CIRANO
Allier savoir et décision

INTERNATIONAL MIGRATION, AGING, AND EXTERNAL IMBALANCES



KEVIN MORAN
JAMES WABENGA YANGO

CS
2026s-24
CAHIER SCIENTIFIQUE

CIRANO
Allier savoir et décision

Les **cahiers de la série scientifique** visent à rendre accessibles les résultats des recherches effectuées par des chercheurs membres du CIRANO afin de susciter échanges et commentaires. Ces cahiers sont rédigés dans le style des publications scientifiques et n'engagent que leurs auteurs.

The purpose of the Working Papers is to disseminate the results of research conducted by CIRANO research members in order to solicit exchanges and comments. These reports are written in the style of scientific publications. The ideas and opinions expressed in these documents are solely those of the authors.

Le CIRANO est un organisme sans but lucratif constitué en vertu de la Loi des compagnies du Québec. Le financement de son infrastructure et de ses activités de recherche provient des cotisations de ses organisations-membres, d'une subvention d'infrastructure du gouvernement du Québec, de même que des subventions et mandats obtenus par ses équipes de recherche.

CIRANO is a private non-profit organization incorporated under the Quebec Companies Act. Its infrastructure and research activities are funded through fees paid by member organizations, an infrastructure grant from the government of Quebec, and grants and research mandates obtained by its research teams.

Les partenaires du CIRANO – *CIRANO Partners*

Partenaires Corporatifs - Corporate Partners	**Partenaires gouvernementaux - Governmental partners**	**Partenaires universitaires - University Partners**
Autorité des marchés financiers	Ministère des Finances du Québec	École de technologie supérieure
Banque de développement du Canada	Ministère de l'Économie, de l'Innovation et de l'Énergie	École nationale d'administration publique
Banque du Canada	Innovation, Sciences et Développement Économique Canada	HEC Montréal
Banque Nationale du Canada	Ville de Montréal	Institut national de la recherche scientifique
Bell Canada		Polytechnique Montréal
BMO Groupe financier		Université Concordia
La Caisse		Université de Montréal
Énergir		Université de Sherbrooke
Hydro-Québec		Université du Québec
Intact Corporation Financière		Université du Québec à Montréal
Mouvement Desjardins		Université Laval
Power Corporation du Canada		Université McGill
Pratt & Whitney Canada		
VIA Rail Canada		

Le CIRANO collabore avec de nombreux centres et chaires de recherche universitaires dont on peut consulter la liste sur son site web. CIRANO collaborates with many centres and university research chairs; list available on its website.

© Octobre 2026. Kevin Moran, James Wabenga Yango. Tous droits réservés. *All rights reserved*. Reproduction partielle permise avec citation du document source, incluant la notice ©. *Short sections may be quoted without explicit permission, if full credit, including © notice, is given to the source.*

Les idées et les opinions émises dans cette publication sont sous l'unique responsabilité des auteurs et ne représentent pas les positions du CIRANO ou de ses partenaires. *The observations and viewpoints expressed in this publication are the sole responsibility of the authors; they do not represent the positions of CIRANO or its partners.*

ISSN 2292-0838 (version en ligne)

International Migration, Aging, and External Imbalances[*]

Kevin Moran[†], *James Wabenga Yango*[‡]

Abstract

This paper resolves analytically a puzzle that confronts life-cycle theory. Although an aging population should draw down its wealth, the world's fastest-aging economies do the opposite, holding the largest and most persistent net foreign asset positions on record, often pairing a current-account surplus with a trade deficit, a combination that existing open-economy frameworks can sign only under implausible parameters, or set aside as an empirical curiosity. We build a two-country overlapping-generations model with working retirees, incomplete annuity markets, pay-as-you-go pensions, and a cost-based financial friction, calibrated to two regions that differ in retirement length, retiree labor productivity, and financial depth. Longevity alone never signs the net foreign position, and the longer-lived region turns creditor only once a produc- tivity gap in retiree labor supply combines with a literature-disciplined financial wedge. Under this mechanism the calibrated model reproduces the joint surplus-deficit configuration observed in Japan and Germany, matching its sign though only a fraction of its empirical magnitude. An exact Shapley decomposition, cross-validated against a one-at-a-time variance split, attributes most of the trade-balance response to population aging rather than migration or productivity, and a companion welfare decomposition shows that the cost of adjustment falls on retirees rather than workers in the aging region, a distributional result invisible to models that track external balances alone.

Résumé

Cet article propose une résolution analytique d'un paradoxe auquel se heurte la théorie du cycle de vie. Alors qu'une population vieillissante devrait puiser dans son patrimoine, les économies qui vieillissent le plus rapidement dans le monde font l'inverse : elles détiennent les positions extérieures nettes les plus importantes et les plus persistantes jamais enregistrées, associant souvent un excédent du compte courant à un déficit commercial. Les modèles existants d'économie ouverte ne permettent de prédire les signes de cette combinaison qu'au prix d'hypothèses paramétriques peu plausibles, ou la considèrent comme une curiosité empirique. Nous développons un modèle à générations imbriquées comprenant deux pays, dans lequel les retraités peuvent continuer à travailler, les marchés de rentes sont incomplets, les régimes de retraite fonctionnent par répartition et une friction financière fondée sur les coûts est prise en compte. Le modèle est calibré pour deux régions qui diffèrent par la durée de la retraite, la productivité du travail des retraités et le degré de développement financier.

[*]The authors would like to thank Benoît Carmichael for all his helpful discussions.

[†] Corresponding Author. Department of Economics, Université Laval, kevin.moran@ecn.ulaval.ca

[‡] Department of Economics, Université Laval, james.wabenga-yango.1@ulaval.ca

L’allongement de l’espérance de vie ne suffit jamais, à lui seul, à déterminer le signe de la position extérieure nette. La région où l’espérance de vie est la plus longue ne devient créancière que lorsqu’un écart de productivité du travail des retraités se combine à une friction financière dont la calibration s’appuie sur la littérature scientifique. Grâce à ce mécanisme, le modèle calibré reproduit la combinaison d’un excédent du compte courant et d’un déficit commercial observée au Japon et en Allemagne. Il en reproduit les signes, mais seulement une fraction de l’ampleur empirique. Une décomposition exacte de Shapley, validée par comparaison avec une décomposition de la variance faisant varier un facteur à la fois, attribue la majeure partie de la réaction du solde commercial au vieillissement démographique plutôt qu’à la migration ou à la productivité. Une décomposition complémentaire du bien-être montre que le coût de l’ajustement est supporté par les retraités, plutôt que par les travailleurs, dans la région vieillissante. Ce résultat distributif échappe aux modèles qui examinent uniquement les soldes extérieurs.

Mots-clés/Keywords : Macroéconomie en économie ouverte, solde du compte courant, balance commerciale, migration internationale, marchés financiers, tendances démographiques / Open economy macroeconomics, Current account balance, Trade balance, International migration, Financial markets, Demographic trends.

Codes JEL/JEL Codes : E21, E44, E62, F22, F32, F41, J11

Pour citer ce document / To quote this document
Moran, K., & Wabenga Yango, J. (2026). International Migration, Aging, and External Imbalances (2026s-24, Cahiers scientifiques, CIRANO.)
https://doi.org/10.54932/WURO2933

1 Introduction

Life-cycle theory predicts that a population growing older should draw down its wealth, yet the demographic frontier of the world economy does the opposite. The fastest-aging societies hold the largest and most persistent net foreign asset positions on record. This divergence between prediction and pattern motivates the two-country overlapping-generations model developed below, which labels its regions "North" and "South" after the one demographic asymmetry the calibration disciplines directly, a longer retirement span in the North (Section 5.1), even where other dimensions of the model depart from the geographic namesake. Retiree labor-force participation, left free to differ across regions in an earlier calibration, is instead equalized here, because participation tends to run higher in lower-income economies and the results below do not rely on that channel to sign external outcomes (Section 5).

Fig. 1, built from the United Nations World Population Prospects 2022, traces how this transition plays out unevenly across the world. Life expectancy is rising everywhere, up roughly 25 years since 1950 in developing countries alone and projected to rise another 20 by 2100. Fertility is falling in both groups but accelerating in developing countries. The old-age dependency ratio (adults aged 65 and over relative to the working-age population) is climbing everywhere, more slowly where fertility remains higher. Sizeable migratory flows accompany this transition, as Fig. 2 shows, running from developing to developed countries and redistributing working-age population across a divide the model treats as structural rather than incidental.

These forces reach every corner of intertemporal decision-making. Aging, falling fertility, and migration bear on household saving, government finance, and firm investment, and because the transition unfolds asynchronously across regions, it reshapes global saving, capital flows, and factor prices (Attanasio et al., 2006, 2007). Existing open-economy frameworks capture pieces of this picture, but typically need implausible parameter values to sign the resulting net foreign asset position, or treat a current-account surplus coexisting with a trade deficit as a special case rather than a structural implication, a gap this paper closes by modeling the demographic, financial, and labor-supply channels jointly rather than in isolation (Section 2 situates this claim against the literature in detail). We develop a life-cycle open-economy model, in the spirit of Gertler (1999), to analyze these consequences in economies that differ in demographic structure but are otherwise standard. Capital markets are imperfectly integrated across the two regions, and international migration enters as a South-to-North flow of young, asset-less workers that shifts relative labor-force growth, not as a wage-elastic individual choice.

Three empirical regularities sharpen the question this paper answers. Mature, capital-abundant economies such as Japan and Germany have become the world's dominant net creditors, accumulating large, persistent positive net foreign asset positions as they age rather than running them down as a simple life-cycle dissaving story would predict (Alfaro et al., 2011; Bárány et al., 2023). Several of these mature creditors simultaneously post current-account surpluses and trade deficits, a combination that looks anomalous in isolation but follows directly once net foreign income on a large external position enters the accounting. And the same decades that saw longevity rise and fertility fall also saw a persistent decline in world real interest rates, consistent with an aging-driven rise in desired saving (Lisack et al., 2019; Carvalho et al., 2016; Bielecki et al., 2020). No existing framework signs all three jointly, whereas the model developed here does, not as three separate stories stitched together but as three faces of the same mechanism.

[1] For each indicator, the "South" series population-weights the United Nations' "Less developed regions, excluding least developed countries" (e) and "Least developed countries" (l) sub-groups, $x_t^{South} = (x_{e,t}P_{e,t} + x_{l,t}P_{l,t}) / (P_{e,t} + P_{l,t})$, where $P_{e,t}$ and $P_{l,t}$ are the sub-groups' populations (UN WPP 2022), rather than simply summing the two sub-group indicators.

[2] Flows are recovered from the UN's published net migration rate $r_{g,t}$ (per 1,000 population) for each group

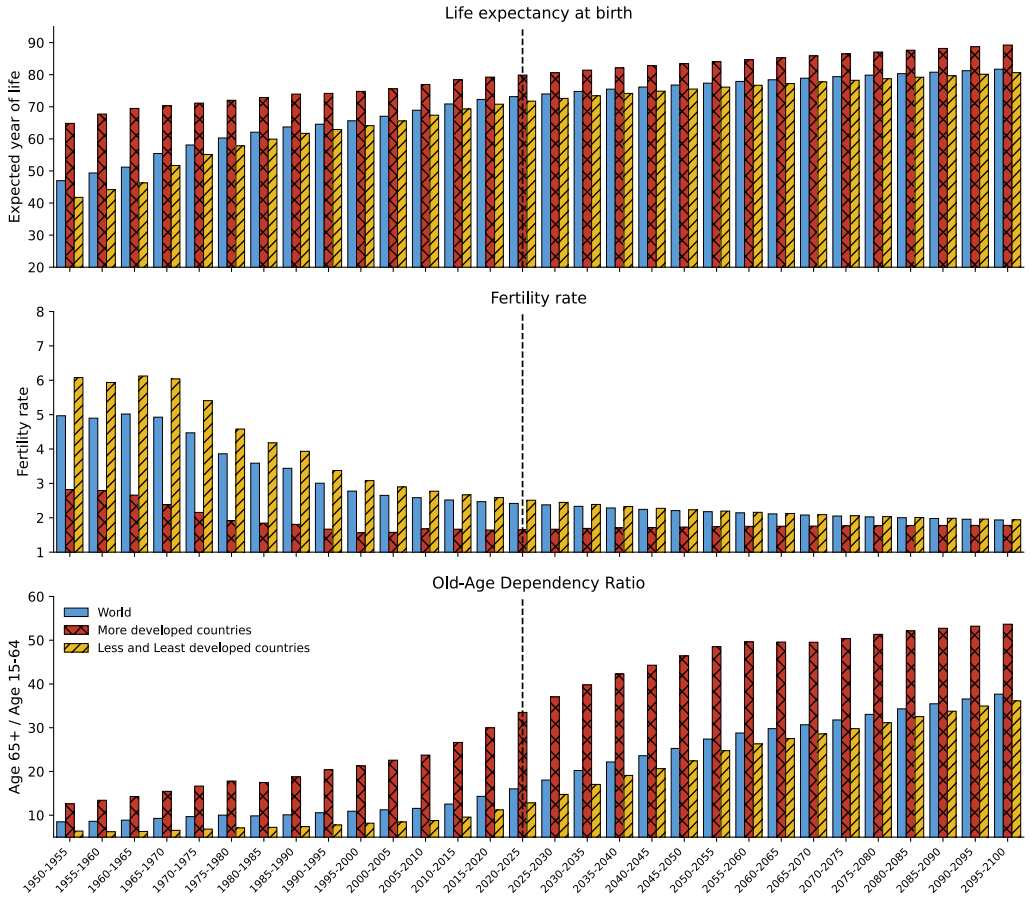


Figure 1: Life expectancy, fertility rate, and old-age dependency ratio trends.[1]

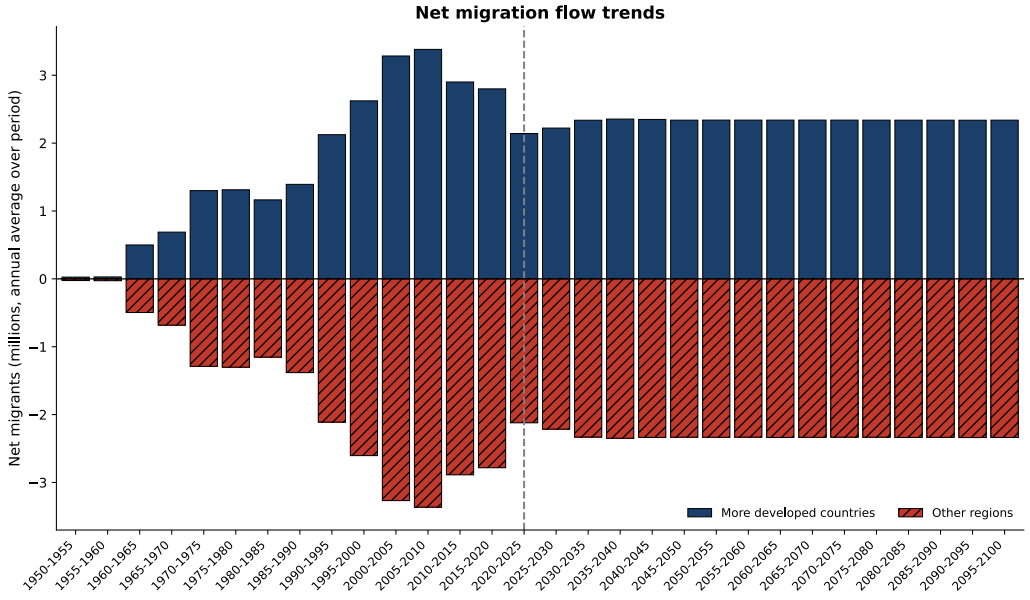


Figure 2: Net migration flow trends.[2]

$g \in \{\text{More developed}, \text{Other regions}\}$ via $M_{g,t} = r_{g,t} \times P_{g,t}/1000$, where $P_{g,t}$ is the group's average population over the period; plotting the rate itself would not produce mirror-image bars, since the two groups' populations differ by an order of magnitude.

We run two numerical experiments to establish this, the first perturbing the steady-state equilibrium to isolate the long-run, structural consequences of a permanent change in longevity, fertility, migration, or productivity for external accounts. The second complements these long-run comparisons with transitory-shock impulse responses, tracing how saving, the real interest rate, and external balances evolve as the economy returns to steady state.

Each experiment maps onto one of the three facts above. The large net creditor position of the calibrated aging region, the North, given its longer average retirement (Section 5.1), is a long-run structural outcome that the steady-state comparison across demographic profiles reads off directly. The current-account surplus coexisting with a trade deficit is likewise a property of the stationary net foreign asset position, through the accounting identity linking the two balances. The joint decline in fertility and rise in longevity alongside a falling world real rate touches both margins at once. In steady state, lower fertility lowers the real interest rate directly (Proposition 5), while higher longevity lowers it through the same excess-saving channel (proof of Proposition 4), quantified for the South's longevity catch-up in Table 5. The impulse responses then show how quickly saving, the interest rate, and external balances reach their new configuration, including the decline in the real interest rate that follows a Northern longevity shock (Fig. 12).

Quantitatively, the calibrated model reproduces the qualitative external profile it was built to explain, a current-account surplus alongside a trade deficit, financed by a large positive net foreign asset position (Table 4). We deliberately calibrate to match only the world real rate and the South's target debt ratio, reading every other external moment, including the level of Japan's and Germany's net foreign asset positions, as an untargeted check rather than a fitted target. Section 5.1 reports where the calibration over- or undershoots that range, a transparency choice that lets the mechanism, not a tuned parameter, carry the empirical burden. A decomposition of the trade-balance path (Table 7) traces most of its movement to demographic forces, chiefly slower population growth, ahead of relative productivity and migration, with longevity accounting for the remainder. Section 7.3 cross-checks this ranking against an exact Shapley allocation under an alternative shock-scaling convention, so the ordering does not rest on a single decomposition method. The experiments carry a normative message as well. Northern retirees bear the welfare cost of demographic adjustment along the transition, losing about 11.7% of permanent consumption when longevity and slower growth are fed in jointly, while a relative-productivity rise lifts both Northern cohorts along the transition. Even with all four drivers active, Northern retirees remain worse off, a loss of about 7.8%, while Northern workers turn slightly positive, so the transition redistributes across generations more than it does across regions. The model treats emigration as exogenous in the benchmark (Section 3), because observed South–North flows are policy-rationed rather than wage-clearing. Section 8 tests this choice directly by letting emigration respond to the North–South wage gap. Feeding the same shock through that wage-elastic rule (Fig. 14) confirms the exogenous specification is not doing the work, since the endogenous response damps, but never reverses, any of the external-balance effects reported above.

The paper proceeds as follows. Section 2 reviews the literature and situates our contribution. Section 3 lays out the two-country life-cycle model, Section 4 derives the analytical determinants of external balances, and Section 5 presents the calibration. Section 6 reports the long-run experiments and welfare effects on workers and retirees in both regions, and Section 7 traces the transitional dynamics of demographic and productivity shocks, including a decomposition into their driving forces and, in Appendix B, the resulting welfare effects along the transition. Section 8 reviews the model's scope and robustness, Section 9 relates these findings to the literature, and Section 10 concludes.

2 Related literature and contribution

Demography reshapes saving decisions, current-account positions, and the stock of net foreign assets, not as a background variable but as a structural force in its own right. Building on Gertler (1999)'s life-cycle framework, the paper sits at the intersection of three literatures, the demographic determinants of saving and the world interest rate, OLG models of international capital flows, and the valuation and financial-friction channels of external adjustment.

The demographic transition is asynchronous, as the North ages earlier while the South retains a younger, faster-growing labor force, redistributing world saving, capital, and factor prices (Attanasio et al., 2006, 2007). Higgins (1998) documents the age profile of saving and current accounts, and Sposi (2022) shows that cross-country demographic differences shape the direction of long-run capital flows. Within the Gertler framework, age-structure shifts move the current account (Ferrero, 2010; Kilponen et al., 2006), depress the equilibrium real rate (Carvalho et al., 2016), and account for much of its secular decline (Lisack et al., 2019; Bielecki et al., 2020; Eggertsson et al., 2019; Rachel and Summers, 2019; Papetti, 2021; Del Negro et al., 2019). A related literature links aging to lower trend growth, investment, and productivity (Aksoy et al., 2019; Basso and Jimeno, 2021; Acemoglu and Restrepo, 2017; Maestas et al., 2023; Jaimovich and Siu, 2009). What unites this work is a simple mechanism, an older population saves more relative to what it invests, pulling the world interest rate down and net foreign assets up in the fastest-aging economies, a force that can also sap global demand once monetary policy is constrained (Fornaro and Romei, 2019).

Divergent aging trajectories in OLG models can rationalize the emergence of mature economies as large net creditors (Bárány et al., 2023). The timing of demographic transitions explains much of regional current-account pressure (Domeij and Floden, 2006; Backus et al., 2014), and dependency ratios account for high Asian saving rates (Curtis et al., 2017; Cooley and Henriksen, 2018). Credit constraints that bind more tightly in fast-growing economies offer a complementary mechanism, generating net capital outflows from the developing world and a falling world interest rate even absent demographic asymmetry (Coeurdacier et al., 2015). Our model instead attributes the direction of the flow to the demographic channel of retirement length, and the two mechanisms need not be mutually exclusive. Either channel speaks to the "allocation puzzle" documented by Gourinchas and Jeanne (2013), whereby capital flows toward, rather than away from, faster-growing developing economies, the opposite of the frictionless neoclassical prediction. A demographic asymmetry in retirement length offers a complementary explanation, orthogonal to the growth-driven productivity and credit-constraint channels the allocation-puzzle literature emphasizes, since even absent a growth differential, an aging region's longer retirement horizon alone can direct capital flows toward it.

Government spending and pensions also shape external balances (Ghosh, 1992; Faruqee and Mühleisen, 2003; Lane, 2010), and pay-as-you-go pension reform redistributes saving across cohorts over the demographic transition (Song et al., 2015). The fiscal channel we sign in Proposition 9 is the two-country analogue of the closed-economy result that immigration eases pay-as-you-go pension financing (Storesletten, 2000). North–South asymmetries also carry consequences for capital mobility and generational welfare (Bryant, 2007; Ludwig et al., 2009; Gannon et al., 2020), and immigration may partly offset aging-driven declines in natural interest rates (Bielecki et al., 2020). Closest to our welfare exercise, Krueger and Ludwig (2007) quantify how demographic change reallocates returns to capital and wealth across cohorts and countries in a multi-region OLG model, a question our welfare decomposition (Theorem 1) also addresses analytically. Ludwig et al. (2009) extend this to a broader multi-region setting without our analytical characterization, and Niemeläinen (2021) studies a two-country life-cycle model of external imbalances between China and the United States close in spirit to ours. Relative to that application, our contribution is the closed-form comparative-

static characterization of every channel (Propositions 4 to 7), with explicit retiree labor-supply and annuity-market-completeness margins, rather than a numerically calibrated pairing.

Once cross-border positions grow large, the current account and the trade balance can diverge, because the former also records net income on the net foreign asset position (Obstfeld and Rogoff, 1996; Lane and Milesi-Ferretti, 2001). This channel matters quantitatively for creditor economies (Gourinchas and Rey, 2007), and valuation and net-income effects are a first-order determinant of who holds a country's external liabilities (Forbes, 2010). Financial frictions are themselves a source of external imbalances, since in a fast-growing but financially underdeveloped economy, domestic saving is channeled abroad and surfaces as an external surplus (Song et al., 2011), close to the financial-integration channel we formalize for the South. We follow Schmitt-Grohé and Uribe (2003) in modeling this with a debt-elastic interest-rate premium, which pins down a well-defined steady-state asset position and tractably represents the frictions separating the two regions.

Against this backdrop, four contributions distinguish the present analysis. Analytically, Propositions 4 to 7 sign every demographic and financial channel of the net foreign asset position, current account, and trade balance in closed form, rather than off simulations alone. Corollary 3 then checks the resulting sign pattern, confirming that a current-account surplus coexisting with a trade deficit is an accounting property of a large creditor position, not an anomaly. In modeling, we embed annuity markets (Hansen and İmrohoroğlu, 2008) and labor-supplying retirees, making retiree saving a primitive responsive to longevity and to annuity-market completeness, a parameter from zero to one nesting the perfect- and no-annuity economies as polar cases, a margin absent from three-period setups. We also replace the recentered, potentially sign-switching risk premium common in the small-open-economy literature with a sign-definite, cost-based intermediation friction. Proposition 7 and Corollary 4 show the wedge is a resource cost that never turns into a subsidy, letting the friction cleanly reorder capital's international allocation. Quantitatively, we derive a closed-form welfare decomposition that holds separately for workers and retirees (Theorem 1), scoring each shock on both external balances and consumption-equivalent welfare across steady states and along the perfect-foresight transition. This transitional measure isolates the welfare borne by cohorts alive when a shock arrives and shows Northern retirees bear more of this adjustment cost than workers, a between-cohort statement, not a claim about which workers are closest to retirement, since the perpetual-youth structure carries no age index within either group. Finally, the calibrated North, which targets only the world real rate and the South's own debt ratio, reproduces the qualitative untargeted external profile of today's aging creditors, a current-account surplus coexisting with a trade deficit on a positive net foreign asset position. Under Table 3 the recomputed magnitude, 20.55% of GDP, sits below the roughly 80% positions of Japan and Germany (Table 4) by construction, since the calibration is disciplined to match the sign and mechanism of the creditor position rather than its level, so a smaller untargeted magnitude is the expected consequence of a deliberately parsimonious calibration rather than a shortfall in fit. A decomposition of the trade-balance path (Table 7) attributes most of its movement to demographics, chiefly slower population growth, ahead of relative productivity and migration, confirming the analytical ordering with numbers the calibration never sought to match.[3]

Our environment builds directly on Gertler (1999). Carvalho et al. (2016) use the same continuous-survival structure in a single closed economy to study the world interest rate. We instead place two such economies in open-economy general equilibrium and characterize the resulting cross-border allocation in closed form. The closest paper in the broader literature is Auclert et al. (2021), who use a sufficient-statistics approach and population forecasts for 25 countries to project how aging lowers the return to wealth and widens global imbalances over the

[3]Tables 4 and 7 have both been recomputed directly under Table 3 ($\mu^\star = 0.65 > \mu = 0.55$, Northern creditor; Sections 5.1, 5.2 and 7.3).

twenty-first century. We instead offer a fully closed-form two-country model with every channel signed analytically (Propositions 4 to 7) rather than backed out as a sufficient statistic, with retiree labor supply, annuity-market completeness, and pay-as-you-go pensions as primitives rather than reduced-form wealth accumulation. The closest open-economy studies with explicit life-cycle structure, Ferrero (2010) and Bárány et al. (2023), show demographic asymmetries move the current account and steer capital toward slower-aging regions, while Backus et al. (2014) document the low-frequency demographic component of capital flows in an accounting framework. Relative to their stylized setups, we add an active retiree saving margin with adjustable annuity completeness, a sign-definite financial friction, and an explicit welfare metric, signed analytically. The model isolates the demographic forces that jointly sign the direction of capital flows at our calibration, namely longer Northern retirements together with the North–South gap in retiree labor-force participation, set to reinforce rather than offset the former (Result 1), from the financial-friction force, which reinforces rather than creates the North's resulting creditor position.

3 Model economy

The analysis rests on a two-country life-cycle model in which demographic change and international migration jointly determine the world's pattern of external imbalances. The world economy comprises two regions, the North and South, representing advanced and developing economies with different demographic dynamics. Each is populated by households (workers and retirees), firms, and a government. Households consume a single, costlessly traded final good, save, and allocate time between work and leisure. Retirees are not idle by assumption, remaining in the labor force, albeit at lower effort than active workers. Individuals enter as workers and move stochastically through a worker-retiree-death life cycle (Blanchard, 1985; Weil, 1989; Gertler, 1999), facing a drop in labor income at retirement, to a fraction of the worker wage supplemented by a government pension, and, once retired, an uncertain time of death.[4] Governments finance an exogenous stream of public spending and pension obligations through debt and labor taxation, and workers may migrate from South to North. This section presents the North's economy, with asterisks denoting Southern variables.

