)
$\sigma_2^{c,\nu}$	4.508	5.789
	(4.117,4.962)	(5.419,6.226)
$\sigma_3^{c,\nu}$	4.584	5.616
	(4.240,5.014)	(5.182,6.075)
$\sigma_4^{c,\nu}$	4.970	5.632
	(4.651,5.330)	(5.255,6.000)
$\sigma_5^{c,\nu}$	27.509	25.410
	(24.955,31.277)	(24.390,26.506)
$\sigma_1^{\tau,\nu}$	0.535	0.495
	(0.416,0.710)	(0.402,0.621)
$\sigma_2^{\tau,\nu}$	0.519	0.583
	(0.419,0.659)	(0.448,0.796)
$\sigma_3^{\tau,\nu}$	0.517	0.643
	(0.419,0.661)	(0.477,0.979)
$\sigma_4^{\tau,\nu}$	0.535	0.556
	(0.427,0.695)	(0.441,0.734)
$\sigma_5^{\tau,\nu}$	0.541	0.591
	(0.428,0.708)	(0.452,0.840)
log-likelihood	668.4275	1055.547

Values in ( )'s are 25th% and 75th% quantiles; σ's are shown multiplied by 10^3.
Subscripts [1,2,3,4,5] refer to releases [1st, 2nd, 3rd, BM, Final] for MFP, and [1st, 2nd, 1 Yr, BM, Final] for OPHA.

Table 9: Trend Kalman Gain, Inverse Gamma Prior

	1st	2nd - 1st	3rd/1yr - 2nd	benchmark - 3rd/1yr	Final - benchmark
MFP	0.072	-0.026	-0.028	-0.028	-0.026
	0.072	-0.026	-0.028	-0.026	
	0.071	-0.025	-0.025		
	0.070	-0.023			
	0.069				
OPHA	0.060	-0.019	-0.038	-0.036	-0.034
	0.060	-0.019	-0.037	-0.033	
	0.058	-0.017	-0.033		
	0.057	-0.012			
	0.055				