3.1 Demographic structure

Each country is at time t populated by workers N_t^w and retirees N_t^r.[5] Workers may migrate from South to North, but not conversely. Conditional on being a worker in period $t-1$, an individual remains in the working stage with probability ω_t in period t and retires otherwise. Once retired, it survives to period t with probability γ_t and dies otherwise. Both transition probabilities are age-independent, which makes aggregation of each group's decisions tractable (Blanchard, 1985; Weil, 1989).

In each period, $(1-\omega_t+n_t^w)N_{t-1}^w$ new workers enter the labor market and $\omega_t N_{t-1}^w$ incumbent workers remain active. Adding the immigrant workers $M_t^{w^\star}$ arriving from the South, the total number of workers in the North evolves as

$$N_t^w = (1-\omega_t+n_t^w)N_{t-1}^w + \omega_t\, N_{t-1}^w + M_t^{w^\star} = \hat{n}_t^w\, N_{t-1}^w, \tag{1}$$

where $\hat{n}_t^w$ denotes the North's gross labor-force growth rate. The total number of workers in the

[4] We neutralize this survival risk, which would otherwise complicate aggregation, via a market for annuities (Hansen and İmrohoroğlu, 2008).

[5] The period of youth preceding labor-market entry is not modeled.

South follows similarly,

$$N_t^{w^\star} = (1 - \omega_t^\star + n_t^{w^\star}) N_{t-1}^{w^\star} + \omega_t^\star \, N_{t-1}^{w^\star} - M_t^{w^\star} = \hat{n}_t^{w^\star} \, N_{t-1}^{w^\star},$$

where $M_t^{w^\star} = m_t^{w^\star} (1 + n_t^{w^\star}) N_{t-1}^{w^\star}$ is the number of emigrant workers going to the North, and $m_t^{w^\star}$, $\hat{n}_t^{w^\star} = (1 - m_t^{w^\star})(1 + n_t^{w^\star})$, and $\omega_t^\star$ denote, respectively, the Southern emigration rate, the growth rate of Southern workers between periods $t-1$ and t, and the probability of remaining in the working stage in period t.[6]

The total number of retirees as of period t is the sum of newly retired workers $(1 - \omega_t) N_{t-1}^w$ and surviving retirees $\gamma_t N_{t-1}^r$, with the law of motion

$$N_t^r = (1 - \omega_t) N_{t-1}^w + \gamma_t N_{t-1}^r.$$

The old-age dependency ratio ψ_t, defined as the time-t number of retirees per worker, $\psi_t = N_t^r / N_t^w$, evolves as

$$\psi_t = \frac{1 - \omega_t}{\hat{n}_t^w} + \frac{\gamma_t}{\hat{n}_t^w} \psi_{t-1}. \tag{2}$$

Setting $\omega_t = \omega$, $\hat{n}_t^w = \hat{n}^w$, and $\gamma_t = \gamma$ at all t and imposing $\psi_t = \psi_{t-1} = \psi$ in (2) yields the steady-state old-age dependency ratio $\psi = (1 - \omega)/(\hat{n}^w - \gamma) > 0$.

Total Northern population, $N_t = N_t^w + N_t^r$, grows at $N_t = \hat{n}_t \, N_{t-1}$, where $\hat{n}_t = (1 + \psi_t)\hat{n}_t^w / (1 + \psi_{t-1})$ is the gross population growth rate. The South's demographic evolution follows the same expressions, absent immigration. World population Pop_t sums the two countries' populations,

$$\text{Pop}_t = N_t + N_t^\star, \tag{3}$$

while world working population Pop_t^w sums North's and South's total worker populations, N_t^w and $N_t^{w^\star}$:

$$\text{Pop}_t^w = N_t^w + N_t^{w^\star}.$$

Because each region's population share evolves with its own demographics, the two regions' relative size is itself time-varying. Let $\kappa_t = N_t / \text{Pop}_t$ denote the North's share of world population and $\kappa_t^w = N_t^w / \text{Pop}_t^w$ its share of world working population, with Southern counterparts $(1 - \kappa_t)$ and $(1 - \kappa_t^w)$. Their laws of motion are, respectively,

$$\kappa_t = \frac{\hat{n}_t \, \kappa_{t-1}}{\hat{n}_t \, \kappa_{t-1} + \hat{n}_t^\star \, (1 - \kappa_{t-1})}, \tag{4}$$

$$\kappa_t^w = \frac{\hat{n}_t^w \, (1 + \psi_{t-1}^\star)}{\hat{n}_t^w \, (1 + \psi_{t-1}^\star) + \hat{n}_t^{w^\star} (1 + \psi_{t-1}) \left(\dfrac{1 - \kappa_{t-1}}{\kappa_{t-1}} \right)}, \tag{5}$$

where the North's gross working-population growth rate is given by

$$\hat{n}_t^w = (1 + n_t^w) + m_t^{w^\star} (1 + n_t^{w^\star}) \left[\frac{1 - \kappa_{t-1}^w}{\kappa_{t-1}^w} \right], \tag{6}$$

as derived in Appendix C.1.1. We treat the emigration rate $m_t^{w^\star}$ as exogenous rather than tied to relative wages, because observed South–North flows are typically rationed by the North's immigration policy and respond to push factors (conflict, instability, climate displacement) largely

[6] An alternative quota-based specification indexed to the North's own labor force gives $\hat{n}_t^w = (1 + m_t)(1 + n_t^w)$ instead. We adopt the emigration-based version above, which ties Northern inflows to Southern demographic pressure.

orthogonal to wage differentials. A further consequence, spelled out in Remark 1 below, is that the model cannot support permanently divergent labor-force growth rates across the two regions, since a stationary world working-age share requires $\hat{n}_t^w = \hat{n}_t^{w^\star}$ in the long run, so migration is pinned down by that requirement whenever the regions' demographic paths diverge on average.

Migrants enter their working life with no accumulated assets and the same productivity as native workers, so a new immigrant is indistinguishable from a newborn domestic worker. Migration is a pure labor-force-growth shock, raising the receiving region's labor supply with no cross-border wealth transfer, and regional wealth positions move only through the general-equilibrium price adjustments studied below. Age-at-arrival heterogeneity and portable wealth would add an asset-transfer channel, left for future work. With two countries, net migration rates are related to the relative share of the world working-age population and the relative growth rate of each country's working population, such that the North's immigration rate satisfies

$$m_t^w = m_t^{w^\star} \left(\frac{1 - \kappa_{t-1}^w}{\kappa_{t-1}^w} \right) \left[\frac{1 + n_t^{w^\star}}{1 + n_t^w} \right], \tag{7}$$

where m_t^w denotes the North's immigration rate as of period t, derived in Appendix C.1.2.

3.2 Households

Preferences for an individual of type $z = \{w, r\}$ (w for workers, r for retirees) take the RINCE (risk-adjusted intertemporal certainty equivalent) recursive form (Farmer, 1990), the risk-neutral special case of the Epstein–Zin recursive non-expected utility family (Epstein and Zin, 1989) in which the certainty-equivalent aggregator over the continuation value is linear rather than a power function:

$$V_t^z = \left\{ \left(d_t^z(c_t^z, \ell_t^z)\right)^\rho + \beta_{t+1}^z \left[\mathbb{E}_t \left(V_{t+1}^z | \; z\right)\right]^\rho \right\}^{1/\rho}, \tag{8}$$

where $d_t^z(.)$ denotes a consumption basket composed of final-good consumption c_t^z and leisure ℓ_t^z. Following Farmer (1990), (8) treats workers and retirees as risk-neutral toward income risk while allowing a nonzero intertemporal elasticity of substitution, governed by ρ, so consumption and saving still respond to interest-rate changes.[7] Risk neutrality toward income fluctuations rules out counterfactual excess saving by workers (Gertler, 1999; Farmer, 1990) and delivers consumption decisions linear in total wealth, keeping aggregation and quantitative simulation tractable.

The conditional expected value function $\mathbb{E}_t(V_{t+1}| \; z)$ and discount factor β^z in (8) are respectively given by

$$\mathbb{E}_t \left(V_{t+1} \mid z\right) = \begin{cases} V_{t+1}^r & \text{if } z = r, \\ \omega_{t+1} \, V_{t+1}^w + (1 - \omega_{t+1}) \, V_{t+1}^r & \text{if } z = w, \end{cases}$$

$$\beta_{t+1}^z = \begin{cases} \beta \, \gamma_{t+1} & \text{if } z = r, \\ \beta & \text{if } z = w. \end{cases}$$

Value functions are conditional on status (w or r) because workers face a potential wage-income drop at retirement and consumption-smoothing responds to that risk, and because workers and retirees face different transition possibilities.[8]

[7] The income effect on labor supply also depends on this elasticity, allowing more realistic labor-supply responses to income variations.

[8] Agents have perfect foresight but can be surprised by unexpected shocks. Along any deterministic path V_{t+1} is known with certainty and enters the recursion without an expectation operator. The impulse responses below trace the transition following a one-time, unanticipated shock, after which agents again foresee the entire path perfectly.

Households' income comes from renting out physical capital, labor services (differing for workers and retirees), dividends from firm ownership, and, for retirees ($z = r$), bequests from the deceased.

3.2.1 Annuity markets for retirees

We assume a market for annuities exists to insure against retirement survival risk (Hansen and İmrohoroğlu, 2008; Ludwig and Vogel, 2010). This market can be imperfect, governed by the fraction of annuitized assets $\zeta \in [0, 1]$, and it pools the survival risk created by the uncertain time of death, keeping retirees' decision rules linear in wealth and aggregation tractable. Let a_t^r denote a surviving retiree's assets. Annuitization redistributes the assets of non-survivors among survivors, so a survivor's assets scale up according to

$$a_t^r + \zeta\, a_t^r \left(\frac{1 - \gamma_{t+1}}{\gamma_{t+1}} \right) = a_t^r \, \frac{\varphi_{t+1}}{\gamma_{t+1}},$$

where the annuity factor[9] is

$$\varphi_{t+1} = \gamma_{t+1} + \zeta\,(1 - \gamma_{t+1}). \tag{9}$$

The annuity market is incomplete if $\zeta \in (0, 1)$ or complete if $\zeta = 1$ (so $\varphi_{t+1} = 1$), in which case the deceased retirees' assets are distributed uniformly among survivors (Yaari, 1965; Blanchard, 1985).

The incomplete market ($\zeta \in (0, 1)$) implies unintended bequests from deceased retirees. We assume the government transfers the accidental bequests left by the $(1 - \gamma_t)N_{t-1}^r$ retirees who die between $t - 1$ and t to working-age households, rather than redistributing them among surviving retirees. The aggregate accidental bequest is therefore

$$BQ_t^r \coloneqq (1 - \zeta)(1 - \gamma_t)\, R_{t-1}\, N_{t-1}^r\, a_{t-1}^r,$$

where $R_t = (1 + r_t)$ denotes the gross rate of return, N_{t-1}^r is the number of retirees and a_{t-1}^r their per-capita financial wealth, so that $A_{t-1}^r = N_{t-1}^r\, a_{t-1}^r$ is aggregate retiree financial wealth. Distributed uniformly across the N_t^w workers, each worker receives the lump-sum transfer

$$bq_t^r = \frac{BQ_t^r}{N_t^w} = (1 - \zeta)(1 - \gamma_t)\, R_{t-1}\, \frac{\psi_{t-1}}{\hat{n}_t^w}\, a_{t-1}^r,$$

where $\psi_{t-1} = N_{t-1}^r / N_{t-1}^w$ is the old-age dependency ratio and $\hat{n}_t^w = N_t^w / N_{t-1}^w$ the gross labor-force growth factor. In the symmetric benchmark calibration we set $\zeta = 1$, so the annuity market is complete, $\varphi_t = 1$, and accidental bequests vanish ($BQ_t^r = 0$), as in Gertler (1999). The general case $\zeta < 1$ is retained throughout the analytical results and activated in the non-symmetric (alternative) calibration of Section 5, where ζ and $\zeta^\star$ take literature-based, region-specific values so workers receive a nonzero flow of accidental bequests.

3.2.2 The retiree's problem

Retirees may work, which provides additional income and reduces the risk of depleting financial resources. A representative retiree supplies l_t^r units of labor each period t, translating into $\mu\, l_t^r$ efficiency units, where $\mu \in [0, 1]$ is retirees' labor productivity relative to workers ($\mu = 0$ meaning retirees do not work).

[9] A survivor keeps a_t^r and collects a fraction ζ of the $(1-\gamma_{t+1})$ non-survivors' assets pooled across the γ_{t+1} survivors; since a_t^r cancels, this is a definitional identity, not a law of motion. Following Hansen and İmrohoroğlu (2008), the gross-up $\varphi_{t+1}/\gamma_{t+1}$ multiplies the gross return on retiree assets in the budget constraint below.

In period t, a retiree chooses consumption bundle d_t^r, consumption c_t^r, leisure ℓ_t^r, and assets a_t^r to solve

$$V_t{}^r = \max_{\{d_t^r,\, a_t^r,\, \ell_t^r,\, c_t^r\}} \left\{ (d_t^r)^\rho + \beta\, \gamma_{t+1}\, [V_{t+1}^r]^\rho \right\}^{1/\rho}$$

subject to

$$a_t^r = \frac{\varphi_t R_{t-1}}{\gamma_t}\, a_{t-1}^r + (1-\tau_t)\mu\, w_t + E_t^r + \pi_t^r - P_t^r\, d_t^r \tag{10}$$

$$d_t^r = \left(c_t^r\right)^\nu \left(X_t\, \ell_t^r\right)^{1-\nu}, \tag{11}$$

$$P_t^r\, d_t^r = c_t^r + (1-\tau_t)\mu\, w_t\, \ell_t^r, \tag{12}$$

$$\ell_t^r = 1 - l_t^r, \tag{13}$$

where budget constraint (10) is written in composite form. The retiree is credited with the full time-endowment value $(1-\tau_t)\mu\, w_t$ and spends $P_t^r\, d_t^r$ on the composite basket, whose price already embeds the value of retained leisure.[10] The remaining resources are the gross annuitized return on financial wealth $R_{t-1}\varphi_t/\gamma_t$, pensions E_t^r, and firm dividends π_t^r, and a terminal condition precludes unpaid debts. The retiree wage is a fraction of the worker wage, $w_t^r = \mu\, w_t$, a policy margin capturing retirees' labor-market participation under aging or pension reforms, with τ_t taxing their labor income. Equation (11) makes d_t^r a Cobb–Douglas aggregate of goods and leisure, X_t labor-augmenting technology.[11] Equation (12) defines P_t^r as the minimal unit cost of the basket, leisure priced at the after-tax retiree wage. This is an expenditure-duality identity, not a second budget constraint, pinning down the goods–leisure split.[12] Finally, (13) splits the retiree's unit time endowment between work l_t^r and leisure ℓ_t^r.

Appendix C.2.1 derives the following intratemporal choices for consumption and leisure:

$$c_t^r = \nu\, P_t^r\, d_t^r \tag{14}$$

$$\ell_t^r = (1-\nu)\, \frac{P_t^r d_t^r}{(1-\tau_t)\mu\, w_t}, \tag{15}$$

where

$$P_t^r = \frac{\mu^{1-\nu}}{\nu^\nu\, (1-\nu)^{1-\nu}} \left[\frac{(1-\tau_t)w_t}{X_t}\right]^{1-\nu}. \tag{16}$$

These intratemporal choices are static, within-period optimality conditions involving no expectations of future variables, unlike the intertemporal margin considered next. Intertemporal choices for d_t^r and a_{t+1}^r imply the following, equivalent to the standard Euler equation from the infinitely-lived-agent model:

$$d_{t+1}^r = \beta^\sigma\, (\varphi_{t+1} R_t)^\sigma \left(\frac{P_{t+1}^r}{P_t^r}\right)^{-\sigma} d_t^r \tag{17}$$

where $\sigma = (1-\rho)^{-1}$ is the intertemporal elasticity of substitution.[13]

[10] This follows from substituting the expenditure identity (12) and the time constraint (13) into the primitive budget, since $(1-\tau_t)\mu\, w_t\, l_t^r - c_t^r = (1-\tau_t)\mu\, w_t - P_t^r\, d_t^r$.

[11] With long-run exogenous growth, the marginal utility of leisure must increase at the same rate as labor productivity for a long-run equilibrium to exist for ℓ_t^r and ℓ_t^w.

[12] The consumption good c_t^r is the numeraire, so all prices, including P_t^r, are in consumption-good units.

[13] If retirees do not work, (14), (15), (16), and (17) simplify to $c_t^r = d_t^r$, $\ell_t^r = 1$, $P_t^r = 1$, $d_{t+1}^r = \beta^\sigma (\varphi_{t+1} R_t)^\sigma d_t^r$.

Conjecture 1 (Guess-and-verify: retiree consumption rule)**.** We conjecture that a retiree's composite consumption expenditure is a fraction of total wealth:

$$P_t^r\, d_t^r = \xi_t^r \left(\frac{\varphi_t\, R_{t-1}}{\gamma_t}\, a_{t-1}^r + h_t^r + s_t^r \right), \tag{18}$$

where ξ_t^r denotes the retiree's marginal propensity to consume out of wealth and h_t^r and s_t^r represent the present discounted values of human wealth and future social security benefits, respectively.[14]

This is a conjecture about the functional form of the retiree's decision rule, verified rather than merely posited by the guess-and-verify argument that follows. Substituting (10), (9), (14), and (18) into (17), the law of motion for the retiree's marginal propensity to consume is

$$\xi_{t+1}^r = \gamma_{t+1}\, \beta^\sigma \left(\varphi_{t+1} R_t\right)^{\sigma-1} \left(\frac{P_{t+1}^r}{P_t^r} \right)^{1-\sigma} \frac{\xi_t^r}{1-\xi_t^r} \qquad \text{if } \mu \in (0,1]. \tag{19}$$

The retiree's marginal propensity to spend thus varies with the interest rate, life expectancy (γ_t), and the path of the retiree's consumption-bundle price.[15] The lifetime discounted values of human wealth and social security benefits follow the recursive structure below. Because leisure is valued at the after-tax wage in (12), capitalizing the budget constraint after substituting out c_t^r makes human wealth capitalize the retiree's full time endowment $(1-\tau_t)\mu w_t$ rather than realized labor income:

$$h_t^r = (1-\tau_t)\mu\, w_t + \pi_t^r + \frac{\gamma_{t+1}}{\varphi_{t+1} R_t}\, h_{t+1}^r \qquad \text{if } \mu \in (0,1] \tag{20}$$

$$s_t^r = E_t^r + \sum_{i=1}^{\infty} \frac{\left(\prod_{k=1}^{i} \gamma_{t+k}\right) E_{t+i}^r}{\prod_{k=1}^{i} \varphi_{t+k} R_{t+k-1}} = E_t^r + \frac{\gamma_{t+1}}{\varphi_{t+1} R_t}\, s_{t+1}^r. \tag{21}$$

The last object to derive is the retiree's value function V_t^r. Conjecturing $V_t^r = \Delta_t^r\, d_t^r$, it is straightforward to show $\Delta_t^r = (\xi_t^r)^{\frac{\sigma}{1-\sigma}}$ (Appendix C.2.1), which closes the guess-and-verify argument. The conjectured linear rules (18) and $V_t^r = \Delta_t^r d_t^r$ satisfy the retiree's Euler and Bellman equations for every admissible (ξ_t^r, Δ_t^r) pair generated by (19), confirming rather than assuming the functional form.

3.2.3 The worker's problem

Workers start life with zero assets, and because a typical worker may become a retiree next period, the continuation value is shaped by the transition probability ω_t. A worker chooses a consumption bundle d_t^w (leisure ℓ_t^w, final-good consumption c_t^w) and assets a_t^w to solve

$$V_t^w = \max_{\left\{d_t^w,\, a_{t+1}^w,\, \ell_t^w,\, c_t^w\right\}} \left\{ (d_t^w)^\rho + \beta \left[\omega_{t+1}\, V_{t+1}^w + (1-\omega_{t+1})\, V_{t+1}^r \right]^\rho \right\}^{1/\rho} \tag{22}$$

subject to

$$a_t^w = R_{t-1}\, a_{t-1}^w + (1-\tau_t) w_t + b q_t^r + \pi_t^w - P_t^w\, d_t^w \tag{23}$$

$$d_t^w = \left(c_t^w\right)^\nu \left(X_t\, \ell_t^w\right)^{1-\nu} \tag{24}$$

$$P_t^w\, d_t^w = c_t^w + (1-\tau_t) w_t\, \ell_t^w \tag{25}$$

$$l_t^w = 1 - \ell_t^w,$$

[14] We state the conjecture on composite expenditure $P_t^r d_t^r$ rather than goods consumption, keeping the marginal propensity ξ_t^r free of ν in the dynamics below.

[15] If retirees do not work ($\mu = 0$), (19) simplifies to

$$\xi_{t+1}^r = \gamma_{t+1} \beta^\sigma \left(\varphi_{t+1} R_t\right)^{\sigma-1} \frac{\xi_t^r}{1-\xi_t^r},$$

while retiree human wealth vanishes, $h_t^r = 0$.

The worker's problem mirrors the retiree's: (23) is in composite form, and (25) defines the composite price P_t^w through the same expenditure-duality identity. Here π_t^w is the dividend transferred by intermediate-goods producers to workers, and bq_t^r the lump-sum accidental-bequest transfer from the government, financed by the assets of retirees who died between $t-1$ and t. Solving the worker's problem yields the intratemporal choices for consumption and leisure:

$$c_t^w = \nu\, P_t^w\, d_t^w, \tag{26}$$

$$\ell_t^w = (1-\nu)\,\frac{P_t^w\, d_t^w}{(1-\tau_t)w_t},$$

$$P_t^w = \frac{1}{\nu^\nu\,(1-\nu)^{1-\nu}}\left[\frac{(1-\tau_t)w_t}{X_t}\right]^{1-\nu}, \tag{27}$$

where P_t^w again denotes the price of the worker's composite basket d_t^w in units of the consumption good c_t^w, the numeraire.[16] The first-order conditions for d_t^w and a_{t+1}^w, the envelope theorem, and the conjecture that V_t^w is linear in composite consumption ($V_t^w = \Delta_t^w\, d_t^w$) deliver the workers' Euler equation:

$$\omega_{t+1}\, d_{t+1}^w + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right) d_{t+1}^r = \left(\beta\, R_t\, \Omega_{t+1}\right)^\sigma \left(\frac{P_{t+1}^w}{P_t^w}\right)^{-\sigma} d_t^w, \tag{28}$$

where

$$\Omega_{t+1} = \omega_{t+1} + (1-\omega_{t+1})\left(\frac{P_{t+1}^w}{P_{t+1}^r}\right)\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)^{\frac{1}{\sigma}}$$

is a weighting factor on the gross return R_t, acting as a supplementary discount premium.

As for retirees, the same guess-and-verify argument shows workers' composite expenditure is a constant fraction of total wealth:

$$P_t^w\, d_t^w = \xi_t^w\left(R_{t-1}\, a_{t-1}^w + h_t^w + s_t^w\right), \tag{29}$$

where ξ_t^w is the worker's marginal propensity to consume out of wealth, and h_t^w and s_t^w the present discounted values of human wealth and social security benefits.[17] The accidental-bequest transfer bq_t^r does not enter (29) directly because, like labor income and dividends, it is capitalized inside human wealth. Defined recursively below, h_t^w embeds the entire discounted stream of current and future bequests through h_{t+1}^w.[18]

Equations (14), (26), (23), and (28) give the representative worker's marginal-propensity-to-consume law of motion,

$$\xi_{t+1}^w = \beta^\sigma\left(\Omega_{t+1}\, R_t\right)^{\sigma-1}\left(\frac{P_{t+1}^w}{P_t^w}\right)^{1-\sigma}\frac{\xi_t^w}{1-\xi_t^w}$$

while h_t^w and s_t^w follow the recursions

$$h_t^w = (1-\tau_t)\, w_t + \pi_t^w + bq_t^r + \frac{\omega_{t+1} h_{t+1}^w + (\Omega_{t+1}-\omega_{t+1})\, h_{t+1}^r}{\Omega_{t+1}\, R_t} \qquad \text{if } \mu \in (0,1] \tag{30}$$

$$s_t^w = \frac{\omega_{t+1} s_{t+1}^w + (\Omega_{t+1}-\omega_{t+1})\, s_{t+1}^r}{\Omega_{t+1}\, R_t}. \tag{31}$$

Using $V_t^w = \Delta_t^w\, d_t^w$ and (22), it is straightforward to show that $\Delta_t^w = \left(\xi_t^w\right)^{\frac{\sigma}{1-\sigma}}$.

[16] Equation (27) and $w_t^r = \mu\, w_t$ together give $P_t^r = \mu^{1-\nu}\, P_t^w$.

[17] As for retirees, the conjecture is stated on composite expenditure $P_t^w d_t^w$ rather than goods consumption, keeping ξ_t^w free of ν; it is confirmed, not assumed, by substitution into the worker's Euler equation (28).

[18] If retirees do not work, the worker's human-wealth recursion simplifies to $h_t^w = (1-\tau_t)\, w_t + \pi_t^w + bq_t^r + \frac{\omega_{t+1} h_{t+1}^w}{\Omega_{t+1}\, R_t}$.

3.2.4 Aggregation and the wealth distribution

Because consumption is linear and marginal propensities to consume out of wealth are common within each type, aggregate goods consumption of retirees and workers is $C_t^r = \nu\, P_t^r\, D_t^r$ and $C_t^w = \nu\, P_t^w\, D_t^w$, where D_t^z, $z = w, r$, denotes aggregate composite consumption, obtained by summing $c_t^z = \nu\, P_t^z d_t^z$ across type-z households (P_t^z common to all). Total goods consumption is then

$$C_t = C_t^r + C_t^w.$$

Summing equations (20) and (21) gives retirees' aggregate discounted human wealth H_t^r and aggregate pension benefits S_t^r:

$$\begin{aligned} H_t^r &= (1-\tau_t)\mu\, w_t\, N_t^r + \Pi_t^r + \frac{\gamma_{t+1}\,\psi_t\, H_{t+1}^r}{\hat{n}_{t+1}^w\,\psi_{t+1}\,\varphi_{t+1}\,R_t} \qquad \text{if } \mu \in (0,1] \\ S_t^r &= \Gamma_t^r + \frac{\gamma_{t+1}\,\psi_t\, S_{t+1}^r}{\hat{n}_{t+1}^w\,\psi_{t+1}\,\varphi_{t+1}\,R_t}, \end{aligned} \tag{32}$$

where the labor-income term capitalizes the full time endowment of the N_t^r retirees, mirroring (20); $\Gamma_t^r = N_t^r\, E_t^r$ is aggregate retiree pension benefits and $\Pi_t^r = N_t^r\, \pi_t^r$ aggregate retiree dividends, and L_t^r retirees' aggregate labor supply, entering aggregate effective labor on the production side.

Summing (30) and (31) across workers similarly gives their aggregate discounted pension benefits S_t^w and human wealth H_t^w:

$$\begin{aligned} H_t^w &= (1-\tau_t)\, w_t\, N_t^w + \Pi_t^w + BQ_t^r + \frac{\omega_{t+1} H_{t+1}^w}{\hat{n}_{t+1}^w\,\Omega_{t+1}\,R_t} + \frac{(\Omega_{t+1}-\omega_{t+1})\, H_{t+1}^r}{\hat{n}_{t+1}^w\,\psi_{t+1}\,\Omega_{t+1}\,R_t} \qquad \text{if } \mu \in (0,1] \\ S_t^w &= \frac{\omega_{t+1} S_{t+1}^w}{\hat{n}_{t+1}^w\,\Omega_{t+1}\,R_t} + \frac{(\Omega_{t+1}-\omega_{t+1})\, S_{t+1}^r}{\hat{n}_{t+1}^w\,\psi_{t+1}\,\Omega_{t+1}\,R_t}, \end{aligned} \tag{33}$$

where $\Pi_t^w = \pi_t^w\, N_t^w$ denotes aggregate dividends and $L_t^w = N_t^w\, l_t^w$ aggregate labor supply for workers; as for retirees, the labor-income term in (33) capitalizes the full time endowment of the N_t^w workers.[19]

Aggregate assets owned by workers evolve with the total savings of the fraction ω_t who remain workers between $t-1$ and t:

$$A_t^w = \omega_t\Big(R_{t-1}\, A_{t-1}^w + (1-\tau_t)\, w_t\, N_t^w + BQ_t^r + \Pi_t^w - \xi_t^w\big(R_{t-1}\, A_{t-1}^w + H_t^w + S_t^w\big)\Big). \tag{34}$$

Aggregate retiree financial wealth equals the total savings of those already retired at t plus the savings of workers who retired between $t-1$ and t:

$$\begin{aligned} A_t^r =& \varphi_t\, R_{t-1}\, A_{t-1}^r + (1-\tau_t)\mu\, w_t\, N_t^r + \Gamma_t^r + \Pi_t^r - \xi_t^r\big(\varphi_t\, R_{t-1}\, A_{t-1}^r + H_t^r + S_t^r\big) \\ &+ (1-\omega_t)\Big(R_{t-1}\, A_{t-1}^w + (1-\tau_t) w_t\, N_t^w + BQ_t^r + \Pi_t^w - \xi_t^w\big(R_{t-1}\, A_{t-1}^w + H_t^w + S_t^w\big)\Big). \end{aligned} \tag{35}$$

Both (34) and (35) are composite individual budgets summed across households, with each type credited with the full value of its time endowment and spends its aggregate composite expenditure $P_t^z\, D_t^z$. Defining $\lambda_t = A_t^r / A_t$ as retirees' share of total wealth, with $(1-\lambda_t)$ owned by workers, combining (34) and (35) gives the law of motion for λ_t:

$$\begin{aligned} \lambda_t A_t = \omega_t\Big[&(1-\xi_t^r)\varphi_t\, R_{t-1}\lambda_{t-1} A_{t-1} + (1-\tau_t)\mu\, w_t\, N_t^r + \Gamma_t^r + \Pi_t^r\Big] \\ &- \omega_t\, \xi_t^r\big(H_t^r + S_t^r\big) + (1-\omega_t) A_t. \end{aligned}$$

[19] If retirees do not work, (32) and (33) simplify to $H_t^r = 0$ and $H_t^w = (1-\tau_t)\, w_t\, N_t^w + \Pi_t^w + BQ_t^r + \frac{\omega_{t+1} H_{t+1}^w}{\hat{n}_{t+1}^w\,\Omega_{t+1}\,R_t}$.

3.3 Firms

A representative firm produces the homogeneous final good with a Cobb–Douglas technology,

$$Y_t = K_{t-1}^{1-\alpha} \left(X_t \, L_t\right)^{\alpha}, \tag{36}$$

where K_{t-1} is the aggregate capital stock, L_t aggregate effective labor (a weighted sum of labor supplied by workers and retirees, imperfect substitutes since retirees may work fewer hours), X_t labor-augmenting technology, and α the labor share absent any wedge. X_t grows exogenously as $X_t = \hat{x}_t \, X_{t-1}$, where $\hat{x}_t$ follows the AR(1) process $\hat{x}_t = (1-\rho_x)\,\hat{x}_{ss} + \rho_x \, \hat{x}_{t-1} + \epsilon_t^x$, with $\hat{x}_{ss}$ its steady-state value and ρ_x its persistence parameter, defined with the other shock persistences in Section 3.7 below.

Factor prices are subject to a constant wedge $\theta > 1$ between the value of each factor's marginal product and its market price,

$$w_t = \alpha \left(\frac{\theta - 1}{\theta}\right) \frac{Y_t}{L_t}, \qquad\qquad r_t^k = (1-\alpha) \left(\frac{\theta - 1}{\theta}\right) \frac{Y_t}{K_{t-1}}, \tag{37}$$

where w_t is the real wage and r_t^k the real rental cost of capital paid to households, so the labor share is $s_L = w_t L_t / Y_t = \alpha(\theta-1)/\theta$. The residual $\pi_t = Y_t/\theta$ accrues to households as profit. Following Bielecki et al. (2020), profits are transferred to retirees and workers in proportion to labor income, so dividends are $\pi_t^r = \pi_t(\mu \, l_t^r)/L_t$ and $\pi_t^w = \pi_t \, l_t^w / L_t$.[20] Setting $\theta \to \infty$ recovers the perfectly competitive, unit-markup benchmark ($s_L = \alpha$, $\pi_t = 0$).[21]

3.4 Investment funds and capital accumulation

Saving is intermediated by perfectly competitive, risk-neutral investment funds (Fujita and Fujiwara, 2016; Schön and Stähler, 2020). Each period the representative fund collects household saving a_t, repays the gross return $R_{t-1} \, a_{t-1}$ on past savings, owns the physical capital stock k_t, which it rents to intermediate-goods firms at rate r_t^k, and finances new investment i_t. Its net cash flow, rebated to households, is

$$d_t^f = a_t - R_{t-1} \, a_{t-1} + r_t^k \, k_t - i_t.$$

The fund chooses $\{i_t, \, k_{t+1}\}_{t \geq 0}$ to maximize the expected present value of net cash flows,

$$\max \; \mathbb{E}_t \sum_{s \geq 0} \Lambda_{t,t+s} \, d_{t+s}^f, \qquad \Lambda_{t,t+s} = \prod_{j=1}^{s} \frac{1}{R_{t+j-1}},$$

where the fund, being competitive and risk-neutral, discounts at the gross risk-free rate $R_t = 1 + r_t$. Following Christiano et al. (2005), capital accumulation is subject to adjustment costs:

$$k_{t+1} = (1-\delta) \, k_t + \Bigg[1 - \underbrace{\frac{\phi}{2}\left(\frac{i_t}{i_{t-1}} - \hat{n}_t \, \hat{x}_t\right)^2}_{= S_t(\cdot)}\Bigg] i_t, \tag{38}$$

[20] If retirees do not work ($\mu = 0$), they receive no dividends but still earn rent on the capital they rent to firms.

[21] The wedge θ has a standard microfoundation, omitted for brevity. Monopolistically competitive intermediate-goods producers under (36) sell differentiated varieties to a competitive final-good aggregator with demand elasticity θ, charging the constant markup $\theta/(\theta-1)$; symmetric in equilibrium, they aggregate to (36)–(37), the standard New Keynesian production block without nominal rigidities. Absent price rigidities the intermediate layer plays no role beyond the constant wedge θ; reintroducing them is left for future work.

where $\delta \in (0,1)$ is the depreciation rate and $\phi > 0$ governs the adjustment cost. The reference $\hat{n}_t\,\hat{x}_t$, the balanced-growth gross growth rate of investment, makes the bracketed term equal one and $S_t(\cdot)$ vanish in the detrended steady state. The multiplier q_t on (38) is the shadow price of installed capital (Tobin's q).

Writing gross investment growth as $\iota_t \equiv i_t/i_{t-1}$ (not to be confused with detrended government spending g_t in Section 3.5), the fund's first-order conditions deliver a no-arbitrage condition on the return to saving and the investment Euler equation,

$$R_t = (1 + r_t) = \frac{r^k_{t+1} + (1-\delta)\,q_{t+1}}{q_t}, \tag{39}$$

$$q_t\Big[1 - S_t(\cdot) - S'_t(\cdot)\,\iota_t\Big] + \frac{q_{t+1}}{R_t}\,\iota^2_{t+1}\,S'_{t+1}(\cdot) = 1, \tag{40}$$

where $S'_t(\cdot) = \phi(\iota_t - \hat{n}_t\hat{x}_t)$ is the marginal adjustment cost. Equation (39), the funds' zero-profit condition, equates the gross return on saving to the return on installed capital (rental yield plus depreciated resale value). Equation (40) governs investment dynamics through q_t. The financing of the capital stock by household saving is imposed at the aggregate level, through the asset-market clearing condition below.

3.5 Fiscal authority

The government purchases final goods in quantity G_t, pays retirees' pensions as social security benefits T^r_t, levies labor income taxes $\tau_t\,w_t\,L_t$, and issues one-period debt B_t to finance its expenditures. Its budget constraint is therefore

$$B_t = R_{t-1}\,B_{t-1} + G_t + T^r_t - \tau_t\,w_t\,L_t, \tag{41}$$

where

$$L_t = L^w_t + \mu\,L^r_t$$

is the aggregate effective labor input, reflecting the lower productivity, and thus lower wages, of retirees who work.

Since this paper focuses on population aging and international migration for external balances, we adopt a standard view of government finances in which spending, pensions, and debt are exogenous while labor taxation adjusts to balance the budget (Gertler, 1999; Carvalho et al., 2016). Specifically, $G_t = \bar{g}\,Y_t$, $T^r_t = \bar{e}\,Y_t$, and $B_t = \bar{b}\,Y_t$ each remain a constant share of output along the balanced-growth path, and labor income taxes adjust to satisfy the government's per-period and intertemporal budget constraints, the latter given by

$$R_t\,B_t = \sum_{s=0}^{\infty} \frac{\tau_{t+s}\,w_{t+s}\,L_{t+s}}{\prod_{v=1}^{s} R_{t+v}} - \sum_{s=0}^{\infty} \frac{T^r_{t+s}}{\prod_{v=1}^{s} R_{t+v}} - \sum_{s=0}^{\infty} \frac{G_{t+s}}{\prod_{v=1}^{s} R_{t+v}}. \tag{42}$$

3.6 World equilibrium

The world equilibrium is a sequence of prices and quantities such that in each country the government budget constraint (42) holds, retirees and workers maximize utility subject to their budget constraints, and firms maximize profits subject to (36) and (37). In each country, households' aggregate total assets equal the sum of outstanding government bonds, the capital stock, and the net foreign asset position, $A_t = q_t\,K_t + B_t + F_t$. Net foreign assets F_t link goods and asset markets: F_t, the economy's net stock of claims on the rest of the world, evolves with net exports as

$$F_{t+1} = R_t\,F_t + \mathrm{NX}_t, \tag{43}$$

where NX_t denotes the trade balance. The current account balance CA_t measures the change in net foreign assets,

$$\mathrm{CA}_t = F_{t+1} - F_t = (R_t - 1)F_t + \mathrm{NX}_t,$$

with the Southern mirror identity $\mathrm{CA}_t^\star = (R_t^\star - 1)F_t^\star + \mathrm{NX}_t^\star$. Finally, the national accounts identity states that

$$Y_t = C_t + I_t + G_t + \mathrm{NX}_t. \tag{44}$$

Analogous equations hold in the other country, with foreign variables denoted $Y_t^\star$, $F_t^\star$, $A_t^\star$, $R_t^\star$, $\mathrm{NX}_t^\star$, $C_t^\star$, $I_t^\star$, $G_t^\star$, etc., so the Southern national-accounts identity mirrors (44) exactly.[22] Because each country's national-accounts identity already clears its own goods market ((44), (45)), world goods-market clearing implies $\mathrm{NX}_t + \mathrm{NX}_t^\star = 0$ by Walras' law. Summing the North's current-account identity with its Southern mirror and using this condition gives the world current-account closure condition

$$\mathrm{CA}_t + \mathrm{CA}_t^\star = 0, \tag{46}$$

so the North's current-account surplus is exactly the mirror image of the South's deficit, $\mathrm{CA}_t = -\mathrm{CA}_t^\star$, at every date.[23]

Complete financial integration, $R_t = R_t^\star$, would carry important consequences for capital–labor ratios and other macro aggregates in a model of this kind. We deliberately stop short of it. Financial integration is limited by a single wedge $\Gamma_t^\star$ per unit borrowed by the South, capturing departures from uncovered interest parity (Senhadji, 1997; Ambler et al., 2004; Bielecki et al., 2020). We call $\Gamma_t^\star$ a "cost-based premium" because, although it occupies the place of a premium in the arbitrage condition, it is not a risk premium but an intermediation cost that Southern financial intermediaries charge on external borrowing and rebate lump-sum to Southern households, redistributing resources within the South rather than destroying them.[24] This wedge drives $R_t^\star$ above R_t so that

$$R_t^\star = (1 + \Gamma_t^\star)\, R_t, \tag{47}$$

where R_t is the North's gross interest rate, exogenous from the South's perspective. The premium is debt-elastic in the sense of Schmitt-Grohé and Uribe (2003), rising with the South's indebtedness relative to a target. We anchor it on the South's net-foreign-asset position $F_t^\star / Y_t^\star$ relative to a target ratio $\bar{f}^\star$, so that

$$\Gamma_t^\star = \chi_0 \exp\!\left(-\kappa_f \left(\frac{F_t^\star}{Y_t^\star} - \bar{f}^\star \right) \right), \tag{48}$$

[22] Explicitly, the Southern national-accounts identity is

$$Y_t^\star = C_t^\star + I_t^\star + G_t^\star + \mathrm{NX}_t^\star. \tag{45}$$

[23] Equation (46) restricts flows, not stocks, and does not require $F_t^\star = -F_t$ period by period. That bilateral identity holds only under complete integration, $R_t = R_t^\star$ (the symmetric benchmark, $F_t = F_t^\star = 0$). Once the premium is active ($R_t^\star > R_t$ by (47)), F_t and $F_t^\star$ accumulate independently under (43) and its Southern analogue, each capitalized at its own return, linked only through (46), since the single traded bond tracks each region's net claim as one aggregate state rather than a bilateral gross position. Proposition 7 and the welfare results of Section 6.4 should thus be read as statements about each region's own position and return, not dollar-for-dollar claims once the friction is active.

[24] Equation (48) also admits an iceberg interpretation, under which the friction destroys real resources at a rate proportional to $\Gamma_t^\star$ rather than merely reallocating them (Schmitt-Grohé and Uribe, 2003; Bielecki et al., 2020). This alternative is not used below; it changes the steady state and is left for future work.

where $\chi_0 > 0$ is the baseline premium at the target ratio $\bar{f}^\star$, and $\kappa_f > 0$ a small elasticity governing how fast the premium responds to deviations of $F_t^\star/Y_t^\star$ from target. This keeps $\Gamma_t^\star > 0$ and $R_t^\star > R_t$ for all t, so the friction is always a cost intermediaries charge on Southern paper, never a subsidy. The frictionless benchmark is recovered as $\chi_0 \to 0$.[25] All steady states and impulse responses below use this rebated, net-foreign-asset-elastic specification.

To make this reallocation explicit, consider the balance sheet of the Southern financial intermediary standing between the South and world capital markets. It funds itself by borrowing $F_t^\star$ on world markets at the North's rate R_t (the exogenous cross-border return) and lends the same amount to Southern households and firms at the domestic rate $R_t^\star = (1 + \Gamma_t^\star)R_t$ of (47). Its wedge income is

$$\Omega_t^\star \equiv (R_t^\star - R_t)\, F_t^\star = \Gamma_t^\star\, R_t\, F_t^\star, \tag{49}$$

which it rebates lump-sum to Southern households, so $\Omega_t^\star$ appears as a transfer receipt alongside the domestic-rate return $R_t^\star$ on their own asset holdings. The country's net international investment position, by contrast, accrues at the cross-border rate R_t alone. It is the intermediary's own liability, not the households' domestic claim. This is why (46) carries no residual $\Gamma_t^\star$ term even though $R_t^\star \neq R_t$. Writing (46) via the CA identities gives $(R_t - 1)F_t + (R_t^\star - 1)F_t^\star = 0$; substituting (47) gives $(R_t - 1)F_t + (R_t - 1)F_t^\star + \Gamma_t^\star R_t F_t^\star = 0$, where the wedge payment $\Gamma_t^\star R_t F_t^\star$ is exactly offset by the rebate $\Omega_t^\star$ once both regions' budget constraints are consolidated, leaving (46) as a restriction on $(F_t, F_t^\star)$ jointly rather than forcing $F_t = 0$ or $F_t^\star = -F_t$.

With non-zero technology growth $\hat{x}_t$ and population growth $\hat{n}_t$, the model admits a steady state for variables expressed in efficiency units, with a variable S_t taking the form $s_t = S_t/(X_t N_t)$, and we detrend all growing variables when solving and simulating the model. Below, a long-term effect refers to the steady state of s_t.

3.7 Exogenous driving processes

Demographic and productivity variables are the exogenous drivers of the simulations. We collect the five stochastic primitives (domestic fertility, longevity, the South's own emigration rate, productivity growth, and relative productivity) in the vector $\mathbf{e}_t = \big(n_t^w,\ \gamma_t,\ m_t^{w^\star},\ \hat{x}_t,\ \Upsilon_t\big)'$, with steady-state values $\bar{\mathbf{e}} = \big(n^w,\ \gamma,\ m^{w^\star},\ \hat{x},\ \bar{\Upsilon}\big)'$. Here $\Upsilon_t \equiv X_t/X_t^\star$ is relative productivity, the ratio of Northern to Southern labor-augmenting technology, with steady-state value $\bar{\Upsilon}$; it also governs the cross-country capital–labor ratio and external balances in the results below (see Proposition 1). They follow the first-order vector autoregression

$$\mathbf{e}_t = (\mathbf{I} - \boldsymbol{\rho})\,\bar{\mathbf{e}} + \boldsymbol{\rho}\,\mathbf{e}_{t-1} + \boldsymbol{\varepsilon}_t,$$

where $\mathbf{I}$ is the identity matrix, $\boldsymbol{\rho} = \mathrm{diag}(\rho_n, \rho_\gamma, \rho_m, \rho_x, \rho_\upsilon)$ the diagonal matrix of persistence parameters, and $\boldsymbol{\varepsilon}_t = \big(\epsilon_t^n, \epsilon_t^\gamma, \epsilon_t^m, \epsilon_t^x, \epsilon_t^\upsilon\big)'$ the vector of zero-mean innovations.

The model contains a single migration shock, though $m_t^{w^\star}$ (the South's own emigration rate) plays two distinct roles. It is the primitive AR(1) driver listed above, with the innovation ϵ_t^m perturbing $m_t^{w^\star}$ directly, and m_t elsewhere in the paper (e.g. in Section 7.3) always refers to this Southern emigration rate rather than to an independently shocked Northern object. It also mechanically determines two accounting objects, the North's gross labor-force growth factor $\hat{n}_t^w$ via identity (6) and the North's own immigration rate m_t^w via (7), neither separately shocked nor calibrated but

[25] An equivalent formulation indexes the premium to the debt-to-output ratio, $\Gamma_t^\star = \chi_0 \exp\big(B_t^\star/Y_t^\star - \bar{b}^\star\big)$ (Schmitt-Grohé and Uribe, 2003), but the fiscal rule pins $B_t^\star/Y_t^\star = \bar{b}^\star$, making it degenerate ($\Gamma_t^\star \equiv \chi_0$). We therefore anchor the premium on the net foreign asset ratio $F_t^\star/Y_t^\star$, a genuine state variable; the two share the same steady state, $\Gamma^\star = \chi_0$.

both deterministic functions, period by period, of the primitive vector $\mathbf{e}_t$ and the population shares $\kappa^w_{t-1}, \kappa^\star_{t-1}$. Every result in Section 4.4 referring to "a change in migration $dm^{w^\star}$" or "an immigration shock" should accordingly be read as a shock to the single Southern primitive $m^{w^\star}_t$, propagating mechanically into $\hat{n}^w_t$ and m^w_t through these two identities rather than as two independent migration shocks hitting the two regions. The probability of remaining a worker responds deterministically to longevity,

$$\omega_t = \omega + \varpi\Big(\tfrac{\gamma_t}{\gamma} - 1\Big),$$

where ϖ is the sensitivity of the retirement age to increases in longevity (Carvalho et al., 2016).

3.8 Welfare

Before turning to the analytical results, we make explicit how welfare is measured for workers and retirees, using each type's recursive value function. Because it is linear in composite consumption, it yields closed-form welfare expressions.

Lemma 1 (Welfare representation of workers and retirees)**.** Let Assumption 1 hold. For each type $z \in \{w, r\}$ the steady-state value function is linear in composite consumption, $V^z = \Delta^z d^z$ with $\Delta^z = (\xi^z)^{\sigma/(1-\sigma)}$ (established for retirees from (17) and for workers from (30)). Using the basket aggregators (24)–(11), the static price relations $c^z = \nu P^z d^z$ from (26) and (14), and the linear expenditure rules already derived (worker conjecture (29), retiree conjecture (18), written compactly as $P^z d^z = \xi^z \mathcal{R}^z$ where $\mathcal{R}^z$ denotes type-z total lifetime resources), welfare admits the closed form

$$V^z = (\xi^z)^{\frac{1}{1-\sigma}} \frac{\mathcal{R}^z}{P^z} = (\xi^z)^{\frac{\sigma}{1-\sigma}} d^z, \qquad z \in \{w, r\}, \tag{50}$$

which is homogeneous of degree one in $\mathcal{R}^z$. The resource term is type-specific. A representative worker holds financial wealth, human wealth and pension wealth, so by (29), $\mathcal{R}^w = R\,a^w + h^w + s^w$. A representative retiree holds financial wealth, residual human wealth and pension wealth, so by (18), $\mathcal{R}^r = \frac{\varphi R}{\gamma} a^r + h^r + s^r$. The consumption-equivalent welfare change of type z between two steady states is $\lambda^z = V^z_1 / V^z_0 - 1$, and population-weighted social welfare is $\mathcal{W} = \frac{1}{1+\psi} V^w + \frac{\psi}{1+\psi} V^r$. This measure compares stationary equilibria and thus isolates long-run welfare differences, abstracting from the redistribution across cohorts alive during the transition.

Proof. See Appendix C.6.1.[26] □

Theorem 1 (Welfare decomposition)**.** Let Assumption 1 hold and let V^z be given by (50), $z \in \{w, r\}$. Then V^z is continuously differentiable in $(\xi^z, \mathcal{R}^z, P^z)$ and to a first order

$$d \ln V^z = \underbrace{\frac{1}{1-\sigma}\, d \ln \xi^z}_{\text{intertemporal-price channel}} + \underbrace{d \ln \mathcal{R}^z}_{\text{lifetime-resources channel}} - \underbrace{d \ln P^z}_{\text{static-price channel}}. \tag{51}$$

The welfare effect of any shock thus decomposes into (i) an intertemporal-price channel through the marginal propensity to consume ξ^z, itself a function of the gross return (R for workers, $R^\star$ for the Southern household) and the survival-adjusted discount factor; (ii) a lifetime-resources channel through total wealth $\mathcal{R}^z$, comprising for the representative worker financial wealth a^w, human wealth h^w (driven by the after-tax wage), and pension wealth s^w, as in Lemma 1; and (iii) a static-price channel through the basket price P^z, which depends on the after-tax wage $(1-\tau)w$ through (16) and (27).

Proof. See Appendix C.6.2. □

[26] The appendix collecting the mathematical derivations and proofs is omitted from this online version and available upon request. Proof references throughout follow its numbering (Appendix C of the full version).

4 Analytical results

The model's first-order and equilibrium conditions yield closed-form results on how imperfect financial integration, demographic change, and pension design shape external balances. These underlie the quantitative analysis of Sections 5–7.

4.1 Financial market integration and external balances

We characterize how imperfect financial integration shapes the capital–labor ratio, trade balance, and net foreign assets.

Proposition 1. Let $\alpha = \alpha^\star$, $\theta = \theta^\star$, $\delta = \delta^\star$, and assume no adjustment costs ($\phi = 0$). Given $\bar{\Upsilon}$, an initial value of relative productivity $X_t/X_t^\star$, and the intermediation premium $\Gamma_t^\star$ governing imperfect financial integration, the North's capital–labor ratio in efficiency units is

$$\frac{k_t}{l_t} = \left(1+\Gamma_t^\star\right)^{\frac{1}{\alpha}} \left[\left(\frac{k_t^\star}{l_t^\star} \right)^{-\alpha} - \frac{(1-\delta)\theta\,\Gamma_t^\star}{(1-\alpha)\,(\theta-1)} \right]^{-\frac{1}{\alpha}}, \tag{52}$$

increasing in $\Gamma_t^\star$. Under imperfectly integrated financial markets ($\Gamma_t^\star \neq 0$), $k_t/l_t > k_t^\star/l_t^\star$, so the North exhibits deeper capital.[27]

Proof. See Appendix C.4.1. □

Proposition 2. In a world economy with differential productivity ($X_t \neq X_t^\star$), the North's trade balance in efficiency units is

$$\mathrm{nx}_t = -\frac{(1+\Gamma_t^\star)(c_t+i_t+g_t) - \dfrac{k_t}{k_t^\star}(c_t^\star+i_t^\star+g_t^\star) + \dfrac{(1-\delta)\Gamma_t^\star\,k_t}{(1-\alpha)\mathrm{mc}_t}}{1+\Gamma_t^\star + \left(\dfrac{k_t}{k_t^\star}\right)\left(\dfrac{\kappa_t}{1-\kappa_t}\right)\Upsilon_t}, \quad \forall t, \tag{53}$$

where

$$\Upsilon_t = \frac{X_t}{X_t^\star} = \bar{\Upsilon} \prod_{i=1}^{t-1} \frac{\hat{x}_{i+1}}{\hat{x}_{i+1}^\star}$$

is relative productivity and $\kappa_t(1-\kappa_t)^{-1}\,\Upsilon_t$ is the North's time-t relative economic size, an increase in which raises the trade balance.

Proof. See Appendix C.4.2. □

The trade balance nx_t falls with Northern absorption $c_t + i_t + g_t$ and rises with Southern absorption $c_t^\star + i_t^\star + g_t^\star$.[28]

We next relate the initial net foreign asset position to the long-run current account and trade balance.

[27] Perfectly integrated financial markets ($\Gamma_t^\star = 0$) equalize the two capital–labor ratios, $k_t/l_t = k_t^\star/l_t^\star$.

[28] With perfect financial markets ($\Gamma_t^\star = 0$), Propositions 1 and 2 give $k_t/l_t = k_t^\star/l_t^\star$ and $\mathrm{nx}_t = -\big[c_t + i_t + g_t - \frac{k_t}{k_t^\star}(c_t^\star + i_t^\star + g_t^\star)\big] \big/ \big[1 + \frac{k_t}{k_t^\star}\frac{\kappa_t}{1-\kappa_t}\Upsilon_t\big]$; with symmetric productivity ($X_t = X_t^\star$, $\Upsilon_t = 1$) the denominator is $1 + \frac{k_t}{k_t^\star}\frac{\kappa_t}{1-\kappa_t}$.

Proposition 3. Let $\alpha = \alpha^\star$, $\theta = \theta^\star$, $\delta = \delta^\star$, $X_t = X_t^\star$, $\hat{n}_t\,\hat{x}_t = (1+n_t)(1+x_t) \simeq 1 + n_t + x_t$, and $\phi = 0$. Assuming $\hat{n}_t\,\hat{x}_t = 1 + n + x$, the North's initial net foreign assets are determined by demographic growth, relative population size, and initial absorption:

$$f_0 = \frac{1}{1+n+x} \sum_{t=1}^{\infty} \left(\frac{1+n+x}{R} \right)^t \times \frac{c_{t-1} + i_{t-1} + g_{t-1} - \dfrac{k_{t-1}}{k_{t-1}^\star}\left(c_{t-1}^\star + i_{t-1}^\star + g_{t-1}^\star\right)}{1 + \dfrac{k_{t-1}}{k_{t-1}^\star}\left(\dfrac{\kappa_{t-1}}{1-\kappa_{t-1}}\right)}.$$

Proof. See Appendix C.4.3. □

Corollary 1. If $\mathrm{nx}_t = \bar{\mathrm{nx}}$ is fixed, Proposition 3 gives the North's fixed-level net exports and current account as

$$\bar{\mathrm{nx}} = (1 + n + x - R)\, f_0,$$
$$\mathrm{ca}_0 = (n + x) f_0.$$

Proof. See Appendix C.4.3. □

Lemma 2. Let $\alpha = \alpha^\star$, $\delta = \delta^\star$, $\theta = \theta^\star$. In a two-country economy with imperfect financial markets, the relative real wage depends on the premium $\Gamma_t^\star$ and the South's capital–labor ratio:

$$\frac{w_t}{w_t^\star} = \left(1 + \Gamma_t^\star\right)^{\frac{1-\alpha}{\alpha}} \left[1 - \frac{(1-\delta)\,\theta\,\Gamma_t^\star}{(1-\alpha)\,(\theta-1)} \left(\frac{k_t^\star}{l_t^\star} \right)^{\alpha} \right]^{\frac{\alpha-1}{\alpha}}. \tag{54}$$

The relative real wage rises with $\Gamma_t^\star$, since a higher premium raises the South's real interest rate above the North's, lowering the South's capital–labor ratio and raising its return on capital, lifting the North's relative wage.

Proof. See Appendix C.4.4 □

Corollary 2. By Lemma 2, when no cost separates the two financial markets ($\Gamma_t^\star = 0$) the relative real wage equals one, $w_t = w_t^\star$.

Proof. Setting $\Gamma_t^\star = 0$ in (54) sends the bracket term to 1, since the term multiplying $\Gamma_t^\star$ vanishes, and the leading factor $\left(1 + \Gamma_t^\star\right)^{\frac{1-\alpha}{\alpha}}$ to 1 as well, so $w_t / w_t^\star = 1$. □

4.2 Comparative statics: demographic change and the financial friction

This subsection signs, in closed form, the effects of longevity, population growth, migration, and the financial friction on saving, the capital–labor ratio, factor prices, and the external accounts. All statements refer to the deterministic steady state in efficiency units, other parameters fixed. Since the steady state is independent of the nominal block and of the investment adjustment cost ($S_t(\cdot) = S_t'(\cdot) = 0$ along the balanced-growth path), these are long-run properties alone, subject to Section 5's regularity conditions.

Assumption 1 (Regularity of the calibrated steady state)**.** The steady state satisfies $\beta \in (0,1)$, $\sigma = (1-\rho)^{-1} > 0$, $\alpha \in (0,1)$, $\theta > 1$, $\omega \in (0,1)$, and the demographic stability condition $0 < \gamma < \hat{n}^w$ that guarantees a positive worker share,

$$\psi = \frac{1-\omega}{\hat{n}^w - \gamma} > 0.$$

The world gross interest rate exceeds the world growth factor, $R > 1 + n + x$ (dynamic efficiency). These conditions, satisfied throughout by Table 3, are the only ones assumed here. The sign of the North–South net foreign position is established separately, in Result 1 below, since it follows from the calibration jointly with Proposition 4 rather than from this list.

Result 1 (Sign of the calibrated net foreign asset position)**.** Two demographic asymmetries enter Table 3. A longer Northern retirement ($\gamma > \gamma^\star$) pushes toward a creditor North (Lemma 3 and Proposition 4), reinforced by the retiree-productivity gap $\mu^\star = 0.65 > \mu = 0.55$ (ILO/OECD evidence, Section 2; Southern retirees participate more, needing less pre-retirement wealth). Annuity-market completeness ($\zeta = 0.95 > \zeta^\star = 0.10$) does not reinforce it. Lemma 5 signs only the retiree flow saving rate at fixed prices, and Remark 3 shows this flow channel is dominated in general equilibrium by a return-on-wealth channel, so $\zeta > \zeta^\star$ alone cannot sign f_0. Signing the North as net creditor further requires a financial-friction parameter below Bielecki et al.'s (2020) point estimate: $\chi_0 = 0.025$ rather than 0.038, within Alfaro et al. (2011)'s $[0, 0.05]$ range. Under this parameterization ($\gamma = 0.9333$, $\gamma^\star = 0.8667$, $\mu = 0.55$, $\mu^\star = 0.65$, $\zeta = 0.95$, $\zeta^\star = 0.10$, $\chi_0 = 0.025$, else as in Table 3), resolving the full 119-equation nonlinear steady-state system gives $f_0 = 0.167$ (20.55% of Northern GDP) and $f_0^\star = -0.204$ (-26.71% of Southern GDP), so the North is net creditor and the South net debtor, $f_0 > 0 > f_0^\star$, with residuals at solver tolerance (Corollary 3's identity $\overline{\mathrm{ca}} = (R-1)f_0 + \bar{\mathrm{nx}}$ holding in both regions). Unlike the structural longevity channel of Proposition 4 alone, this sign is calibration-dependent, and Lemma 5 and Remark 3 rule out signing it from the demographic asymmetries and $\zeta > \zeta^\star$ in isolation. Results below stated "under Assumption 1" that also rely on the sign of $f_0^\star$ rely jointly on this Finding, flagged where used rather than folded into Assumption 1, since it need not hold away from Table 3. All quantitative results below are computed under this parameterization; the note opening Section 6 details its scope.[29]

Lemma 3 (Longevity and the propensity to save)**.** In steady state, the retiree marginal propensity to consume out of wealth is $\xi^r = 1 - \beta^\sigma \gamma \, (\varphi R)^{\sigma-1}$, strictly decreasing in longevity,

$$\frac{\partial \xi^r}{\partial \gamma} = -\beta^\sigma R^{\sigma-1} \varphi^{\sigma-2} \left[\zeta + \sigma\gamma(1-\zeta)\right] < 0$$

(recall $\varphi = \gamma + \zeta(1-\gamma)$), so the retiree saving rate $1 - \xi^r$ rises with γ. The worker's marginal propensity to consume takes the parallel steady-state form

$$\xi^w = 1 - \beta^\sigma \left(\Omega R\right)^{\sigma-1},$$

where $\Omega = \omega + (1-\omega)\,(P^w/P^r)\,(\Delta^r/\Delta^w)^{1/\sigma}$ is the survival-adjusted horizon factor and $\Delta^z = (\xi^z)^{\sigma/(1-\sigma)}$. Longevity enters ξ^w only through Ω, and, mirroring the retiree case,

$$\frac{\partial \xi^w}{\partial \gamma} = \beta^\sigma (1-\sigma)\, R^{\sigma-1}\, \Omega^{\sigma-2}\, \frac{\partial \Omega}{\partial \gamma} < 0, \qquad \frac{\partial \Omega}{\partial \gamma} = \frac{1-\omega}{1-\sigma}\, \frac{P^w}{P^r} \Big(\frac{\Delta^r}{\Delta^w}\Big)^{1/\sigma} \frac{1}{\xi^r}\, \frac{\partial \xi^r}{\partial \gamma} < 0.$$

A longer expected retirement raises the value of wealth carried into old age, lowering ξ^r and, with it, Δ^r and the horizon factor Ω. Since $\sigma < 1$, a lower Ω raises the worker saving rate $1 - \xi^w = \beta^\sigma (\Omega R)^{\sigma-1}$, since anticipating a longer, asset-financed old age leads workers to front-load their saving. The direct channel through Δ^r dominates, the feedback through Δ^w being second-order and sign-preserving. Aggregate household saving pools the two cohorts, workers saving a fraction $1 - \xi^w$ and retirees $1 - \xi^r$ of their resources, retirees weighted by the dependency ratio ψ. Longevity thus raises aggregate saving through three reinforcing margins, namely higher worker

[29] The magnitudes reported here and in the sections that follow correspond to the asymmetric calibration of Table 3 as implemented in the accompanying code (South retirement $\gamma^\star = 0.8667$, 7.5 years; wedge $\chi_0 = 0.025$; South debt ratio $\bar{b}^\star = 60\%$), computed by homotopy continuation from the symmetric benchmark to the asymmetric steady state.

and retiree saving rates and the composition shift toward high-saving retirees as ψ rises. For given factor prices, longevity therefore raises aggregate household saving unambiguously.

The South's propensities take the parallel steady-state form, obtained by starring every object and replacing R with the South's domestic gross return $R^\star = (1+\Gamma^\star)R$ of (47) (Proposition 10(iii)):

$$\xi^{r\star} = 1 - \beta^\sigma \gamma^\star \left(\varphi^\star R^\star\right)^{\sigma-1}, \qquad \varphi^\star = \gamma^\star + \zeta^\star(1-\gamma^\star),$$

$$\xi^{w\star} = 1 - \beta^\sigma \left(\Omega^{w\star} R^\star\right)^{\sigma-1}, \qquad \Omega^{w\star} = \omega^\star + (1-\omega^\star)\,\frac{P^{w\star}}{P^{r\star}}\left(\frac{\Delta^{r\star}}{\Delta^{w\star}}\right)^{1/\sigma},$$

where $\Omega^{w\star}$ is the South's survival-adjusted horizon factor, the starred counterpart of Ω; it should not be confused with the wedge income $\Omega_t^\star$ of (49), a distinct, time-dated transfer internal to the South's financial intermediation sector. Holding $R^\star$ fixed, the same argument as above gives

$$\frac{\partial \xi^{r\star}}{\partial \gamma^\star} = -\beta^\sigma (R^\star)^{\sigma-1}(\varphi^\star)^{\sigma-2}\left[\zeta^\star + \sigma\gamma^\star(1-\zeta^\star)\right] < 0, \qquad \frac{\partial \xi^{w\star}}{\partial \gamma^\star} = \beta^\sigma(1-\sigma)(R^\star)^{\sigma-1}(\Omega^{w\star})^{\sigma-2}\,\frac{\partial \Omega^{w\star}}{\partial \gamma^\star} < 0,$$

so Southern longevity raises Southern saving through the identical three margins, higher $1-\xi^{r\star}$, higher $1-\xi^{w\star}$, and a composition shift toward retirees via $\psi^\star$, for given $R^\star$.

Retiree productivity μ (South: $\mu^\star$) leaves all four propensities unaffected. The steady-state law of motion (19) evaluates the price ratio P^r_{t+1}/P^r_t at unity regardless of μ, so the closed forms above for $\xi^r, \xi^w, \xi^{r\star}, \xi^{w\star}$ contain no μ or $\mu^\star$ term:

$$\frac{\partial \xi^r}{\partial \mu} = \frac{\partial \xi^w}{\partial \mu} = \frac{\partial \xi^{r\star}}{\partial \mu^\star} = \frac{\partial \xi^{w\star}}{\partial \mu^\star} = 0 \qquad \text{holding } R \text{ (resp. } R^\star\text{) fixed.}$$

Retiree productivity instead operates on the level of resources being split, entering retiree human wealth linearly through the full time endowment credited to retirees, $h^r_t = (1-\tau_t)\mu\, w_t + \pi^r_t + \ldots$ ((20); South: $h^{r\star}_t = (1-\tau^\star_t)\mu^\star w^\star_t + \ldots$), so a rise in μ (resp. $\mu^\star$) raises h^r (resp. $h^{r\star}$) and hence retiree expenditure $\xi^r h^r$ one-for-one with human wealth, leaving the propensity itself unchanged. Both parameters do act indirectly on all four propensities through the world interest rate, since a rise in $\mu^\star$ raises Southern retiree resources and saving (Result 1), moving R (and, through (47), $R^\star$) in general equilibrium, which then feeds back into every propensity via $\partial\xi^r/\partial R, \partial\xi^w/\partial R > 0$ ($\sigma < 1$), the same interest-rate channel isolated for $\gamma^\star$ in the footnote below. This zero direct effect together with a sign-ambiguous, calibration-dependent indirect effect through R is what Result 1 exploits, whereby $\mu^\star > \mu$ reinforces the North's creditor position through higher Southern human wealth and consumption levels, not through any difference in marginal propensities to consume.

Proof. See Appendix C.5.1. □

Proposition 4 (Longevity, external balances, and growth)**.** Let Assumption 1 hold and consider a permanent increase $d\gamma > 0$ in the North's longevity, the South held fixed. In the long run: (i) the North's aggregate saving rate rises (Lemma 3); (ii) its old-age dependency ratio $\psi = (1-\omega)/(\hat{n}^w - \gamma)$ rises, since $\partial\psi/\partial\gamma > 0$; (iii) the North accumulates net foreign assets ($f_0 > 0$) and runs a current-account surplus, $\mathrm{ca}_0 = (n+x)\, f_0 > 0$; (iv) it runs a long-run trade deficit, $\bar{\mathrm{nx}} = (1+n+x-R)\, f_0 < 0$ whenever $R > 1+n+x$. Aggregate output continues to grow at the rate $\hat{n}^w\hat{x}$, while output per worker rises through capital deepening.

Proof. See Appendix C.5.2. □

Corollary 3 (Reconciling the current account and the trade balance)**.** Since $\overline{\mathrm{ca}} = (R-1)f_0 + \bar{\mathrm{nx}}$ with $\bar{\mathrm{nx}} = (1+n+x-R)f_0$, a creditor economy ($f_0 > 0$) displays a long-run trade deficit alongside a current-account surplus. The wedge between them equals net foreign income $(R-1)f_0$, large

precisely when the net foreign asset position is large, an equilibrium property of economies with sizeable external positions, not an anomaly.[30]

Proof. See Appendix C.5.3. □

Proposition 5 (Decrease in population growth)**.** Let Assumption 1 hold. A permanent decline $d\hat{n}^w < 0$ in the labor-force growth rate (i) raises the old-age dependency ratio $\psi = (1-\omega)/(\hat{n}^w - \gamma)$; (ii) raises the steady-state capital–labor ratio, since for a given world interest rate $k/l \propto (\hat{n}^w \hat{x})^{-(1-\alpha)/\alpha}$; (iii) raises desired saving relative to domestic investment, improving the net foreign asset position and generating a current-account surplus; (iv) lowers aggregate output growth one-for-one ($g_Y = \hat{n}^w \hat{x}$) while raising output per worker.

Proof. See Appendix C.5.4. □

Proposition 6 (Immigration)**.** Net immigration to the North (i) raises its gross labor-force growth $\hat{n}^w = (1+n^w) + m^{w^\star}(1+n^{w^\star})\big[(1-\kappa^w)/\kappa^w\big]$ and its share κ^w of the world working-age population; (ii) lowers its old-age dependency ratio, $\partial\psi/\partial\hat{n}^w < 0$, partially offsetting aging; (iii) by lowering ψ and raising $\hat{n}^w$, reduces the aggregate saving rate and raises investment demand, weakening the North's net foreign asset position and current account in the long run.

Proof. See Appendix C.5.5. □

Proposition 7 (Financial friction and the international allocation of capital)**.** Under the cost-based premium $\Gamma^\star = \chi_0 \exp\big(-\kappa_f(F^\star/Y^\star - \bar{f}^\star)\big) > 0$ of (48), an increase in χ_0 (i) raises $\Gamma^\star$ and the South's interest rate $R^\star = (1+\Gamma^\star)R$ for every net foreign asset position; (ii) raises the North's capital–labor ratio relative to the South and the North's relative wage (Lemma 2); (iii) lowers Southern investment, which in partial equilibrium would release resources to the North, but $R^\star$ is also the return on the South's own accumulated wealth, a compounding channel (Remark 3) that in general equilibrium dominates, so that at the calibrated baseline of Result 1 a higher χ_0 instead improves the South's net creditor position and current account and weakens the North's (established numerically); (iv) raises the intermediation cost $\Gamma^\star > 0$ borne by Southern borrowers regardless of the sign of the South's net foreign asset position (Corollary 4).

Proof. See Appendix C.5.6. □

Corollary 4 (Sign-definiteness of the premium)**.** Because $\Gamma^\star_t > 0$ for every $F^\star_t/Y^\star_t$ under (48), the South always faces $R^\star > R$, and Proposition 7's capital-deepening and external-balance effects hold regardless of the sign of the South's net position, unlike a recentered risk-premium wedge that can turn into a subsidy. This concerns the wedge $\Gamma^\star$ itself, not the level of $f^\star_0$. Under Result 1 the South is net debtor ($f^\star_0 = -0.1754 < 0$), but the corollary holds with equal force were the South a net creditor, as under the earlier $\mu^\star = 0.05$ convention of Section 6's magnitudes. It is consistent with Proposition 7(iii), since this corollary signs the price of external finance, $\Gamma^\star$, while Proposition 7(iii) signs the quantity of net foreign assets through Remark 3's compounding-return channel; the two need not move together.

Proof. See Appendix C.5.7. □

[30] This corollary is the standard intertemporal balance-of-payments identity $\mathrm{ca} = (R-1)f + \mathrm{nx}$ (Obstfeld and Rogoff, 1996), evaluated at our model's steady-state net foreign asset position; the same wedge is documented empirically for large creditor and debtor economies by Lane and Milesi-Ferretti (2001). We record it as Corollary 3, without claiming novelty, because it recurs below (Appendix B.3 and Section 8) and provides an internal-consistency check, since any computed steady state with $\overline{\mathrm{ca}} - \overline{\mathrm{nx}} \neq (R-1)f_0$ signals a coding or detrending error.

Table 1: Long-run qualitative effects on the North.

Shock	k/l	w	R	Saving	F (NFA)	NX	CA
Higher longevity ($\gamma \uparrow$)	+	+	−	+	+	−	+
Decrease in population growth ($\hat{n}^w \downarrow$)	+	+	−	+	+	−	+
Immigration to North ($m^w \uparrow$)	−	−	+	−	−	+	−
Stronger financial friction ($\chi_0 \uparrow$)	+	+	−	−	−	+	−

Long-run Northern responses (Lemma 3, Propositions 4 to 7, and Corollary 3). Since CA $= (R-1)F + \text{NX}$, a creditor North ($F > 0$) pairs a trade deficit with a current-account surplus (Corollary 3), matching Result 1. The financial-friction row differs, in that a higher χ_0 lowers the North's own rate R, weakening rather than strengthening its creditor position (Proposition 7(iii), Remark 3); established numerically, not in closed form. Southern CA/NX flows mirror the North exactly, since $\Gamma^\star$ is rebated lump-sum (Footnote 23); NFA stocks need not mirror, as $\Gamma^\star > 0$ drives $R^\star \neq R$.

4.3 Decomposing the external accounts into saving and investment

Building on the welfare measure of Section 3.8, we decompose external accounts into saving and investment.

Lemma 4 (Saving–investment identity)**.** In steady state the trade balance equals national saving minus investment, $\text{NX} = S - I$, where $S \equiv Y - C - G$ and $I \equiv \text{inv}$. The current account is $\text{CA} = (R-1)F + \text{NX} = (R-1)F + S - I$.

Proof. See Appendix C.6.3. □

Theorem 2 (Demographic decomposition of the external accounts)**.** Let Assumption 1 hold. The response of the current account to a demographic shock decomposes as

$$d\text{CA} = \underbrace{dS}_{\text{saving}} - \underbrace{dI}_{\text{investment}} + \underbrace{(R-1)\,dF + F\,dR}_{\text{valuation}}. \tag{55}$$

Writing aggregate consumption per worker as $C/N^w = c^w + \psi\, c^r$, the saving response splits into a between-cohort (dependency-ratio) term $-\,c^r\,d\psi$ and a within-cohort term $-(dc^w + \psi\, dc^r)$. The investment response splits into capital widening ($\hat{n}^w \hat{x}\, dk$ induced by $d\hat{n}^w$) and capital deepening (dk at given $\hat{n}^w$ induced by dR). Under Lemma 3, a rise in longevity or a fall in population growth raises S (through $d\psi > 0$ and the higher retiree saving rate) and lowers I (through deepening), so $d\text{CA} > 0$.

Proof. See Appendix C.6.4. □

Theorem 3 (Decomposition of the migration effect)**.** An immigration shock to the North affects the external accounts through three channels. A labor-force-growth (capital-widening) channel raises investment I, a composition channel through the working-age share κ^w and the dependency ratio ψ lowers national saving S, and a relative-size channel through κ shifts the world distribution of absorption in (53). At the calibration of Table 3 the first two channels dominate and $d\text{CA} < 0$.[31] On welfare, immigration operates through (51), lowering human wealth h^w by reducing the wage through capital dilution but raising net resources by lowering the dependency ratio and hence the labor-income tax τ. The net welfare effect is the sign of this wage-versus-fiscal difference.

Proof. See Appendix C.6.5. □

[31] As in Proposition 8(ii), Assumption 1 fixes each channel's direction, not which dominates. Dominance is a property of Section 5's calibrated values, verified numerically, and could in principle reverse with a sufficiently flat dependency-ratio response.

4.4 Population growth, pensions, and North–South symmetry

The comparative statics above treat longevity, population growth, migration, and the financial friction separately. We now (i) collapse the demographic block into a small set of sufficient statistics, (ii) sign the effects of population growth on consumption, the real interest rate, the trade balance, and total retiree wealth, (iii) split labor-force growth into a domestic-fertility part and an immigration part, (iv) characterize the effects of immigration on pensions and the retirement margin, and (v) state every result symmetrically for North and South. Assumption 1 is maintained throughout, all statements refer to the steady state in efficiency units, and proofs are collected in Appendix C.7.

Although the model has four demographic primitives, the household block collapses at the steady state onto a three-dimensional set of sufficient statistics, so that any two configurations inducing the same triple induce the same long-run household equilibrium. With $\sigma = (1-\rho)^{-1} \in (0,1)$ the intertemporal elasticity of substitution, the retiree survival probability γ, gross labor-force growth $\hat{n}^w$, staying-worker probability ω, and net migration rate $m^{w^\star}$ (entering only through $\hat{n}^w$ via the identity (7)) determine the steady-state household block at a given world gross interest rate R only through the triple

$$(\psi,\ \hat{n}^w,\ \xi^r), \qquad \psi = \frac{1-\omega}{\hat{n}^w - \gamma}, \qquad \xi^r = 1 - \beta^\sigma \gamma\,(\varphi R)^{\sigma-1}, \tag{56}$$

namely the dependency ratio ψ, labor-force growth $\hat{n}^w$, and the retiree marginal propensity to consume ξ^r. Each is differentiable on $\{\hat{n}^w > \gamma\}$, with sign-definite partial derivatives

$$\frac{\partial \psi}{\partial \gamma} = \frac{\psi}{\hat{n}^w - \gamma} > 0, \qquad \frac{\partial \psi}{\partial \hat{n}^w} = -\frac{\psi}{\hat{n}^w - \gamma} < 0, \qquad \frac{\partial \psi}{\partial \omega} = -\frac{1}{\hat{n}^w - \gamma} < 0,$$

for the dependency ratio, and

$$\begin{aligned} \frac{\partial \xi^r}{\partial \gamma} &= -\beta^\sigma R^{\sigma-1}\varphi^{\sigma-2}\left[\zeta + \sigma\gamma(1-\zeta)\right] < 0, \\ \frac{\partial (1-\xi^r)}{\partial \gamma} &= \ \ \beta^\sigma R^{\sigma-1}\varphi^{\sigma-2}\left[\zeta + \sigma\gamma(1-\zeta)\right] > 0, \\ \frac{\partial \xi^r}{\partial R} &= -\beta^\sigma \gamma\,\varphi^{\sigma-1}(\sigma-1)\,R^{\sigma-2} > 0, \end{aligned}$$

for the retiree marginal propensity to consume. Two implications recur below. First, longevity and immigration act on ψ with opposite signs, so migration is the demographic offset to aging. Second, because $\sigma < 1$, $\partial \xi^r / \partial R > 0$, so the retiree saving rate $1-\xi^r = \beta^\sigma \gamma (\varphi R)^{\sigma-1}$ falls with R, and a lower world interest rate raises retiree saving, the accumulation channel of Proposition 8(iv). Derivation in Appendix C.7.1.

Proposition 8 (Population growth: consumption, the real interest rate, the trade balance, and retiree wealth)**.** Let Assumption 1 hold and let the world capital market satisfy the Walrasian stability condition that world excess saving $\Sigma(R) \equiv S + S^\star - I - I^\star$ is increasing in R, $\Sigma'(R) > 0$. A permanent decline $d\hat{n}^w < 0$ in the North's labor-force growth has the following long-run effects.

(i) The world real interest rate falls, $dR/d\hat{n}^w > 0$, since lower growth reduces the capital-widening component of investment ($I = (\hat{n}^w \hat{x} - (1-\delta))k$) by more than it reduces desired saving, so $\Sigma_{\hat{n}^w} < 0$ and $dR/d\hat{n}^w = -\Sigma_{\hat{n}^w}/\Sigma'(R) > 0$.

(ii) Aggregate consumption per worker $C/N^w = c^w + \psi c^r$ moves through a positive between-cohort term $c^r\, d\psi > 0$, a higher dependency ratio placing more weight on high-consuming retirees, and a negative within-cohort term $dc^w + \psi\, dc^r < 0$, as the lower interest rate raises the retiree

saving rate $1-\xi^r$ and, through the human-wealth and price channels of (51), lowers each cohort's consumption. The within-cohort force dominates in the calibration, so C/Y falls.[32]

(iii) Higher saving relative to investment raises net foreign assets, $df_0 > 0$; with $R > 1+n+x$ the North runs a long-run trade deficit $\overline{\mathrm{nx}} = (1+n+x-R)f_0 < 0$ alongside a current-account surplus $\overline{\mathrm{ca}} = (n+x)f_0 > 0$ (Corollary 3).

(iv) Aggregate retiree wealth per worker $\tilde{a}^r = \psi\, a^r$ rises, $d\tilde{a}^r/d\hat{n}^w < 0$, through two reinforcing channels: a composition channel ($d\psi > 0$, more retirees per worker) and an accumulation channel (a higher retiree saving rate $1-\xi^r = \beta^\sigma\gamma(\varphi R)^{\sigma-1}$, itself raised by the lower R of part (i)).

A symmetric decline $d\hat{n}^{w^\star} < 0$ in the South produces the mirror effects on Southern variables.

Proof. See Appendix C.7.2. □

Theorem 4 (Fertility versus immigration decomposition of the external accounts and retiree wealth)**.** Let Assumption 1 hold. Because the North's gross labor-force growth is

$$\hat{n}^w = (1+n^w) + m^{w^\star}(1+n^{w^\star})\big[(1-\kappa^w)/\kappa^w\big],$$

a change in labor-force growth splits additively into a domestic-fertility part and an immigration part,

$$d\hat{n}^w = \underbrace{dn^w}_{\text{fertility}} + \underbrace{\Xi\, dm^{w^\star}}_{\text{immigration}}, \qquad \Xi = (1+n^{w^\star})\,\frac{1-\kappa^w}{\kappa^w} > 0. \tag{57}$$

For any external aggregate $\mathcal{X} \in \{\,\overline{\mathrm{ca}},\ \overline{\mathrm{nx}},\ f_0,\ \tilde{a}^r\,\}$ the total response decomposes as

$$d\mathcal{X} = \underbrace{\frac{\partial \mathcal{X}}{\partial \hat{n}^w}\, dn^w}_{\text{fertility: growth channel only}} + \underbrace{\frac{\partial \mathcal{X}}{\partial \hat{n}^w}\,\Xi\, dm^{w^\star} + \frac{\partial \mathcal{X}}{\partial \kappa^w}\,\frac{\partial \kappa^w}{\partial m^{w^\star}}\, dm^{w^\star}}_{\text{immigration: growth channel + relative-size channel}}. \tag{58}$$

Fertility and immigration share the labor-force-growth channel $\partial\mathcal{X}/\partial\hat{n}^w$, since both raise $\hat{n}^w$, lower ψ, dilute capital, and hence lower f_0, $\overline{\mathrm{ca}}$, and $\tilde{a}^r$ while raising $\overline{\mathrm{nx}}$. Immigration carries an additional relative-size channel through the working-age share κ^w, shifting the world distribution of absorption in the trade balance (53). At the calibration of Table 3 the growth channel dominates, so both forces reduce the North's net foreign assets, current account, and total retiree wealth, differing only through the immigration-specific κ^w term.[33]

Proof. See Appendix C.7.3. □

Proposition 9 (Immigration, pensions, and the retirement margin)**.** Let Assumption 1 hold and let pensions be pay-as-you-go with outlay $T^r = \bar{e}\,Y$ (Section 3.5). Net immigration to the North, $dm^{w^\star} > 0 \Rightarrow d\hat{n}^w > 0 \Rightarrow d\psi < 0$, has the following long-run effects.

(i) The per-retiree pension replacement ratio, pension relative to output per worker, equals $\bar{e}/\psi$ and is strictly decreasing in the dependency ratio, so immigration raises it: $\dfrac{d(\bar{e}/\psi)}{dm^{w^\star}} = -\dfrac{\bar{e}}{\psi^2}\dfrac{\partial\psi}{\partial\hat{n}^w}\,\Xi > 0$. The per-retiree pension level rises whenever this channel dominates the capital-dilution fall in output per worker.

[32] Unlike parts (i), (iii), (iv), this comparison is not implied by Assumption 1 alone, since the two terms have opposite signs in general, their relative size a property of Section 5's calibrated values.

[33] As in Proposition 8(ii), Assumption 1 signs each channel of (58) individually but does not guarantee the growth channel outweighs the relative-size channel; this dominance reflects the calibrated κ^w and its migration sensitivity.

(ii) How aging is shared between taxpayers and pensioners depends on the pension rule. Under the paper's specification, pension outlay is a fixed share of output, $T^r = \bar{e}\,Y$, and the balanced-budget labor tax

$$\tau = \frac{1}{s_L}\Big[\bar{g} + \bar{e} + \bar{b}\big(\tfrac{R}{1+n+x} - 1\big)\Big], \qquad s_L \equiv \frac{wL}{Y}, \tag{59}$$

is independent of the dependency ratio, $\partial\tau/\partial\psi = 0$, so the entire aging burden falls on the per-retiree pension $\bar{e}/\psi$ of part (i) rather than on the tax rate, and immigration relieves that burden by raising $\bar{e}/\psi$. Under a defined-benefit rule instead, immigration would lower τ, raising the after-tax wage $(1-\tau)w$ and, through (51), workers' welfare (the fiscal channel of Theorem 3); Corollary 5 details how the two regimes bracket the fiscal response to demographics.

(iii) If the retirement rule holds the dependency ratio at a target $\bar{\psi}$ by adjusting the staying-worker probability ω (equivalently the statutory retirement age, monotone increasing in ω), immigration permits an earlier retirement, $\frac{d\omega}{d\hat{n}^w}\Big|_{\psi=\bar{\psi}} = -(1-\omega)/(\hat{n}^w - \gamma) < 0$, while greater longevity requires a later one, $\frac{d\omega}{d\gamma}\Big|_{\psi=\bar{\psi}} = (1-\omega)/(\hat{n}^w - \gamma) > 0$.

Proof. See Appendix C.7.4. □

Proposition 10 (Two-country and world symmetry)**.** Let Assumption 1 hold. Every long-run demographic result for the North has a Southern counterpart obtained by starring all variables, the two economies linked by migration and world market clearing.

(i) The emigration flow that raises the North's labor-force growth lowers the South's, $\hat{n}^{w^\star} = (1 - m^{w^\star})(1 + n^{w^\star})$ with $\partial\hat{n}^{w^\star}/\partial m^{w^\star} = -(1 + n^{w^\star}) < 0$. Emigration therefore acts on the South exactly as a negative population-growth shock (Proposition 5), raising $\psi^\star$, $k^\star/l^\star$, Southern saving, and total Southern retiree wealth $\tilde{a}^{r\star}$.

(ii) World goods-market clearing implies that the population-weighted current accounts sum to zero exactly, $\kappa\,\overline{\mathrm{ca}} + (1-\kappa)\,\overline{\mathrm{ca}}^\star = 0$, regardless of the sign or size of the financial friction $\Gamma^\star \geq 0$ (Proposition 7). Since $\Gamma^\star_t$ is rebated lump-sum within the South, it enters no term of (46) (Footnote 23), so a demographic force improving the North's current-account (flow) position worsens the South's by the exact mirror amount. This is a statement about current accounts, not the underlying stocks. Once $\Gamma^\star > 0$ so that $R^\star \neq R$, F_0 and $F_0^\star$ need not be mirror images ($F_0^\star \neq -F_0$ in general), even though their flows always are (Footnote 23).

(iii) Parts (i)–(iv) of Proposition 8, Lemma 3 and Propositions 4 and 5, and Theorem 4 hold verbatim for the South with starred variables and the world rate $R^\star = (1 + \Gamma^\star)R$ in place of R in the retiree saving rate $1 - \xi^{r\star} = \beta^\sigma\gamma^\star(\varphi^\star R^\star)^{\sigma-1}$.

Proof. See Appendix C.7.5. □

Corollary 5 (Tax response to demographics)**.** Let Assumption 1 hold. Under the paper's fiscal rule $T^r = \bar{e}\,Y$, the balanced-budget labor tax given by (59) is independent of the dependency ratio, $\partial\tau/\partial\psi = 0$, so aging is absorbed one-for-one by the per-retiree pension $\bar{e}/\psi$, which falls with ψ, and demographics affect τ only through R (debt service) and the labor share s_L, both second-order in the calibration. Under a defined-benefit rule instead, $T^r = \rho_{\text{rep}}\,w\,\psi L$, $\partial\tau/\partial\psi = \rho_{\text{rep}} > 0$, so a rise of the old-age dependency ratio from, say, 0.25 to 0.45 would raise the labor tax by $0.20\,\rho_{\text{rep}}$. The two rules bracket the empirical response; the model's fixed-outlay choice routes the cost of aging to benefits rather than taxes.

Proof. See Appendix C.7.6. □

Corollary 6 (Open economy versus closed economy)**.** Let Assumption 1 hold and compare the North as modeled, a large open economy facing the world rate R, with the same region in financial autarky.

(i) In autarky the North's rate is pinned by domestic clearing $S = I$, so a decline $d\hat{n}^w < 0$ moves it by $dR/d\hat{n}^w = -\Sigma^c_{\hat{n}^w}/\Sigma^{c\prime}(R)$ with $\Sigma^c \equiv S - I$. Openness replaces Σ^c by the world excess-saving schedule $\Sigma = S + S^\star - I - I^\star$ of Proposition 8; because the world schedule aggregates a partly offsetting South, $|dR/d\hat{n}^w|$ is strictly smaller under openness, capital mobility dampening the interest-rate swing.

(ii) The margin unavailable to the closed economy is the external one, whereby openness lets the North run $f_0 \neq 0$, so aging-induced excess saving is exported ($df_0 > 0$, a current-account surplus $\overline{\text{ca}} = (n + x)f_0 > 0$) rather than fully absorbed by a domestic fall in R and rise in k/l. The welfare gain from openness is the standard consumption-smoothing value of this margin, largest when domestic and world demographic shocks are least correlated.

(iii) All comparative-statics signs of Propositions 8 and 9 and Theorem 4 are common to the closed and open economy; openness changes their magnitudes and adds the external-balance responses $(f_0, \overline{\text{ca}}, \bar{\text{nx}})$, identically zero under autarky.

Proof. See Appendix C.7.7. □

Corollary 7 (A North closed to immigration)**.** Let Assumption 1 hold and suppose the North admits no migrants, $m^{w^\star} = 0$. Northern labor-force growth then collapses to domestic fertility, $\hat{n}^w = 1 + n^w$, strictly below its migration-augmented value, so by Equation (56) the dependency ratio rises and every effect of Proposition 8 operates at full strength, undamped by the migration offset. Symmetrically (Proposition 10), the South, retaining its would-be emigrants, has higher labor-force growth than under emigration, a lower dependency ratio, and a diluted capital stock. Closing the border thus reproduces Proposition 5's aging experiment without the migration offset.

Proof. See Appendix C.7.8. □

Table 2: Long-run demographic effects: North and South.

Shock	ψ	R	C/Y	$\tilde{a}^r$ (retiree wealth)	NX	CA
North fertility ↓ (n^w ↓)	+	−	−	+	−	+
North longevity ↑ (γ ↑)	+	−	−	+	−	+
Immigration to North ($m^{w^\star}$ ↑)	−	+	+	−	+	−
South emigration ($m^{w^\star}$ ↑, South view)	+	·	−	+	−	+

Steady-state responses (Equation (56), Propositions 8 to 10, and Theorem 4). R: world real rate; "·": South's own rate $R^\star = (1+\Gamma^\star)R$. Immigration to North = emigration for South; by adding-up (Proposition 10) the two current accounts mirror exactly regardless of $\Gamma^\star$, though NFA stocks need not (Footnote 23).

Remark 1 (Existence of the two-country steady state and the constancy of population shares)**.** Two features guarantee a common-growth steady state despite demographic asymmetry. The migration rule (7) fixes $m^{w^\star}$ so the North grows at its target $\hat{n}^w$, since the identity $\hat{n}^w = (1 + n^w) + m^{w^\star}(1 + n^{w^\star})[(1 - \kappa^w)/\kappa^w]$ has a stationary κ^w precisely when $\hat{n}^w = \hat{n}^{w^\star}$ in gross terms, at which point κ^w is constant in (5) and both regions detrend by $X_t N_t$. If $n^w \neq n^{w^\star}$, $m^{w^\star}$ adjusts endogenously, and a stationary κ^w still exists as long as the required $m^{w^\star} \in [0, 1)$, respected by the calibration. The strictly positive premium $\Gamma^\star > 0$ of (48) further keeps the South's rate above the North's,

removes any residual unit root in the two-country net foreign asset position (Schmitt-Grohé and Uribe, 2003), and, with $B^\star/Y^\star = \bar{b}^\star$ fixed, delivers a determinate steady-state real-rate differential and a locally stable steady state.

Remark 2 (Other North–South differences and what can be abstracted from)**.** Beyond the retirement-length difference $\gamma \neq \gamma^\star$, the calibration retains two further North–South differences, labor-force growth $\hat{n}^w \neq \hat{n}^{w^\star}$ (the South younger and faster-growing) and the financial friction $\Gamma^\star > 0$ borne only by the South. By Equation (56) each enters through the same sufficient statistics $(\psi, \hat{n}^w, \xi^r)$ and the world rate R, so their effects are additive and sign-definite, each a first-order channel the data identify. Abstracted differences (human-capital composition, sectoral structure, idiosyncratic income risk) do not interact with the demographic block at steady state and would shift levels without altering Table 2's signs, though they would matter for a welfare or distributional analysis. The North–South gap in retiree labor-force participation is retained on the ILO/OECD evidence of Section 2 underlying Result 1.

5 Calibration

We simulate the model under two calibrations, a symmetric one where the two countries share identical demographic and macroeconomic parameters and a non-symmetric one where these parameters differ.

Table 3 reports parameter values and steady-state exogenous variables for the symmetric benchmark. The model treats the North as a proxy for developed OECD economies and the South as representative of developing countries. In the symmetric calibration, individuals enter the labor force at age 20, work 45 years on average, retire at 65, and live 15 years after retirement, matching the roughly 80-year average life expectancy in OECD economies.[34] The North/South labels track the demographic channel, retirement length, not financial development, and Section 5.1 shows it is the North whose calibrated external position matches aging OECD creditor economies, since Table 3 sets retiree labor productivity higher in the South than the North ($\mu^\star = 0.65 > \mu = 0.55$, per the ILO/OECD evidence in Section 2), reinforcing rather than offsetting the North's longevity advantage in signing the net foreign asset position (Result 1 and Proposition 4).

Following Carvalho et al. (2016) and Gertler (1999), we set working-population growth to $n^w = 0.55\%$. We target a world asset-market-clearing real interest rate of 4%, which, given the model's demographic structure, implies a discount factor $\beta = 0.9631$. We set the elasticity of intertemporal substitution to $\sigma = 0.5$ following Auerbach and Kotlikoff (1987), pinning down $\rho = 1 - \sigma^{-1} = -1$ in (8), with $\sigma < 1$ pinning the sign of several aging comparative statics. We set the consumption share in the composite basket to $\nu = 0.8$, following Kilponen et al. (2006).[35] This configuration is dynamically efficient, since $R = 1.04$ exceeds the world growth factor $1 + n + x \approx 1.0155$. The regularity condition $R > 1 + n + x$ underlying the long-run trade-deficit-with-current-account-surplus result (Corollary 3) then holds. Using a standard product markup of 1.23, the implied demand elasticity for intermediate goods is $\theta = 1.23/0.23 \approx 5.35$. We set the retiree labor-productivity parameter to $\mu = 0.55$ following Gertler (1999), so the retiree wage is $w^r = \mu\, w = 0.55\, w$. The remaining parameters come from Kiguchi and Mountford (2019), Carvalho et al. (2016), and Gertler (1999). Table 3 presents the calibration for the non-symmetric steady

[34] OECD. *Health at a Glance 2025: Life Expectancy at Birth*. Technical report, OECD Publishing, Paris, 2025, https://www.oecd.org/en/publications/health-at-a-glance-2025_8f9e3f98-en/full-report/life-expectancy-at-birth_8a8dee48.html, accessed 2026.

[35] The implied labor-supply (Frisch) elasticity is 2.03, within the 1 to 3 range common in the RBC literature, computed from the worker's intratemporal first-order condition at the calibrated steady state, depending on ν, σ, and the leisure share.

state. In the North, individuals enter the labor force at age 20, work 45 years on average, retire at 65, and live 15 years after retirement. In the South, retirees live 7.5 years after retirement on average.[36] We set retiree labor productivity to $\mu = 0.55$ in the North, following Gertler (1999), and $\mu^\star = 0.65 > \mu$ in the South, following the cross-country evidence on elderly labor-force participation below.[37] Unlike the symmetric benchmark, the non-symmetric economy departs from the complete-annuity-market assumption, setting $\zeta < 1$ in both regions, lower in the South, so the accidental-bequest-to-workers channel (the transfer bq_t^r) is active and asymmetric across regions. Remaining parameters are unchanged. Table 3 consolidates both calibrations in a single table, adding North and South columns to the symmetric-benchmark values rather than repeating them in a separate table.

5.1 External-sector calibration

Because the South's net foreign asset position and the North–South interest-rate differential are central to the analysis, we treat them as explicit targets. In the symmetric benchmark the two regions are identical, so $f_0^\star = 0$, $R^\star = R$, and external accounts are balanced, isolating the demographic asymmetries introduced later. We target a world real rate $R - 1 = 4\%$, standard in the OLG literature (Auerbach and Kotlikoff, 1987; Gertler, 1999). In the non-symmetric and financially imperfect economies, the differential is governed by the intermediation premium (48), $R^\star - R = \Gamma^\star$, equal to χ_0 at the target ratio $\bar{f}^\star$. We vary $\chi_0 \in [0, 0.05]$, spanning the North–South return differentials documented by Alfaro et al. (2011). The premium is centered self-referentially on $\bar{f}^\star = -0.2384$ (about -23.8% of Southern GDP), equal to the model's own steady-state $f_0^\star/y^\star$, a numerical anchor rather than an independent target, since that position is pinned down by the demographic and productivity primitives of Section 5.2. The elasticity $\kappa_f = 0.1$ plays a similar role, since smaller values, e.g. 0.01, leave $f_t^\star$ near a unit root and make the stacked-time Jacobian singular to working precision, so 0.1 is the smallest value keeping the transition well conditioned, two orders of magnitude above the elasticity in Schmitt-Grohé and Uribe (2003)'s single-country calibration ($c_2 \approx 0.00074$), a citation retained only for the functional form.[39] Table 3 collects these

[36] A stylized value set to half the North's 15-year retirement horizon, capturing the shorter old-age survival of a lower-income South. It lies below UN/WHO remaining-life-expectancy estimates at age 65 for Sub-Saharan Africa (roughly 12.6–14.9 years by subregion, United Nations *World Population Prospects 2024*) and is therefore a deliberately conservative lower bound on the North–South longevity gap rather than a region-matched estimate.

[37] Here μ is retiree labor productivity relative to a prime-age worker ($w^r = \mu w$ in (41)), not a participation rate; an earlier calibration set $\mu^\star = 0.05 \ll \mu = 0.55$, which we no longer retain. The cross-country evidence points the other way, with senior (65+) labor-force participation averaging 39% in Africa versus 7% in Europe and Central Asia in 2015 (International Labour Organization, Department of Statistics. *What about Seniors? A Quick Analysis of the Situation of Older Persons in the Labour Market.* Spotlight on Work Statistics No. 1, ILO, Geneva, May 2018), with 31 of 36 countries worldwide having male 65+ participation above 70% located in Africa, against 6.6% in Western Europe (A. Samorodov. *Ageing and Labour Markets for Older Workers.* Employment and Training Papers No. 33, ILO, Geneva, 1999). The pattern holds beyond Africa, where 2020 ILO estimates put 65+ participation at 7.4% (Germany), 10.7% (United Kingdom), and 13.8% (Canada) against 35.5% (Korea) and 41.7% (Indonesia) (International Labour Organization. *Labour Force Participation Rate by Sex and Age, ILO Modelled Estimates.* ILOSTAT, Geneva, 2023, https://ilostat.ilo.org), and the OECD reports a 26.4% employment rate at ages 65–69 in 2024 (OECD. *Pensions at a Glance 2025.* OECD Publishing, Paris, 2025). On earnings, Lagakos et al. (2018) find experience-wage profiles about twice as steep in rich as in poor countries; their sample (roughly ages 18–58) cannot be extrapolated to the 65+ margin, so we use it only as support for the direction $\mu^\star > \mu$, not the magnitude. We accordingly set $\mu^\star = 0.65$, disciplining the gap's direction to this evidence and treating its size as illustrative, since Result 1's sign depends only on $\mu^\star > \mu$, so the result is robust where the evidence is thin and sensitive only to the direction, where it is strong.

[38] OECD. *Pension Markets in Focus 2025.* Technical report, OECD Publishing, Paris, 2025.

[39] A companion sensitivity sweep varies $\bar{f}^\star \in [0, 4]$ and $\kappa_f \in \{0.01, 0.05, 0.1, 0.2, 0.5\}$ at $\chi_0 = 0.038$, gauging the level's sensitivity to $\bar{f}^\star$ and the range of κ_f keeping the Jacobian well conditioned. This sweep was computed at $\chi_0 = 0.038$; since it varies only $\bar{f}^\star$ and κ_f, both of which govern conditioning rather than the sign results, it bears on magnitudes alone, and its re-execution at Table 3's $\chi_0 = 0.025$ is left to the final quantitative pass.

Table 3: Symmetric-benchmark and non-symmetric steady-state parameter values.

Parameter	Corresponding concept	Symmetric benchmark	North	South
$(1-\gamma)^{-1}$	Average retirement period (years)	15	15	7.5
$(1-\omega)^{-1}$	Average working period (years)	45	–	–
$\hat{n}^w$	Working-age population growth	1.0055	–	–
ζ	Fraction of annuitized assets (annuity-market completeness)	1	0.95	0.10
β	Discount factor	0.9631	–	–
μ	Retiree labor productivity	0.55	0.55	0.65
σ	Intertemporal elas. of substitution	0.50	–	–
ν	Consumption share in composite basket	0.80	–	–
α	Labor share	0.67	–	–
δ	Depreciation rate	0.10	–	–
ϕ	Investment adjustment costs	0.2	–	–
x	Growth rate of technology	0.01	–	–
θ	Demand elas. for intermed. goods	5.35	–	–
$\bar{g}$	Government spending (% GDP)	20	–	–
$\bar{b}$	Government debt (% GDP)	60	60	60
$\bar{e}$	Pension rate (% GDP)[a]	2	–	–
χ_0	Wedge parameter[b]	0	0.025	0.025
κ_f	NFA-premium elasticity	0	0.1	0.1
$\bar{b}^\star$	South target debt-to-output ratio (% GDP)	60	–	–
Υ	Steady state for relative productivity	1	1	1

[a] $\bar{e} = 2\%$ is net PAYG outlay $T_t^r = \bar{e}\, Y_t$ (41), not gross public pension spending, whereas OECD spends 8–9% of GDP (Clements et al., 2016; Whiteford and Whitehouse, 2006), so it is conservative. Chosen jointly with $\bar{g} = 20\%$ and $\bar{b} = 60\%$ (benchmark, North, and South) to keep τ (59) empirically plausible. Under the earlier calibration this target was re-solved for $\bar{e} \in \{4\%, 6\%, 8\%\}$ ($\bar{g}, \bar{b}$ fixed) with Table 1's signs and Section 6's results preserved; the same robustness check under Table 3's revised parameterization is pending re-execution.

[b] North/South columns report a value only where the non-symmetric calibration departs from column 3; dash = column-3 value retained in both regions. $\gamma = 0.9333$ (North, 15y) and $\gamma^\star = 0.8667$ (South, 7.5y; Footnote 36); $\mu^\star = 0.65 > \mu = 0.55$ reinforces the longevity-driven sign of Result 1. Financial friction $\chi_0 = 0.025$ (revised from Bielecki et al. (2020)'s 0.038; see discussion after Finding 1), inside Alfaro et al. (2011)'s $[0, 0.05]$ range swept in Section 6.1. $\zeta = 0.95$, $\zeta^\star = 0.10$ proxy annuity-market completeness, matched in magnitude to the OECD funded-pension-assets/GDP ratio ($\approx 92\%$, end-2024)[38] against lower Sub-Saharan figures (Nyang'oro and Njenga, 2022). $\zeta > \zeta^\star$ alone does not sign the NFA position (Lemma 5, Remark 3); the sign depends jointly on $(\gamma, \gamma^\star, \mu, \mu^\star, \chi_0)$. Unlisted parameters take column-3 values in both regions.

targets. Since $\overline{\text{ca}}^\star = (R^\star - 1) f_0^\star + \bar{\text{nx}}^\star$, the size of $f_0^\star$ scales the current-account–trade-balance wedge (Corollary 3), and results are qualitatively invariant to $f_0^\star$. The calibrated wedge is a plausible magnitude relative to large creditor economies. Japan and Germany hold net foreign assets of 50–80% of GDP, reference points rather than targets, since each reflects country-specific factors outside this mechanism. The North's position is large and positive, and with the retiree-productivity gap reinforcing the longevity channel ($\mu^\star = 0.65 > \mu = 0.55$), Proposition 4 signs the North's longer retirement toward a creditor North (Result 1), so f_0 is set positive but conservative, with only the sign and comparative statics meant to be read literally. The numerical magnitudes here and in Section 5.2 are now recomputed directly under Table 3. Tables 3 and 4 accordingly report the North's untargeted moments against the data, not the South's, since Result 1 makes the North, not the South, the calibrated large creditor.

5.2 Untargeted moments: model versus data

A calibrated two-country model can reproduce a qualitative sign pattern almost mechanically, so the exercise's empirical content rests on moments it did not target. Table 4 separates the two. Panel A lists the targeted moments, the world real rate and the South's target debt ratio of Table 3. Panel B confronts the model's Northern external accounts, equilibrium outcomes rather than targets, with

the positions of the aging, capital-abundant creditor economies invoked. As Section 5.1 explains, the North, not the South, has the large positive net foreign asset position, since both the longevity advantage and higher relative retiree productivity sign it toward a creditor North (Result 1). We report end-2024 data. Current-account balances run about 4.8% of GDP for Japan and 5.8% for Germany. Net foreign asset positions stand near 80% of GDP for both Japan and Germany. For Japan, a trade deficit coexists with a record current-account surplus sustained by a primary-income surplus of about 41.7 trillion yen.[40]

The comparison is qualitative, since a single stylized North cannot match any one country exactly, so the relevant test is whether its external accounts fall inside the creditors' range and reproduce the joint current-account surplus and trade deficit. This coexistence, exactly Japan's 2024 configuration, is not imposed but follows analytically from $\overline{\mathrm{ca}} = (R-1)f_0 + \bar{\mathrm{nx}}$ (Corollary 3) once the net foreign asset position is large. The model column of Table 4 now describes the creditor Northern bloc directly under Table 3 ($\mu^\star = 0.65 > \mu = 0.55$, Result 1). It shows a net foreign asset position of about 20.55% of GDP, a current-account surplus of about 0.32%, and a trade deficit of about -0.86%, with net foreign income $(R-1)f_0 \approx 1.18\%$ of GDP turning the deficit into a surplus (Corollary 3). Relative to the earlier $\mu^\star = 0.05$ illustrative convention, under which the creditor bloc (there the South) carried a net foreign asset position of about 275% of GDP, the recomputed Table 3 creditor position is an order of magnitude smaller. The qualitative mechanism, and the joint surplus/deficit coexistence, are unchanged, but the reported magnitudes are markedly more modest.

Table 4: Targeted and untargeted external moments: model versus data.

	Data	**Model (North)**
Panel A. Targeted moments		
World real interest rate $R-1$	4%	5.73%†
South debt-to-output ratio $\bar{b}^\star$	60%	60%
Panel B. Untargeted moments		
Current account / GDP	4.8 to 5.8%	0.32%
Trade balance / GDP	-1 to 6%	-0.86%
Net foreign assets / GDP	50 to 80%	20.55%
Surplus with trade deficit	Japan, 2024	Yes (Corollary 3)

Notes. Panel A's debt ratio matched by construction (Table 3). †World rate targeted at 4% only in the symmetric benchmark (Table 3, $\zeta = 1$) used to calibrate β; under the asymmetric Table 3 economy ($\zeta = 0.95$ North, $\zeta^\star = 0.10$ South, $\chi_0 = 0.025$) the rate is a general-equilibrium outcome of the same β, landing at 5.73%, reported rather than concealed. Panel B: end-2024 data for Japan and Germany (IMF, national statistics); model column is the Table 3 Northern steady state (calibrated creditor bloc, Result 1). Japan's fiscal-2024 trade deficit (-1% of GDP) with a 4.8% current-account surplus illustrates Corollary 3.

Table 4 is best read as a diagnostic of the calibration's plausibility, not a validation of the model or of the $\bar{f}^\star/\kappa_f$ pair discussed in Section 5.1. Panel A's debt ratio is hit exactly by construction and carries no independent information; the world real rate, by contrast, is not re-targeted in the asymmetric economy and instead emerges at 5.73%, a genuine, undisguised departure from the 4% benchmark value that we report rather than mask (see the table note). The content is in Panel B. Because $\bar{f}^\star$ is set equal to the model's own steady-state ratio (Section 5.1) rather than an

[40] Sources: IMF and Japan's Ministry of Finance for Japan's international investment position and fiscal-2024 balance of payments (a record current-account surplus of 30.4 trillion yen alongside a 4 trillion yen trade deficit); Deutsche Bundesbank for Germany's net external assets at end-2024 (just over 80% of GDP). Positions are relative to nominal GDP.

external empirical target, Panel B is not an out-of-sample test in the usual sense; it checks whether the mechanism, once the demographic and productivity primitives are fixed, lands in an empirically plausible region, not whether an independently chosen target was matched. With that caveat, under Table 3 the model places the North's net foreign asset position, 20.55% of GDP, below the roughly 80% positions of Japan and Germany rather than inside them. This is the expected consequence of a deliberately parsimonious calibration. Because the exercise targets only the world real rate and the South's debt ratio (Table 3), it is disciplined to reproduce the sign and mechanism of the creditor position, not its level, so a smaller untargeted magnitude confirms the calibration's parsimony rather than exposing a failure of fit. The joint occurrence of a current-account surplus (0.32% of GDP) with a trade deficit (−0.86% of GDP) still holds, reproducing Japan's 2024 qualitative configuration, but the current-account surplus itself sits well below the 4.8 to 5.8% observed range. This follows analytically from Corollary 3, since with a much smaller creditor position than under the earlier illustrative $\mu^\star = 0.05$ convention, net foreign income $(R-1)f_0 \approx 1.18\%$ of GDP is only just large enough to turn a small trade deficit into a small surplus, so Table 3's recomputed magnitudes should be read as confirming the mechanism's sign and qualitative structure rather than its earlier, more striking quantitative magnitudes.[41]

6 Quantitative analysis

Table 3 sets $\mu^\star = 0.65 > \mu = 0.55$, under which Result 1 and Proposition 4 identify the North, not the South, as net creditor via the reinforcing retiree-productivity channel. All tables and figures below (Tables 3 to 7, Figs. A.8 to A.10 and A.12 to A.14, Fig. 3, Figs. 4 and 5, Figs. 10 to 13 and B.12) are computed under Table 3, which also surfaces two findings. First, the North's creditor status flips sign at $\sigma^{\star\star} \approx 0.4583$, close to the calibrated $\sigma = 0.5$ (Fig. A.2). Second, the financial friction is itself large enough to overturn the pre-shock creditor position before determining the welfare transfer (Section 6.4). Statements independent of which region is creditor (e.g. Proposition 5) read the same regardless.

We report two exercises. The first is comparative statics, comparing steady states after a permanent parameter change. The second is simulations, perturbing a steady state and tracing the transition. Sections 6.1 and 6.4 and Appendix B.3 are computed directly under Table 3. The simulations of Section 7 run under two calibrations, each figure overlaying both, a symmetric "Benchmark" ($\mu = \mu^\star = 0.55$, $\zeta = \zeta^\star = 1$, $\chi_0 \approx 0$) and the asymmetric "Alternative" economy of Table 3 ($\mu^\star = 0.65$, shorter Southern longevity, $\chi_0 = 0.025$). They isolate a population-growth, life-expectancy, net-migration, and productivity shock, the demographic shocks combined, and all shocks together.

The baseline reproduces the North's and South's long-run demographic and national-accounting moments (dependency ratios, saving rates, net-foreign-asset ratios). Each experiment perturbs it one block at a time. Comparative statics compare two deterministic steady states, measuring permanent effects. Dynamic experiments trace a one-percent unanticipated shock along the saddle path, and, shocks being mutually orthogonal, the joint response is to first order their sum.

6.1 Long-term effects of increasing the cost-based premium

A partial closing of Southern financial markets, modeled as a rise in the cost-based intermediation premium χ_0 ((48)), has the following steady-state consequences. The swept range up to $\chi_0 = 0.05$ is a deliberately large illustrative upper bound, baseline experiments using $\chi_0 = 0.025$ (Table 3).

[41] A numerical check on Corollary 3 gives $(R-1)f_0 + \bar{\text{nx}} = 1.18\% - 0.86\% = 0.32\%$ of GDP, matching the reported current-account surplus of 0.32% up to second-decimal rounding of the three inputs.

Table 5 (row block 1) reports the steady-state changes as χ_0 rises from a practical floor of 0.002 (singular at 0) to 0.05. A higher χ_0 raises the Southern interest rate and return on capital, widens $\Gamma^\star$ from about 0.002 to 0.05, raises Southern wealth and saving, and lowers Southern investment, capital, output, and employment. Since capital falls by more than labor, the Southern capital–labor ratio declines while the North's rises ((52)), lowering Southern real wages ((54)). The resulting rise in Southern saving-to-output flips the South from net debtor into net creditor as χ_0 rises (Proposition 7(iii)), capital flowing South to North, the South's trade balance moving from surplus into an increasingly large deficit as its net foreign income rises, and the North's current account swinging from small surplus into deficit. At the extreme upper end the Southern net-foreign-asset ratio reaches nearly 193% of GDP and its consumption-to-output ratio approaches unity, reflecting the large χ_0 rather than an empirical position. Over the plausible baseline range both stay within empirical bounds.

6.2 Long-term effects of population aging

Two further experiments trace the steady-state consequences of population aging, a rise in Southern life expectancy ($\gamma^\star$ rising from 0.8667 to 0.9333, 72.5 to 80 years, North's γ fixed at 0.9333) and a decline in Northern labor-force growth. Table 5 (row blocks 2–3) reports the changes.

Higher Southern longevity lowers the world real interest rate, since Southern retirees anticipate a longer retirement, lower their marginal propensity to consume, and raise saving, so the South's net-debtor position narrows toward balance as $\gamma^\star$ rises toward the Northern value, and its capital–labor ratio rises. The North, with higher life expectancy throughout, remains the creditor region and runs a trade deficit financing higher consumption. As the longevity gap narrows, its creditor position and current-account surplus shrink in tandem toward the South's.

Rising Southern longevity raises the share of financial assets held by working retirees, whose marginal propensity to consume exceeds workers', $\xi^r > \xi^w$. With $\xi^r = 1-\gamma\beta^\sigma(\varphi R)^{\sigma-1}$, $\xi^w = 1-\beta^\sigma(\Omega R)^{\sigma-1}$, and $\Omega = \omega+(1-\omega)(P^w/P^r)\varepsilon^{1/(1-\sigma)}$ ($\varepsilon \equiv \xi^r/\xi^w$), the gap $\xi^r-\xi^w = \beta^\sigma R^{\sigma-1}(\Omega^{\sigma-1}-\gamma\,\varphi^{\sigma-1})$ is strictly positive at the calibrated values, since $\sigma<1$ makes $x \mapsto x^{\sigma-1}$ decreasing while $\Omega \approx 1$ and $\gamma\varphi^{\sigma-1} < 1$. Since Northern longevity exceeds the South's ($\gamma > \gamma^\star$, $\beta = \beta^\star$), the same mechanism implies a lower baseline retiree MPC in the North.[42]

A decline in the North's labor-force growth raises old-age dependency, deepens capital intensity, and raises saving relative to investment. The North becomes net creditor and runs a current-account surplus (Proposition 5), and since $R > 1+n+x$, net foreign income exceeds this surplus, so by Corollary 3 the North also runs a long-run trade deficit, reinforced by wealthier working retirees' higher consumption-to-output ratio. Lower population growth also lowers investment, hours, and employment, as in Kilponen et al. (2006). Table 5 (row block 3) reports each channel's direction (Fig. A.6).

6.3 Long-term effects under an asymmetric North–South calibration

The comparative statics above hold the two regions symmetric except for the swept parameter, so their steady-state ratios largely coincide. To bring out the regional dimension, we adopt Table 3's

[42] From $\xi^{r^\star} = 1-\gamma^\star\beta^\sigma(\varphi^\star R^\star)^{\sigma-1}$,

$$\frac{d\xi^{r^\star}}{d\gamma^\star} = \underbrace{-\beta^\sigma(R^\star)^{\sigma-1}(\varphi^\star)^{\sigma-2}\big[\zeta^\star+\sigma\gamma^\star(1-\zeta^\star)\big]}_{<0\ \text{(direct)}} + \underbrace{\frac{\partial\xi^{r^\star}}{\partial R^\star}\frac{dR^\star}{d\gamma^\star}}_{<0\ \text{(interest rate)}} < 0,$$

since $\partial\xi^{r^\star}/\partial R^\star > 0$ ($\sigma<1$) and $dR^\star/d\gamma^\star < 0$. The higher a region's retirement longevity, the more its net position, current account, and consumption shape the world rate and investment, in line with Ferrero (2010); Ludwig et al. (2009); Carvalho et al. (2016); Bielecki et al. (2020).

Table 5: Steady-state comparative statics: selected long-run elasticities.

Shock	Variable	Initial	Final
Cost-based premium χ_0: 0 to 0.05	South: gross real interest rate $R^\star$	1.083	1.102
	South: capital–labor ratio $k^\star/l^\star$	1.80	1.56
	North: capital–labor ratio k/l	2.02	2.56
	South: consumption/output	0.59	0.91
	South: net foreign assets/output	−2.08	1.93
	South: current account/output	−3.2%	3.0%
	South: trade balance/output	14.1%	−16.7%
	North: current account/output	3.0%	−1.9%
Southern life expectancy $\gamma^\star$: 0.8667 to 0.9333	South: gross real interest rate $R^\star$	1.093	1.081
	South: capital–labor ratio $k^\star/l^\star$	1.66	1.83
Northern labor-force growth $\hat{n}^w$: decline	North: creditor position, ca/y, k/l	all rise(qualitative; see Fig. A.6)	

Changes as each parameter sweeps column 1's range, else fixed at Table 3 (North net creditor). Cost-premium sweep's initial column uses the practical floor $\chi_0 = 0.002$, not the singular 0 (Section 6.1); South starts as net debtor ($f^\star/y^\star < 0$), crossing into net-creditor territory as χ_0 rises (Proposition 7(iii)). Rows without a numerical entry are qualitative directions, not point estimates; full magnitudes in Appendix A.2.

explicitly asymmetric calibration, a long-lived North ($\mu = 0.55$, $\gamma_{ss} = 0.9333$) against a shorter-lived South with more productive working retirees ($\mu^\star = 0.65$, $\gamma^\star_{ss} = 0.8667$, $\chi_0 = 0.025$), and sweep χ_0, Southern life expectancy $\gamma^\star_{ss}$, and Southern retiree productivity $\mu^\star$ (Figs. A.8 to A.10, appendix). In the external-balance panels, North and South are stacked bars around zero, total height measuring the North–South gap.

A higher cost-based premium widens the North–South divergence. The South's net-foreign-asset and current-account positions strengthen relative to the North's, its capital–labor ratio falls while the North's rises, and the consumption-to-output ratios move apart, the stacked bars lengthening monotonically with χ_0 (Fig. A.8). Rising Southern life expectancy works the other way, deepening Southern saving and its creditor position so the gaps converge as $\gamma^\star_{ss}$ approaches the Northern value (Fig. A.9). Raising Southern retiree productivity $\mu^\star$ likewise moves Southern aggregates toward their Northern counterparts, confirming the gap reflects structural, not parameter-specific, asymmetries (Fig. A.10).

Annuity-market completeness $\zeta \in [0, 1]$ governs the annuitized share of a retiree's assets and hence the annuity factor $\varphi = \gamma + \zeta(1 - \gamma)$. Its effect on retiree saving is analytical. With $\xi^r = 1 - \beta^\sigma \gamma\,(\varphi R)^{\sigma-1}$, a higher ζ raises φ and, since $\sigma < 1$, raises the retiree marginal propensity to consume and lowers saving $1 - \xi^r$. Fig. A.11 (appendix) decomposes these closed-form objects across ζ. The factor φ splits into the survival component γ and the annuitized increment $\zeta(1 - \gamma)$, rising from γ at $\zeta = 0$ to unity at $\zeta = 1$. As ζ rises, ξ^r and $1 - \xi^r$ shift toward consumption, and transferable wealth $(1 - \gamma)$ splits into the annuitized part and the accidental-bequest intensity $(1 - \zeta)(1 - \gamma)$, which shrinks to zero under a complete market. Finally, $|\partial\xi^r/\partial\gamma|$ increases with ζ. The baseline sets $\zeta = 1$, so accidental bequests vanish and $\varphi = 1$.

The general-equilibrium sweep in Fig. A.12 (symmetric baseline, both regions' ζ moved jointly) confirms a less complete market raises retiree saving and strengthens the region's net-foreign-asset and current-account positions. The asymmetric sweep in Fig. A.13 (Table 3, $\zeta = \zeta^\star$ moved jointly, feasible grid $\{0.2, 0.4, 0.6, 0.8, 1\}$, below which the joint anchor fails to resolve) reproduces the same pattern. Isolating the Southern market alone (Fig. A.14, sweeping only $\zeta^\star$ with the North's ζ fixed at 0.95, full grid $\zeta^\star \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$) shows a more complete Southern market lowers

the long-run real interest rate (North from 5.6% to 4.9%) and raises the capital–labor ratio in both regions. The sign is the general-equilibrium return-on-wealth channel of Remark 3, which dominates the opposite flow-saving (precautionary-bequest) reading: a more complete market raises the return embedded in retiree wealth faster than it lowers the flow saving rate, so accumulated wealth and the capital stock rise.

6.4 Long-run welfare effects of each shock

Having signed how each shock moves the external accounts, we now ask who gains and who loses across stationary equilibria. Using Lemma 1's welfare representation and Theorem 1's decomposition, we quantify the consumption-equivalent welfare change of workers and retirees, in both regions, from each of five structural shocks. Comparing two stationary equilibria isolates the long-run, structural welfare gain or loss of a newborn member of each cohort, abstracting from redistribution across cohorts alive during the transition.

For type $z \in \{w, r\}$ the value function is $V^z = (\xi^z)^{\sigma/(1-\sigma)} d^z$, and social welfare is $\mathcal{W} = \frac{1}{1+\psi} V^w + \frac{\psi}{1+\psi} V^r$. A permanent shock moving the economy from steady state 0 to steady state 1 is summarized by the consumption-equivalent variation (CEV)

$$\lambda^z = \frac{V_1^z}{V_0^z} - 1, \tag{60}$$

the permanent percentage change in the consumption basket that makes a type-z household indifferent between the two equilibria ($\lambda^z > 0$ denotes a gain). By Theorem 1, each λ^z admits the exact three-way decomposition

$$\ln \frac{V_1^z}{V_0^z} = \underbrace{\frac{1}{1-\sigma} \Delta \ln \xi^z}_{\text{intertemporal-price}} + \underbrace{\Delta \ln \mathcal{R}^z}_{\text{lifetime-resources}} - \underbrace{\Delta \ln P^z}_{\text{static-price}}, \tag{61}$$

with $\mathcal{R}^z = c^z/\xi^z$ and P^z the type-z basket price. We compute (60)–(61) by re-solving the steady state under each permanent perturbation (longevity γ, population growth $\hat{n}^w$, migration m, productivity $\hat{x}$, cost-based premium χ_0), for both the symmetric benchmark and the asymmetric economy. Table 6 summarizes the qualitative pattern, with magnitudes in Fig. 3, interpreted below.[43]

Table 6: Qualitative welfare effects (consumption-equivalent variation).

	North			South		
Shock	Workers	Retirees	Social	Workers	Retirees	Social
Higher longevity ($\gamma \uparrow$)	$-$	$-$	$-$	$-$	$-$	$-$
Slower population growth ($\hat{n}^w \downarrow$)		$\approx 0^{\ddagger}$			$\approx 0^{\ddagger}$	
Immigration to North ($m \uparrow$)	$-$	$-$	$-$	$+$	$+$	$+$
Higher productivity ($\hat{x} \uparrow$)		$\approx 0^{\ddagger}$			$\approx 0^{\ddagger}$	
Stronger financial friction ($\chi_0 \uparrow$)	$-$	$-$	$-$	$+$	$+$	$+$

Signs of the CEV (60) from the decomposition (61), computed under Table 3 for one illustrative shock per parameter (exact sizes in Section 6.4); signs hold at this magnitude/calibration, not swept, and could turn "$\pm$" at a different size. Exact magnitudes: Fig. 3. ‡Across stationary equilibria the labour-force-growth and productivity-growth experiments have negligible welfare effect: the migration rule offsets a change in fertility at the long-run target $\hat{n}^w$ (Remark 1), and a change in the productivity growth rate detrends out of the stationary per-capita equilibrium. Both act through the transition instead, where their effects are sizeable (Appendix B.3).

[43] $V^w, V^r, \mathcal{W}$ and their Southern counterparts are appended to the equilibrium system, and (60)–(61) evaluated across the corresponding stationary equilibria.

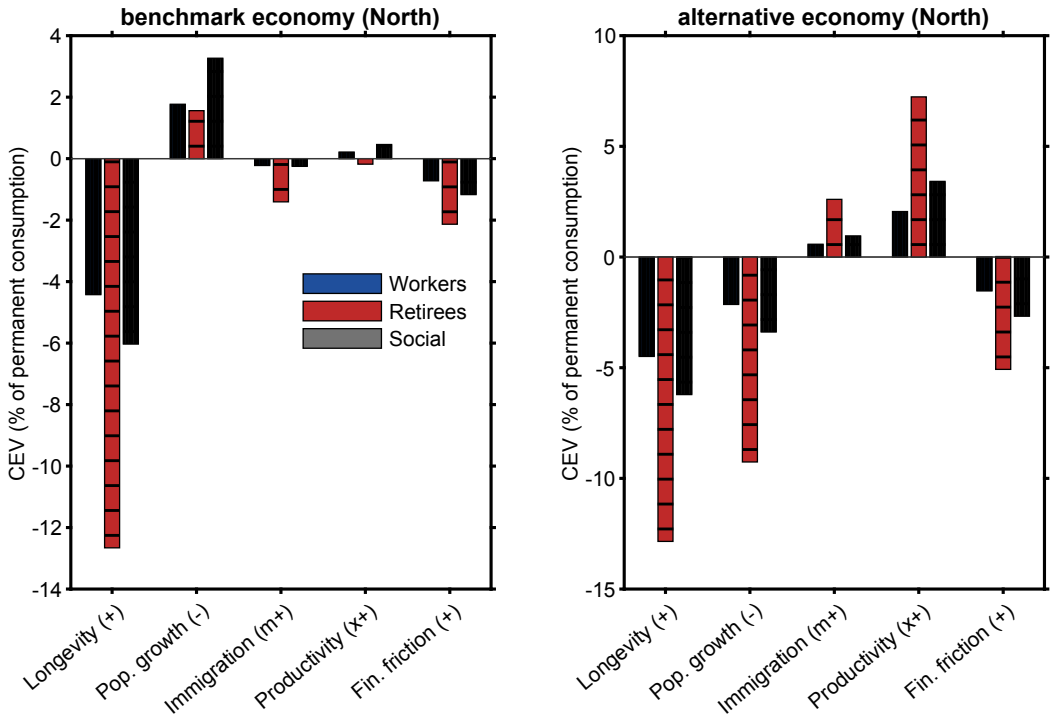


Figure 3: Long-run consumption-equivalent welfare effects of each shock, workers versus retirees, North versus South, computed directly under Table 3 (the exact perturbations are listed in Section 6.4). (Left: benchmark economy, North; right: alternative economy, North.)

Longevity is welfare-reducing for both Northern cohorts. Through (61) the intertemporal-price channel dominates. The lower real interest rate and longer expected retirement depress ξ^r, and with $\sigma < 1$ this cuts measured welfare per unit of resources. The lifetime-resources channel pushes the other way, since capital deepening (Propositions 4 and 5) raises wages and pension income, but not enough to offset it. Retirees, more exposed to the interest-rate channel, lose more. Under Table 3 (a 0.35-point rise in γ), Northern workers lose about -1.50% and retirees -4.37%, and the Southern spillover is negative for both cohorts (-0.17% workers, -0.16% retirees, an order of magnitude smaller) as the world interest-rate decline lowers Southern lifetime resources too.

A permanent decline in Northern labour-force growth has essentially no effect across stationary equilibria. The migration rule holds long-run labour-force growth at its target by adjusting immigration (Remark 1), so a fall in fertility is offset one-for-one by higher inflows, leaving the steady state, and hence steady-state welfare and the net foreign position, unchanged. The experiment bites only with the border closed (Corollary 7) or along the transition before migration adjusts. The transitional response (Appendix B.3) lowers both Northern cohorts' welfare on impact (workers about -0.91%, retirees -1.18%), as the smaller future workforce raises saving and depresses the real interest rate before output adjusts, and raises both Southern cohorts' (workers 0.13%, retirees 0.95%).

Immigration lowers welfare for both Northern cohorts, more for retirees. The inflow dilutes capital per worker and lowers the wage, cutting workers' human wealth; retirees, more exposed to the fall in the return on accumulated wealth, lose more, despite the lower old-age dependency ratio it brings via Corollary 5. Under Table 3 (net-migration rate up by 0.3 percentage points), Northern workers lose about -0.76% and retirees -1.06%. The resulting Southern emigration raises capital per remaining worker and the wage (as in a slower-population-growth shock, Proposition 10), so both Southern cohorts gain (workers 1.64%, retirees 0.90%), and the inflow sharply erodes the

North's creditor position (f/y from about 20.55% to 3.2%, $f^\star/y^\star$ from -26.7% to -6.8%).

A permanent rise in Northern productivity growth leaves the stationary per-capita equilibrium almost unchanged, since a change in the technology growth rate detrends out: steady-state welfare and the net foreign position barely move (f/y stays near 20.2%). Its welfare effect is transitional. Along the transition (Appendix B.3) both Northern cohorts gain (workers about 4.41%, retirees 3.30%) as real wages and human wealth rise ahead of the capital adjustment, while both Southern cohorts lose modestly (workers -0.03%, retirees -1.48%) as capital is drawn North. This is the one shock whose steady-state and transitional welfare diverge sharply, a reminder that a growth-rate change reshapes the adjustment path rather than the stationary cross-section.

The cost-based premium is the clearest welfare-transfer shock. The transfer follows whichever region is the net creditor after the shock, not the presumption that a higher cost of capital hurts the region facing it, nor its creditor status before. Raising χ_0 lifts the Southern interest rate $R^\star$ (Proposition 7). Under Table 3 (a rise in χ_0 from 0.025 to 0.035), South gains (workers 7.96%, retirees 7.22%) and North loses (workers -3.36%, retirees -8.97%). The compounding-return channel of Remark 3 and Proposition 7(iii) raises Southern saving and returns enough to flip the South to net creditor, from $f^\star/y^\star \approx -26.7\%$ to 65.7%, the North moving symmetrically into debt. Because the reported effects use the rebated friction specification (Footnote 24), which reallocates resources within the South without destroying any, the shock is close to a pure inter-regional transfer rather than global deadweight loss. Only under the iceberg interpretation, where $\Gamma^\star > 0$ consumes real resources, would the creditor's gain fall short of the debtor's loss.

Two regularities emerge across the stationary equilibria. First, retirees are consistently more exposed than workers to interest-rate-moving shocks, so longevity, immigration, and the friction weigh on retirees more than workers (e.g. -8.97% versus -3.36% for the North under the friction), while every shock moves a region's two cohorts together. Second, the three shocks with a steady-state effect split cleanly: longevity costs both regions, since a longer retirement adds claimants on a fixed pension outlay, while immigration and the financial friction cost the North and benefit the South. Labour-force growth and productivity growth, by contrast, leave the stationary equilibrium essentially unchanged (the migration rule offsets the first, Remark 1; growth-rate detrending neutralizes the second) and act only along the transition (Appendix B.3). The financial friction is large enough to overturn the baseline creditor position on its own, flipping the South to net creditor (Remark 3 and Proposition 7(iii)), so post-shock creditor identity governs that transfer. Demographic structure and financial development thus drive external imbalances and determine who bears their cost, a conclusion to read alongside Table 6's note on shock-size dependence.

7 Transition dynamics of external imbalances

We trace North–South transition dynamics of the trade balance, net foreign assets, and current account following four Northern shocks, a 1% rise in productivity growth, in life expectancy, and in immigration, and a 1% decline in population growth. Impulse responses report nx, f, and ca as deviations from the pre-shock steady state.

7.1 Sign and ranking of the shocks' effects on external accounts

In the benchmark, a negative population-growth shock improves the North's trade balance, as fewer workers and a larger retiree share cut labor supply and consumption while raising saving, so exports exceed imports. Longevity works through the same channel, a longer retirement lengthening the saving horizon, but reinforces the surplus less. Immigration and productivity push the balance the other way, enlarging consumption demand and wealth respectively, both producing a Northern deficit. The immigration effect reverses after about eight periods, once immigrants' output

contribution overtakes their demand contribution, the South mirroring throughout (Fig. 4). The non-symmetric economy confirms the ranking, population growth, then longevity, then immigration (Fig. 5), with demographic shocks dominating productivity in driving the North's trade surplus.

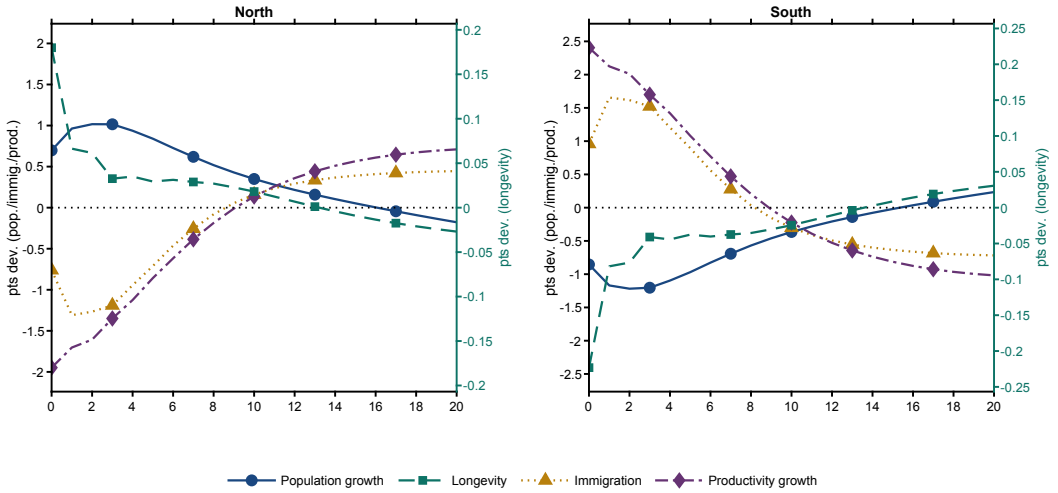


Figure 4: Impacts of demographic and productivity shocks on trade balances (Benchmark) (computed under Table 3, with the benchmark's symmetric, frictionless parameters, $\zeta = \zeta^\star = 1$, $\mu^\star = \mu$, $\gamma^\star_{ss} = \gamma_{ss}$, $\chi_0 \approx 0$) (Left: North; right: South.) Responses are deviations from the initial steady state, expressed as a share of GDP.

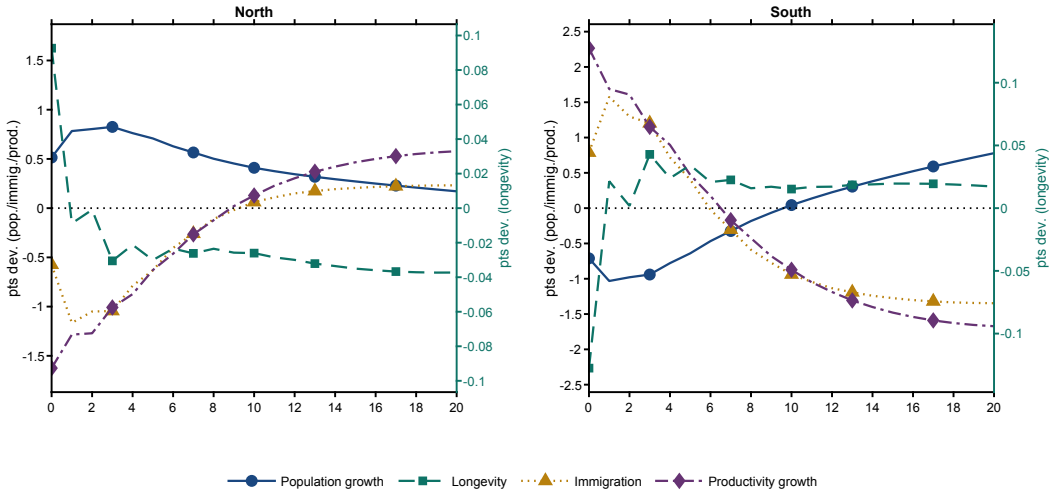


Figure 5: Impacts of demographic and productivity shocks on trade balances (Alternative model) (computed under Table 3) (Left: North; right: South.) Responses are deviations from the initial steady state, expressed as a share of GDP.

Net foreign assets follow the same demographic ranking in the benchmark. Population growth and longevity drive the North's accumulation, immigration and productivity the South's (Fig. 6). A smaller Northern workforce lowers output and raises precautionary saving, raising the North's net foreign assets while lowering the South's. By the adding-up constraint (Footnote 23) the South's deficit mirrors the North's surplus, and at this calibration (Result 1) its position is also negative, since the intermediation friction $\Gamma^\star > 0$ wedges $R^\star$ from R, so $F^\star_0$ need not equal $-F_0$ (Fig. 8). The alternative model reorders this, productivity leading, then immigration, longevity, and population growth, driving the North's negative net-foreign-asset position and current-account deficit, with the South reversed (Figs. 7 and 9).[44]

[44] A current-account deficit combined with a trade surplus arises for a net-debtor economy. With $F < 0$, net

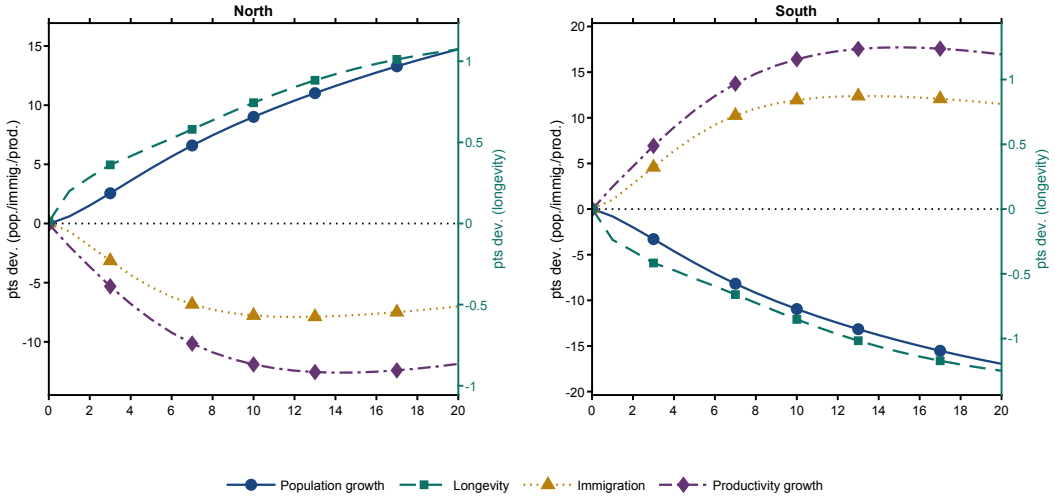


Figure 6: Impacts of demographic and productivity shocks on net foreign assets position (Benchmark) (computed under Table 3, with the benchmark's symmetric, frictionless parameters, $\zeta = \zeta^\star = 1$, $\mu^\star = \mu$, $\gamma^\star_{ss} = \gamma_{ss}$, $\chi_0 \approx 0$) (Left: North; right: South.) Responses are deviations from the initial steady state, expressed as a share of GDP.

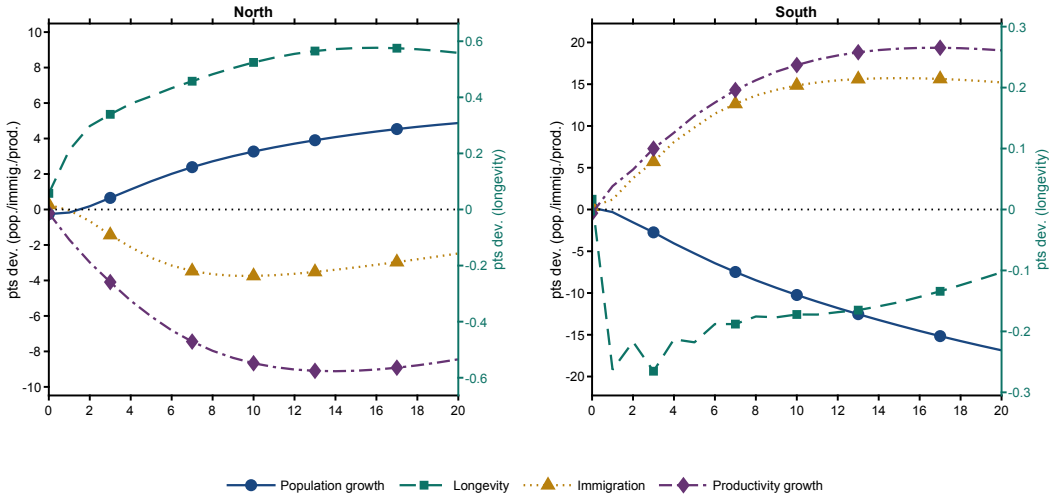


Figure 7: Impacts of demographic and productivity shocks on net foreign assets position (Alternative model) (computed under Table 3) (Left: North; right: South.) Responses are deviations from the initial steady state, expressed as a share of GDP.

7.2 Impulse-response dynamics of demographic and productivity shocks

We examine each shock in turn, in the benchmark and in the non-symmetric economy where Northern retirees are longer-lived and active, Southern retirees are not.

A rise in Northern productivity growth temporarily raises real wages and hence the human wealth of Northern workers and retirees (Fig. B.2), lifting marginal propensities to consume and consumption relative to the South (Fig. 10). The accompanying fall in the marginal productivity of capital (Fig. B.1) leads firms to cut capital and investment. The capital–labor ratio troughs at about -3.5% roughly ten years out, investment-to-output drops about 0.15 point on impact, and profits, employment, net immigration, and financial wealth decline next period (Fig. B.2). With full financial

investment income $(R-1)F < 0$ can outweigh a trade surplus $\mathrm{NX} > 0$, the empirically relevant case for net external debtors such as Australia, Brazil, and South Africa, servicing liabilities via trade surpluses.

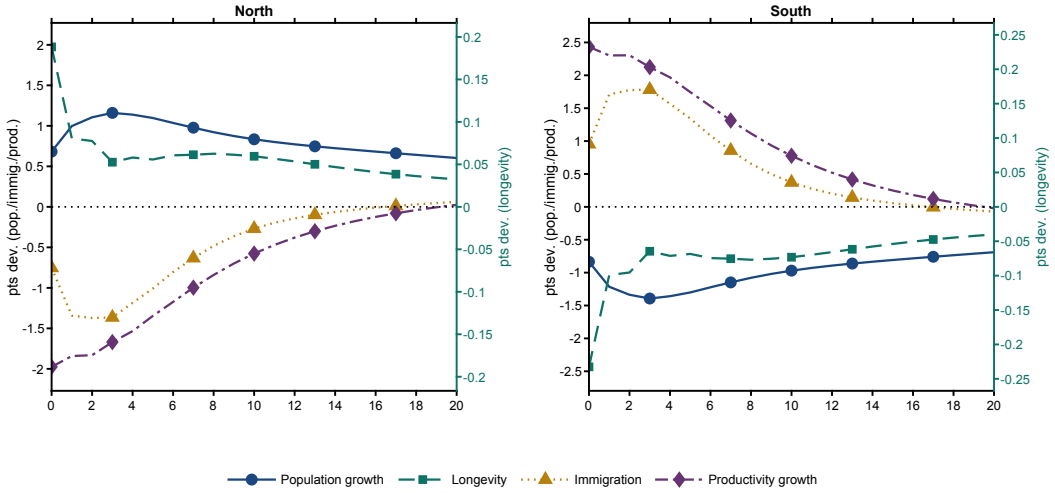


Figure 8: Impacts of demographic and productivity shocks on current account balances (Benchmark) (computed under Table 3, with the benchmark's symmetric, frictionless parameters, $\zeta = \zeta^\star = 1$, $\mu^\star = \mu$, $\gamma^\star_{ss} = \gamma_{ss}$, $\chi_0 \approx 0$) (Left: North; right: South.) Responses are deviations from the initial steady state, expressed as a share of GDP.

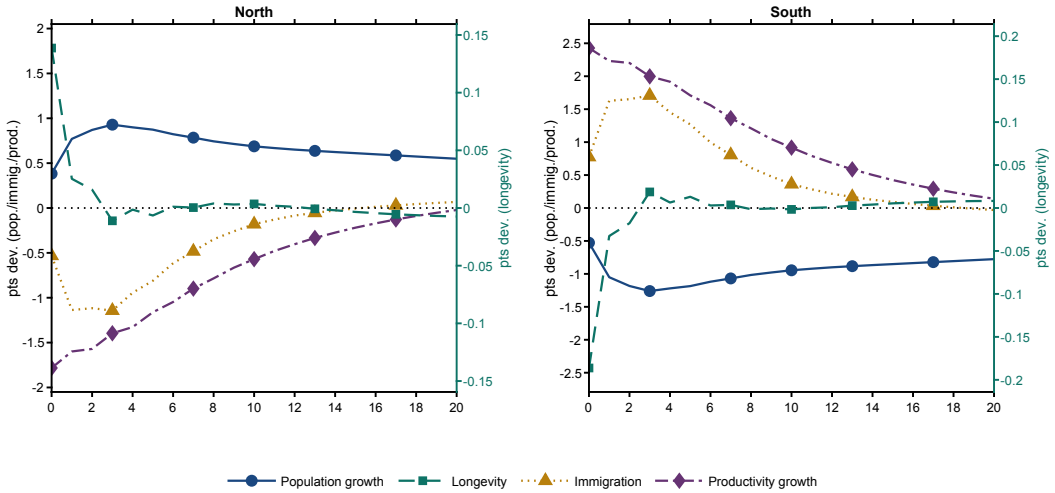


Figure 9: Impacts of demographic and productivity shocks on current account balances (Alternative model) (computed under Table 3) (Left: North; right: South.) Responses are deviations from the initial steady state, expressed as a share of GDP.

integration, lower output and saving turn the North into a net debtor running a current-account deficit (Fig. 8), with the current-account-to-output ratio troughing at about 1.8 points within four years before partly recovering, while the South's net-foreign-asset position rises to about 17 points of output as capital flows North. Lower saving raises the gross real interest rate by about 0.2 point on impact, peaking near 0.46 point in year two, further depressing investment (Figs. 10 and B.1). Output falls less than employment, so labor productivity rises, while the higher marginal propensity to consume raises the consumption-to-GDP ratio by about 1.8 points on impact, fading into a small deficit of about 0.85 point by year 20. Rising consumption with falling production raises imports, so the trade-balance-to-output ratio falls by about 1.7 points on impact before recovering to a small surplus by year 20 as capital adjusts (Fig. 4). Faster productivity growth draws capital northward to equalize returns (Figs. 6 and 8), so the benchmark predicts a Northern trade deficit and negative net foreign assets.

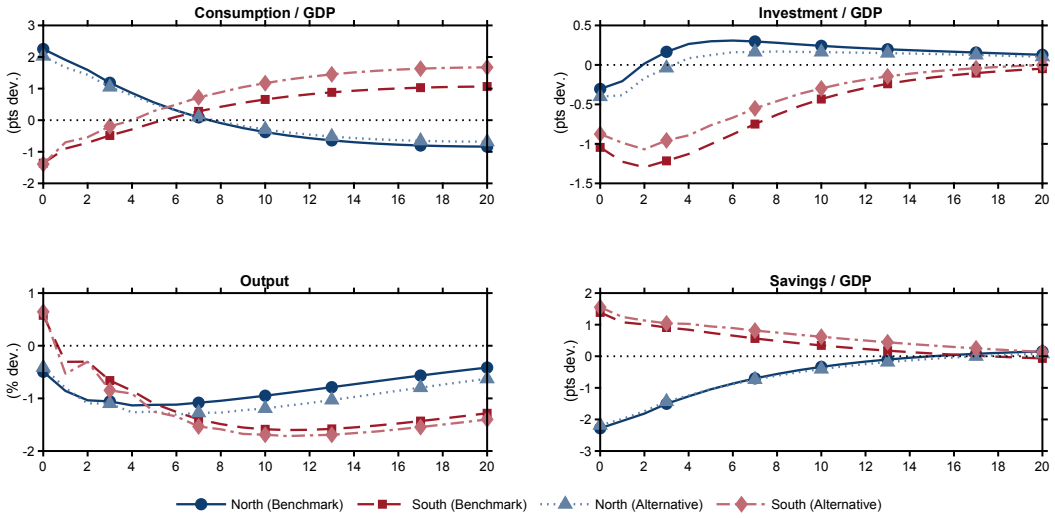


Figure 10: Productivity shock: consumption, investment, output and savings, Benchmark and Alternative (computed directly under Table 3, from the linearized companion solve of Section 7.3) (Panels, top to bottom, left to right: consumption/GDP; investment/GDP; output; savings/GDP.)

The alternative model reaches the same outcome differently. Higher immigration and demographic growth pressure the Northern labor market, lowering the real wage, human wealth, consumption, saving, and total wealth (Fig. B.2), raising the real interest rate (Fig. B.1) and again lowering net foreign assets and widening the current-account deficit despite rising investment (Fig. 9), so capital flows South to North. Weaker demand trims imports and lifts the trade balance, though it stays in deficit for several years before a small surplus (Fig. 5). The South shows the opposite dynamics.

In the benchmark, a lower Northern population-growth shock shrinks the working-age share, lowers the region's world-population weight, and raises the old-age dependency ratio (Fig. B.5). It cuts worker labor supply, employment, and pensions while raising real wages, elderly labor supply, retiree human wealth, the retiree share of wealth, and overall wealth next period (Fig. B.4). The North's interest rate rises on impact (about 0.5 point) before higher saving pulls it below its initial level, to about -0.3 point by year 20 (Fig. 11). Migration here is not wage-driven. The net migration rate $m^{w^\star}$ is set so the Northern labor force grows at its calibrated 0.55% target via the normalization $\hat{n}^w = (1+n^w)+m^{w^\star}(1+n^{w^\star})[(1-\kappa^w)/\kappa^w]$ of (6)–(7), so the comovement of declining population growth and migration in Fig. B.5 is an accounting artifact, not wage-driven relocation. Fewer workers and lower employment raise the capital–labor ratio, depress the marginal productivity of capital, and cut investment and output (Figs. 11 and B.3), with the ratio rising about 2% on impact, building to roughly 3% above steady state over two decades, and investment-to-output falls by about 0.9 point on impact, widening to about 1.5 points within four years. Declining population growth also raises total Northern assets (Fig. B.3), as agents anticipating lower future pensions save more than in the South. Higher assets lift saving, financial wealth, net foreign assets, and the current account (Figs. 6 and 8), while the wealth gain raises consumption only gradually, from essentially zero on impact to about 1.5 points of GDP by year 20. Higher consumption with weaker investment and a smaller Northern population share yields a trade surplus of about 0.9 point on impact, peaking near 1.3 points within three years, before fading into a small deficit of about 0.2 point by year 20.

In the alternative model, with longer-lived, active Northern retirees and inactive Southern ones, the same shock predicts a trade surplus on impact and a positive, steadily rising Northern net-foreign-asset position (Figs. 5 and 7), lower employment, and higher worker wealth (Fig. B.4),

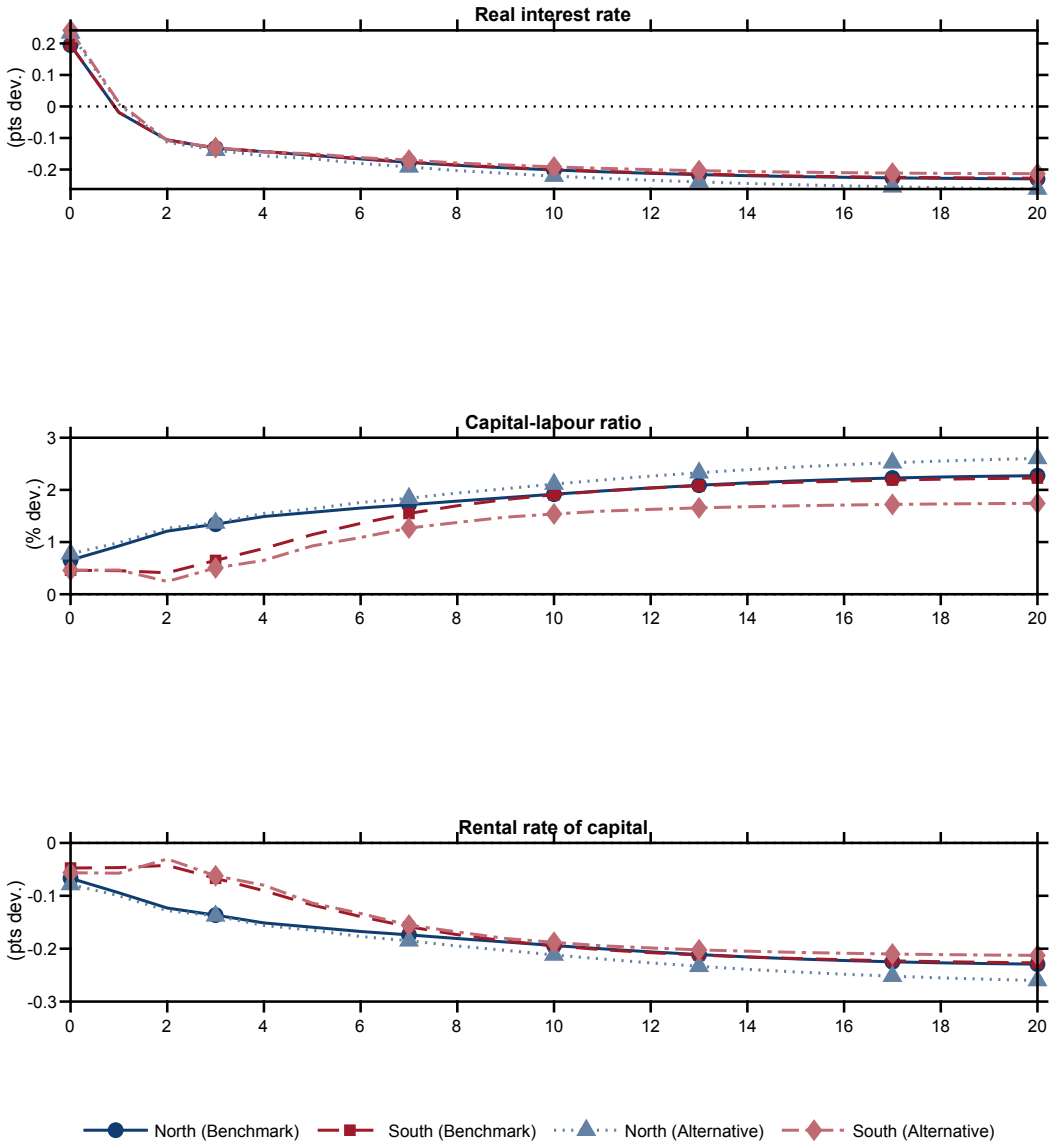


Figure 11: Fertility shock: interest rate and capital, Benchmark and Alternative (computed directly under Table 3, from the linearized companion solve of Section 7.3) (Panels, top to bottom: real interest rate; capital–labor ratio; rental rate of capital.)

alongside a smaller working population and higher old-age dependency ratio (Fig. B.5). Fewer workers cut labor supply and raise the real wage. Under pay-as-you-go, lower pensions push retirees to work more, lowering their marginal propensity to consume and raising saving (Figs. B.3 and B.4). The real interest rate rises on impact before higher retiree saving pulls it below its initial level (Fig. 11). Consumption for both groups rises less than in the benchmark, since inactive, shorter-lived Southern retirees have a higher marginal propensity to consume. Lower employment raises the capital–labor ratio and reduces the marginal productivity of capital and output (Figs. 11 and B.3). The initial trade surplus narrows into a small deficit as consumption catches up with lower production, while sustained higher saving keeps lifting the North's net foreign assets and current account further into surplus, so capital continues to flow North to South. The South shows the mirror image on impact, lower output, a trade deficit, higher consumption relative to GDP, lower worker financial wealth, and higher investment demand, but its net-foreign-asset position moves further negative over time rather than turning positive, the counterpart of the North's growing

creditor position (Figs. 5, 7, B.3 and B.4).

A positive Northern life-expectancy shock in the symmetric benchmark raises retiree labor supply and lowers worker labor supply, delivering, after an initial dip, a durably higher capital–labor ratio, a lower real interest rate, and higher output next period, with a higher old-age dependency ratio (Fig. B.8), more retiree employment, and lower real wages (Figs. B.6 and B.7). Higher retiree employment and human wealth raise worker human wealth and the retiree wealth share (Fig. B.7). The resulting wealth and saving gain lowers the real interest rate (Fig. 12), persistently raises investment, moderating over time (Fig. B.6), and cuts consumption demand via lower marginal propensities to consume. Higher production with lower domestic consumption yields a Northern

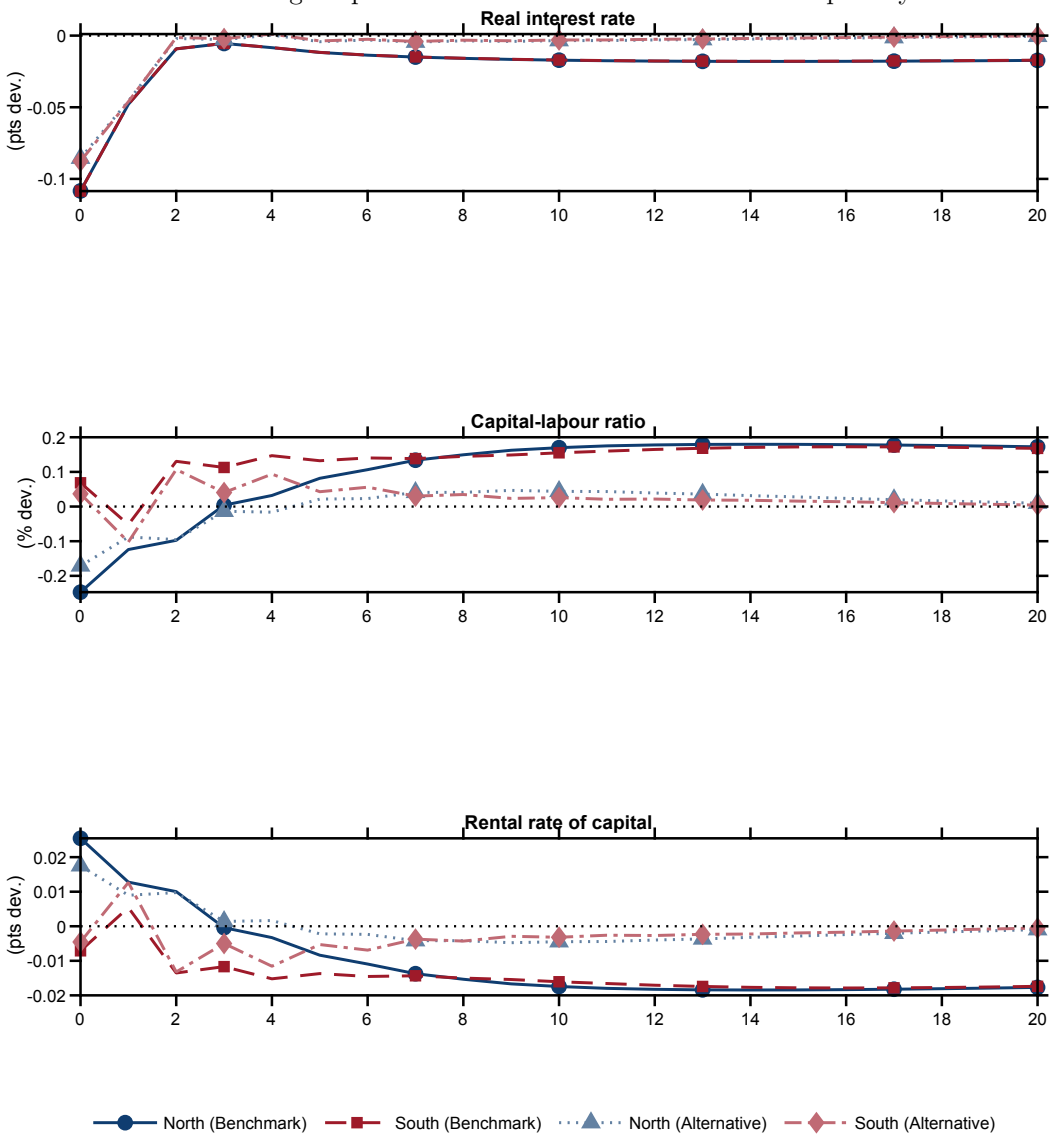


Figure 12: Longevity shock: interest rate and capital, Benchmark and Alternative (computed directly under Table 3, from the linearized companion solve of Section 7.3) (Panels, top to bottom: real interest rate; capital–labor ratio; rental rate of capital.)

trade surplus (Figs. 4 and B.6). As the North accumulates capital, it flows North to South and the North runs a current-account surplus as creditor. In the non-symmetric economy, where Southern retirees are inactive and shorter-lived, the same shock again yields a trade surplus, but with greater

wealth decumulation and lower saving, the North's net-asset position dips briefly negative on impact before recovering to a modest positive position, so the North remains a net creditor overall, albeit with a smaller current-account surplus than in the benchmark (Fig. 7).

In the benchmark, higher South-to-North immigration lowers the old-age dependency ratio (Fig. B.11) and temporarily raises Northern worker supply (Fig. B.10), lowering the real wage, raising the present value of social security and the marginal productivity of capital, lowering the capital–labor ratio (Fig. 13), and temporarily boosting output (Fig. B.9). The real interest rate dips on impact before rising above its initial level within a few years (Fig. 13). Lower real wages

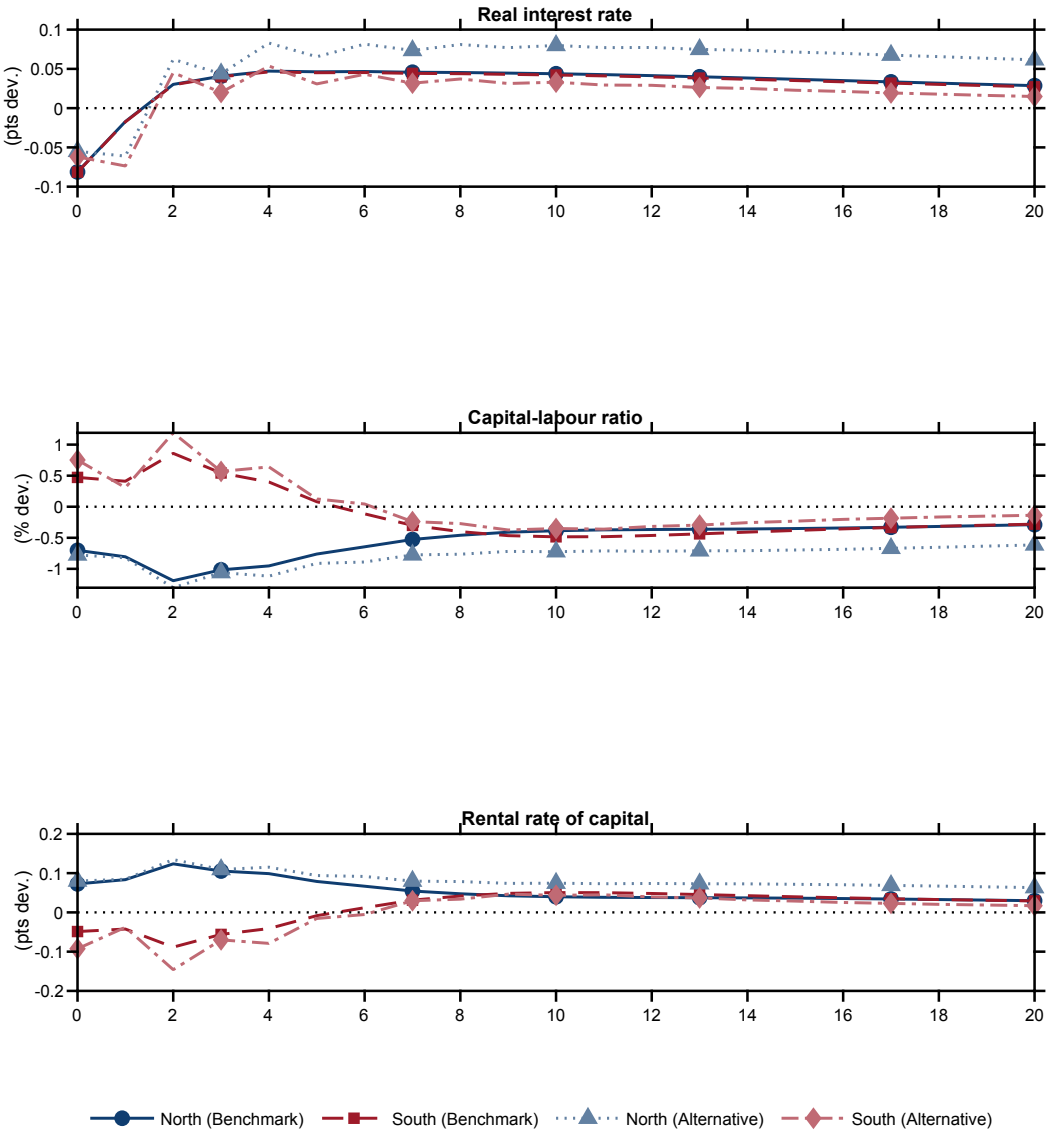


Figure 13: Immigration shock: interest rate and capital, Benchmark and Alternative (computed directly under Table 3, from the linearized companion solve of Section 7.3) (Panels, top to bottom: real interest rate; capital–labor ratio; rental rate of capital.)

reduce retiree human wealth, yet total wealth of both retirees and workers still rises as worker human wealth rises, raising consumption (Figs. B.9 and B.10). The resulting demand rise pushes the North into trade deficit, while lower national saving turns its net-foreign-asset position increasingly negative, reaching about -12% of output within fifteen years (Figs. 4 and 6). Immigration

also reshapes the South. Emigration lowers Southern labor supply, raises real wages, and boosts retiree human wealth, but lowers worker human and financial wealth. The resulting fall in total wealth, consumption, and saving raises the South's real interest rate, after a similar initial dip, and cuts investment demand. Since investment falls more than saving, the South records a trade and current-account surplus (Figs. 4 and 8).

7.3 A contribution decomposition of the trade balance

We now make explicit the shock ranking asserted in the introduction. Since the model is a deterministic, perfect-foresight transition rather than a stochastic system, a standard conditional variance decomposition does not apply. Instead we feed the exogenous processes one block at a time and measure each one's share of the trade-balance path.

Let $\{\mathrm{nx}_t\}_{t=1}^{H}$, $H = 200$, denote the equilibrium trade-balance path when all four exogenous drivers, longevity γ_t, population growth n_t, net migration m_t, and relative productivity x_t, follow their calibrated transition paths. For each driver j we recompute the transition with only j active, others held at their initial steady-state values, obtaining a counterfactual path $\{\mathrm{nx}_t^{(j)}\}$, with contribution share

$$\mathcal{S}_j = \frac{\sum_{t=1}^{H} \left(\mathrm{nx}_t^{(j)} - \mathrm{nx}_0\right)^2}{\sum_k \sum_{t=1}^{H} \left(\mathrm{nx}_t^{(k)} - \mathrm{nx}_0\right)^2}, \tag{62}$$

the fraction of summed squared deviation of the trade balance from its initial steady state nx_0 attributable to driver j over the $H = 200$-period horizon. Each driver is perturbed by a common shock size of 0.01 in absolute value, so Equation (62) compares drivers on a common scaling convention, not historically estimated volatilities, and Table 7's ranking is conditional on it. Because the drivers interact, the one-at-a-time paths need not sum to the joint path, so Table 7 also reports the residual interaction term, alongside a Shapley-value allocation redistributing that residual symmetrically across the four drivers rather than leaving it undecomposed.[45] The shock processes are the calibrated demographic and productivity transition paths used above, with the persistence and scale reported there.

Table 7 confirms the ranking under either method, with demographic forces dominant. The joint γ-and-n solve alone slightly exceeds the sum of all four one-at-a-time parts (101.9% of that sum, a signature of nonlinear reinforcement between longevity and population growth), and accounts for 118.7% of the trade-balance response under the exact Shapley allocation, the excess over 100% absorbed by the negative Shapley shares of migration and productivity. The two methods disagree on the split within the demographic bloc. One-at-a-time shares attribute almost the entire demographic contribution to population growth (87.2% against longevity's 0.3%), whereas the Shapley values, which fully price in each driver's interaction with the other three, credit longevity with a larger share (8.0% against population growth's 110.7%), more than twentyfold higher once its interaction with the other drivers is counted. Under Table 3 the one-at-a-time metric ranks productivity (8.5%) above migration (4.1%). The Shapley values turn both negative, productivity (-12.7%) more so than migration (-6.1%), since along the transition each partly offsets rather than reinforces the dominant demographic drivers. Fed jointly, the one-at-a-time interaction/residual is -85.0% of the four drivers' joint squared deviation, the individual parts over-explaining the joint

[45] Both columns of Table 7 are computed from the model's full 119-equation nonlinear system at the Table 3 steady state, solved with a perfect-foresight solver (1000-period horizon), not a linearization. Col. 1 follows (62). Each driver is fed its calibrated ± 0.01 transition shock one at a time, all others held at steady state, giving $\mathcal{S}_j$. The "Demographic" row is a separate joint γ-and-n solve, not the arithmetic sum of the two rows below it, and can exceed that sum when the drivers reinforce nonlinearly. Col. 2 reports the exact Shapley value $\phi_j = \sum_{S \subseteq N \setminus \{j\}} \frac{|S|!(n-|S|-1)!}{n!} \big(v(S \cup \{j\}) - v(S)\big)$ with $v(S) = \sum_{t=1}^{H} \left(\sum_{j \in S} \mathrm{nx}_t^{(j)} - \mathrm{nx}_0\right)^2$, one nonlinear solve per non-empty subset; by the efficiency property the four values sum exactly to the joint four-driver $v(\{\gamma, n, m, x\}) = 0.0222873$, verified against the direct joint solve.

Table 7: Contribution of each shock to the trade-balance path: one-at-a-time shares and Shapley values (computed under Table 3, the paper's calibration; see Section 7.3's footnote for methodology).

Driver	**Share $\mathcal{S}_j$ (%)** (one-at-a-time)	**Shapley ϕ_j (%)** (Shapley)
Demographic $(\gamma + n)$	–	118.7
of which longevity γ	0.3	8.0
of which population growth n	87.2	110.7
Productivity x	8.5	−12.7
Net migration m	4.1	−6.1
Total (individual shares)	100	100
Memorandum: interaction/residual	−85.0	0.0 (exact)

Notes. Both columns are computed under Table 3 from the model's nonlinear perfect-foresight transition (Section 7.3, footnote), directly comparable driver by driver. Col. 1 reports one-at-a-time shares $\mathcal{S}_j$ (62) ($H = 200$, common shock 0.01); the four driver rows sum to 100 by construction, while the joint $\gamma{+}n$ solve (101.9) exceeds 100% because γ and n reinforce along the transition, making it a joint-solve diagnostic rather than an additive share. Col. 2 reports exact Shapley values ϕ_j, allocating the full interaction term and summing to 100 by efficiency; the demographic entry (118.7) is the additive subtotal $\phi_\gamma + \phi_n = 8.0 + 110.7$, and the one-at-a-time interaction/residual is −85.0% of the four-driver joint squared deviation, the individual parts over-explaining it through cancellation. Both productivity's and migration's negative Shapley shares are not errors: along the transition each partly offsets the dominant demographic drivers, an interaction Col. 1 cannot represent.

path through cancellation, consistent with Table 3's $\zeta, \zeta^\star, \chi_0$ parameterization reshaping how the shocks' trade-balance responses interact along the transition. Population growth raises capital per worker and desired saving directly and on impact, while longevity works more gradually through the retirement-span saving margin, dampened by the fixed-outlay pension rule (Corollary 5). Demographic forces remain, under either method, the paper's dominant driver of the trade-balance path. This ordering carries through the dynamic experiments above. The exact split between longevity and population growth, the ranking of migration against productivity, and the sign and magnitude of the interaction term should be read as conditional on the decomposition method and on the calibration (Section 7.3).

Appendix B.2 in Appendix B extends this decomposition to a population-weighted world perspective that accounts for the North's declining population share over 2025–2100 (UN WPP projections). Population growth remains the dominant driver of the world-weighted path, and only migration works against it once cross-country offsetting between the North and South is taken into account.

Appendix B.3 in Appendix B complements the long-run welfare comparison of Section 6.4 by measuring transitional consumption-equivalent welfare, λ^z_{tr}, for cohorts alive when each shock hits rather than only the newborn scored there. The relative-productivity shock benefits the Northern cohorts on impact while weighing slightly on the South. Both aging shocks impose transitional losses concentrated on Northern retirees, compounding to about an 11.7% loss for Northern retirees when longevity and slower population growth are fed jointly. Immigration raises Northern welfare on impact while lowering Southern welfare.

8 Robustness

The results above rest on a deliberately parsimonious set of modeling choices, each adopted because it isolates the mechanism of interest without an auxiliary channel that would blur it. Each choice is defended below against the evidence that it does not compromise the paper's results.

Section 3.3 models production as a single Cobb–Douglas technology with a constant factor-price wedge θ ((36)–(37)) rather than an explicit New Keynesian block, which plays no active role here. Along the balanced-growth path the adjustment cost vanishes ($S(\cdot) = S'(\cdot) = 0$) and the markup enters factor prices only as the constant scalar in (37), leaving Section 4.2's comparative statics unchanged. Transitions are driven by real shocks, adjustment costs, and life-cycle saving, and nominal rigidities are a natural extension.

The RINCE preferences in (8) make households risk-neutral toward income risk while allowing a non-unit intertemporal elasticity σ, keeping consumption and labor rules linear in wealth. The Cobb–Douglas consumption–leisure aggregator implies a Frisch elasticity near 2.03, within the standard real-business-cycle range of 1 to 3, governing the magnitude but not the sign of the wage response to demographic change.[46] We discipline this value to the macro-consistent RBC range rather than to micro intensive-margin estimates for older workers. Chetty et al. (2011) show the gap between the two is driven largely by extensive-margin responses, including retirement timing, and since retirees here adjust along exactly that margin, a macro-consistent elasticity is the parameter the theory calls for, not a second-best substitute. Structural evidence on retirement behavior (French, 2005) corroborates the direction, though it is reported as an extensive-margin retirement-timing elasticity and no directly comparable intensive-margin estimate for older workers yet exists. Pinning down the exact position within Chetty et al. (2011)'s range is left for future calibration work.

Fiscal closure depends on the pension rule. Under the benchmark fixed-outlay rule $T^r = \bar{e}\,Y$, the labor tax clearing (42) is invariant to the dependency ratio ($\partial\tau/\partial\psi = 0$, Corollary 5), so aging is absorbed by the per-retiree pension $\bar{e}/\psi$ rather than net wages. A defined-benefit rule would instead raise τ with ψ, reinforcing the fall in labor supply and rise in saving. The benchmark thus lets external balances reflect household saving rather than an endogenous fiscal tightening.

The open-economy structure is essential. In a closed economy demographic forces would show up entirely in R, k/l, and factor prices, external balances zero by construction, whereas North–South asymmetries here generate the capital flows central to the paper. Two simplifications bound the framework. A single traded good fixes the terms of trade, so the entire current-account–trade-balance wedge is net foreign income $(R-1)F$. A single traded bond confines valuation effects to that term, excluding the portfolio revaluations Lane and Milesi-Ferretti (2001) emphasizes, and clears the world bond market as a flow restriction $\mathrm{CA}_t + \mathrm{CA}^\star_t = 0$ ((46)) rather than a bilateral stock identity (Footnote 23). Neither restriction overturns results resting on this identity, and richer goods and asset structures are left for future work.

The two regions differ mainly through average retirement span, though also in productivity, financial development, fertility, retiree participation, and pension generosity. Three enter the calibration explicitly, relative productivity Υ, the financial friction $\Gamma^\star$, and the retiree-productivity gap $\mu^\star > \mu$, set from ILO/OECD participation evidence (Section 2, Section 5) rather than left at $\mu = \mu^\star$. Higher Southern fertility and labor-force growth would, by Propositions 5 and 6, partly offset the friction (Proposition 7), while pension-generosity gaps operate through the fiscal channel above. These asymmetries rescale, rather than overturn, Table 1.

[46] A companion sweep re-solves the steady state and comparative statics of Table 1 for $\sigma \in \{0.2, 0.35, 0.5, 0.65, 0.8\}$, spanning Frisch elasticities of roughly 1 to 3 and reproducing ≈ 2.03 at $\sigma = 0.5$; every sign survives, larger elasticities damping the response.

Migration enters the benchmark as an exogenous South-to-North shifter rather than a wage-elastic choice, for the identification reasons given after (7). To quantify that choice, we nest a wage-elastic rule $m_t^{w^\star} = \bar{m} + \eta(w_t - w_t^\star)$, $\eta \geq 0$, with $\eta = 0$ reproducing the exogenous benchmark exactly at the steady state and along the full nonlinear transition.[47] Fig. 14 feeds the net-migration shock e_m from Section 7 through the benchmark and the variant at $\eta \in \{0.15, 0.30\}$ under the calibration of Section 5.1. Immigration lowers the Northern and raises the Southern wage (Proposition 6), and the elastic rule feeds this back, damping but not reversing the external-balance movement. Every entry in the migration row of Table 1 keeps its sign for any $\eta \geq 0$ (Theorem 3). A structural treatment with moving costs and skill selection is left for future work.

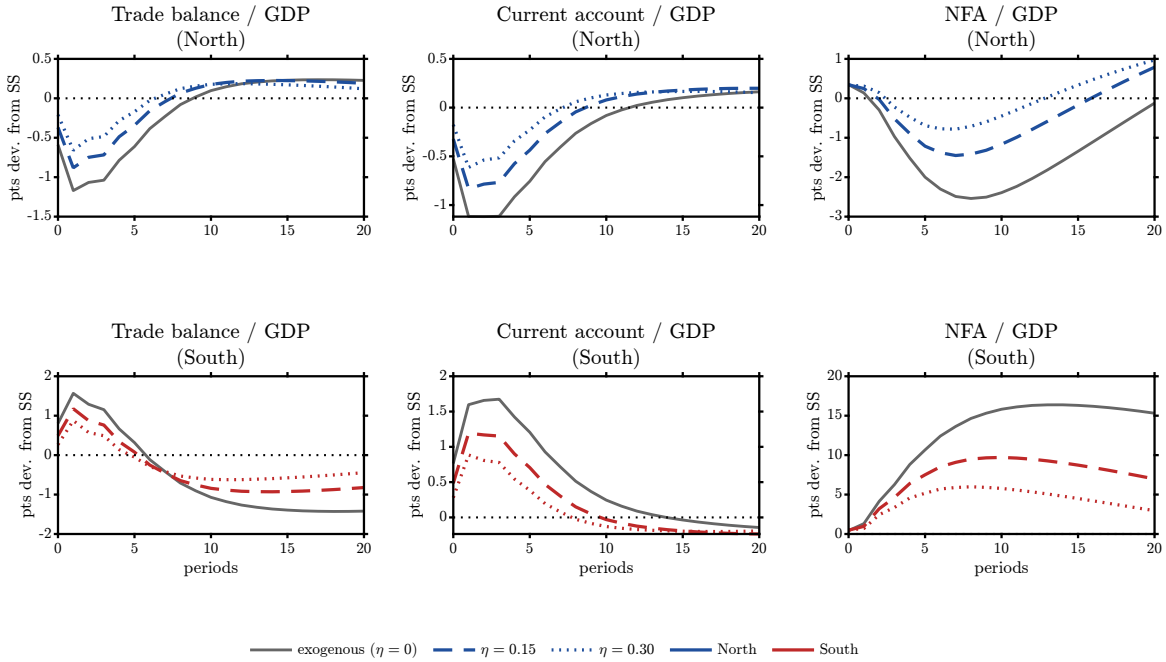


Figure 14: Trade balance, current account, and net foreign assets: benchmark ($\eta = 0$) vs. wage-elastic migration rule ($\eta = 0.15, 0.30$) (computed under Table 3, the current calibration convention; see Section 5.1). (Top row: North; bottom row: South; left to right: trade balance, current account, net foreign assets, as a share of GDP.)

9 Discussion

This section draws the formal and numerical results together, asking what the model delivers on each central question and how each answer sits relative to Section 2.

Propositions 1 to 3 and Lemma 2 establish that any wedge separating the two regions' capital markets, here the South's cost-based intermediation premium, deepens the North's capital–labor ratio, lifts its relative wage, and reshapes the trade balance through relative size and absorption. This is the model's version of the small-open-economy lesson that imperfect capital mobility, not demographic asymmetry alone, shapes the cross-country distribution of capital intensity and factor prices. Where that literature imposes a wedge to match an observed differential, Proposition 1 derives its capital–labor and relative-wage consequences in closed form, so the size of the friction, not merely its presence, is read off the calibration (Section 5.1) and traced through Section 4.2.

[47] The wage gap is measured around each region's own benchmark steady-state wage, so the rule alters only the dynamic response, never the steady state, for any $\eta \geq 0$.

The calibrated model reproduces mature, aging economies as large net creditors (Result 1), consistent with Japan and Germany (Alfaro et al., 2011; Bárány et al., 2023). The contribution is less the sign, anticipated by Bárány et al. (2023) and, in a related life-cycle setting, by Ferrero (2010), than the transparency with which the model attributes it to two separately identified channels, a longevity asymmetry and a retiree-labor-productivity asymmetry, while showing a third plausible channel, annuity-market completeness, does not by itself sign the position (Lemma 5 and Remark 3). Disentangling longevity from participation clarifies which piece of the transition does the work, and shows the result is not a foregone conclusion of any aging asymmetry, since the annuity channel alone would not have delivered it.

Lemma 3 and Propositions 4 and 5 deliver the central comparative statics. Longer expected retirement and slower labor-force growth both raise saving relative to investment, turning an aging economy into a net creditor running a current-account surplus alongside a trade deficit. This confirms in closed form the aging–saving–low-rate nexus of Carvalho et al. (2016), Lisack et al. (2019), and Bielecki et al. (2020), and the demographically timed current-account pressure of Domeij and Floden (2006) and Backus et al. (2014). The contribution is analytical, in that Theorem 2 isolates in closed form the separate saving and investment (capital-widening versus capital-deepening) channels through which a demographic shock moves the current account, rather than reporting only its net simulated response, and clarifies why longevity and population growth move it at different speeds, longevity through the retirement-span saving margin (dampened by the fixed-outlay pension and complete annuities), population growth more directly through capital-widening, a distinction Section 7.3 confirms quantitatively.

A large creditor economy can run a current-account surplus alongside a trade deficit purely as net foreign income on its external position, not an anomaly (Corollary 3). This is the intertemporal balance-of-payments identity underlying Obstfeld and Rogoff (1996) and documented by Lane and Milesi-Ferretti (2001). Its role here is to discipline the paper's numerical steady states and to account, without any additional friction, for the second empirical regularity of Section 2.

South-to-North migration lowers the North's old-age dependency ratio, yet also raises investment demand and lowers the aggregate saving rate enough (Proposition 6 and Theorem 3) that at this calibration it weakens rather than strengthens the North's net foreign asset position and current account. This qualifies the reading of immigration as a structural offset to aging (Bielecki et al., 2020; Gannon et al., 2020). Migration relieves the dependency-ratio channel while failing to relieve the saving channel, since a younger immigrant population also raises investment needs. It also speaks to the "allocation puzzle" of Gourinchas and Jeanne (2013) and the credit-constraint channel of Coeurdacier et al. (2015). Here the direction of capital flows is set by the demographic asymmetry in retirement length, and migration runs, if anything, counter to a pure growth-differential story, raising rather than lowering the receiving region's labor-force growth.

Proposition 7 and Corollary 4 show the cost-based premium, unlike the recentered debt-elastic wedge standard since Schmitt-Grohé and Uribe (2003), never turns into a subsidy. The South always faces a higher rate than the North regardless of the sign or size of its net position. Because the wedge is sign-definite, the same friction can represent a South that is a net debtor at one calibration and a net creditor at another, as it becomes once χ_0 rises far enough (Section 6.1), without an economically awkward sign switch. That a higher premium can, in general equilibrium, strengthen rather than weaken the South's net position (Proposition 7(iii), via the compounding-return channel of Remark 3) also qualifies a partial-equilibrium reading of frictions in financially underdeveloped economies (Song et al., 2011). A friction taxing new borrowing can still leave the South wealthier in net terms once its effect on already-accumulated wealth is counted.

At the steady state, the demographic block collapses onto a three-dimensional sufficient statistic, the dependency ratio, labor-force growth, and the retiree marginal propensity to consume (Equa-

tion (56) and Proposition 10), and every long-run Northern result has an exact Southern mirror. This is close to the sufficient-statistics approach of Auclert et al. (2021), but derives the statistic analytically from microfoundations rather than backing it out empirically, at the cost of two representative regions rather than a large cross-section. Corollary 5 and Proposition 9 extend the logic to the fiscal side. Immigration's effect on pension financing depends on the pension rule, the two-country analogue of Storesletten (2000)'s result that immigration eases pay-as-you-go financing.

Calibrated to match only the world real rate and the South's own target debt ratio, the model reproduces, as an untargeted check, the external profile of today's aging creditors (Table 4), letting the mechanism rather than a tuned parameter carry the empirical burden. The trade-balance decomposition of Section 7.3, cross-checked against an exact Shapley allocation, ranks the drivers, with demographic forces, chiefly slower population growth, dominating relative productivity and migration. This connects the paper to the literature attributing the secular decline in the world rate and the shift in global saving to demographics (Lisack et al., 2019; Carvalho et al., 2016; Eggertsson et al., 2019; Rachel and Summers, 2019), offering a model-consistent rather than reduced-form decomposition. The population-weighted extension of Appendix B.2, built on UN projections, shows the ranking survives, and even strengthens, once the North's shrinking share of world population is accounted for.

The welfare decomposition of Theorem 1, across steady states (Section 6.4) and along the transition (Appendix B.3), delivers the clearest distributional result. Northern retirees bear more of the cost of demographic adjustment than Northern workers across nearly every shock, while a relative-productivity rise, acting mainly along the transition, lifts the Northern cohorts. This is a between-cohort, not within-cohort, statement, since the perpetual-youth structure carries no age index within either group. It is closest to Krueger and Ludwig (2007) and complements Ludwig et al. (2009)'s generational accounting in a broader multi-region setting. The contribution is a closed-form three-way decomposition into intertemporal-price, lifetime-resources, and static-price channels, separately for workers and retirees, rather than a single simulated number. A large enough shock, the financial friction in particular, can overturn the pre-shock creditor position before it signs the welfare transfer, so Result 1's baseline sign should not be extrapolated mechanically, a caution for any welfare exercise reading a transfer's direction off the initial calibration rather than the post-shock equilibrium.

10 Conclusion

Global demographic divergence, with aging and slowing growth in the North set against younger, faster-growing populations in the South, has reshaped saving, capital flows, and external balances, yet its channels are typically studied piecemeal and numerically. To address this, we build a two-country life-cycle model with working pensioners, annuity markets, fiscal policy, and a sign-definite financial friction, and derive in closed form the long-run effects of longevity, population growth, migration, and the intermediation premium.

Longer retirements and slower population growth turn the North into a high-saving net creditor, matching the aging–saving–low-rate nexus of Gertler (1999), Carvalho et al. (2016), and Lisack et al. (2019), while isolating capital widening from capital deepening in a single decomposition (Theorem 2), relative to Domeij and Floden (2006), Backus et al. (2014), and Bárány et al. (2023). Southern retiree labor productivity is set above the North's, $\mu^\star = 0.65 > \mu = 0.55$, following ILO/OECD evidence on elderly labor-force participation. This reinforces the longevity asymmetry (Table 3 and Result 1), so the North runs a current-account surplus alongside a trade deficit while the South runs the mirror-image deficit, reflecting net foreign income on large external positions, $\overline{\mathrm{ca}} = (R-1)f_0 + \bar{\mathrm{nx}}$ (Corollary 3), in the spirit of Lane and Milesi-Ferretti (2001) rather than an

anomaly. Financial development reinforces this, since a less integrated South pays a permanently higher return, deepens its capital less, and channels the released resources abroad as financial claims rather than domestic investment (Proposition 7). Because the intermediation premium is sign-definite (Corollary 4), this channel holds at any level of Southern indebtedness, unlike the recentered debt-elastic wedge standard since Schmitt-Grohé and Uribe (2003).

Migration rejuvenates the North's age structure, offsetting its rising dependency ratio, but also raises investment demand and lowers aggregate saving, so on net it erodes rather than restores the North's external balance (Proposition 6 and Theorem 3), qualifying the "migration as a demographic fix" reading of Bielecki et al. (2020) and Gannon et al. (2020). A welfare decomposition into intertemporal-price, resource, and fiscal channels (Theorem 1) shows the burden of demographic adjustment falls mainly on the North's already-retired cohorts rather than its workers, while a relative-productivity rise lifts the Northern cohorts along the transition. The decomposition is closed-form, evaluated separately for workers and retirees across steady states and along the transition, unlike much of the largely positive quantitative literature (Attanasio et al., 2006), and complements Ludwig et al. (2009)'s generational accounting.

The paper's contribution is to unify these mechanisms in a single tractable framework and to provide analytical decompositions of the external accounts and welfare, making transparent why demographic structure drives global imbalances, in the spirit of Sposi (2022) but characterized analytically rather than through simulation. Three extensions would push the analysis further, the first letting fertility and migration respond endogenously to wage and return differentials. The second would relax the North–South asymmetries retained by the calibration to match a wider range of country pairs, and the third would introduce nominal rigidities so the model could speak to the short-run interaction between demographic trends and monetary policy.

References

Daron Acemoglu and Pascual Restrepo. Secular stagnation? The effect of aging on economic growth in the age of automation. *American Economic Review*, 107(5):174–179, 2017.

Yunus Aksoy, Henrique S Basso, Ron P Smith, and Tobias Grasl. Demographic structure and macroeconomic trends. *American Economic Journal: Macroeconomics*, 11(1):193–222, 2019.

Laura Alfaro, Sebnem Kalemli-Ozcan, and Vadym Volosovych. Sovereigns, upstream capital flows, and global imbalances. Technical report, National Bureau of Economic Research, 2011.

Steve Ambler, Ali Dib, and Nooman Rebei. Optimal taylor rules in an estimated model of a small open economy. Technical report, Bank of Canada, 2004.

Orazio Attanasio, Sagiri Kitao, and Giovanni L Violante. Global demographic trends and social security reform. *Journal of monetary Economics*, 54(1):144–198, 2007.

Orazio P Attanasio, Sagiri Kitao, and Giovanni L Violante. Quantifying the effects of the demographic transition in developing economies. *Advances in Macroeconomics*, 6(1):1–44, 2006.

Adrien Auclert, Hannes Malmberg, Frédéric Martenet, and Matthew Rognlie. Demographics, wealth, and global imbalances in the twenty-first century. NBER Working Paper 29161, National Bureau of Economic Research, 2021.

Alan J Auerbach and Laurence J Kotlikoff. Evaluating fiscal policy with a dynamic simulation model. *The American Economic Review*, 77(2):49–55, 1987.

David Backus, Thomas Cooley, and Espen Henriksen. Demography and low-frequency capital flows. *Journal of International Economics*, 92:S94–S102, 2014.

Zsófia L Bárány, Nicolas Coeurdacier, and Stéphane Guibaud. Capital flows in an aging world. *Journal of International Economics*, 140:103707, 2023.

Henrique S Basso and Juan F Jimeno. From secular stagnation to robocalypse? Implications of demographic and technological changes. *Journal of Monetary Economics*, 117:833–847, 2021.

Marcin Bielecki, Michał Brzoza-Brzezina, and Marcin Kolasa. Demographics and the natural interest rate in the Euro area. *European Economic Review*, 129:103535, 2020.

Olivier J Blanchard. Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–247, 1985.

Ralph C. Bryant. Demographic influences on saving-investment balances in developing and developed economies. Technical Report Global Economy and Development Working Paper No. 12, The Brookings Institution, Washington, DC, 2007.

Carlos Carvalho, Andrea Ferrero, and Fernanda Nechio. Demographics and real interest rates: Inspecting the mechanism. *European Economic Review*, 88:208–226, 2016.

Raj Chetty, Adam Guren, Day Manoli, and Andrea Weber. Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, 101(3):471–475, 2011.

Lawrence J Christiano, Martin Eichenbaum, and Charles L Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45, 2005.

Mr Benedict J Clements, Mr David Coady, Frank Eich, Mr Sanjeev Gupta, Mr Alvar Kangur, Baoping Shang, and Mauricio Soto. *The challenge of public pension reform in advanced and emerging economies*. International Monetary Fund, 2016.

Nicolas Coeurdacier, Stéphane Guibaud, and Keyu Jin. Credit constraints and growth in a global economy. *American Economic Review*, 105(9):2838–2881, 2015.

Thomas Cooley and Espen Henriksen. The demographic deficit. *Journal of Monetary Economics*, 93:45–62, 2018.

Chadwick C Curtis, Steven Lugauer, and Nelson C Mark. Demographics and aggregate savings in japan, China, and India. *Journal of Macroeconomics*, 51:175–191, 2017.

Marco Del Negro, Domenico Giannone, Marc P Giannoni, and Andrea Tambalotti. Global trends in interest rates. *Journal of International Economics*, 118:248–262, 2019.

David Domeij and Martin Floden. Population aging and international capital flows. *International Economic Review*, 47(3):1013–1032, 2006.

Gauti B Eggertsson, Neil R Mehrotra, and Jacob A Robbins. A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics*, 11(1):1–48, 2019.

Larry G Epstein and Stanley E Zin. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, pages 937–969, 1989.

Roger EA Farmer. Rince preferences. *The Quarterly Journal of Economics*, 105(1):43–60, 1990.

Hamid Faruqee and Martin Mühleisen. Population aging in Japan: demographic shock and fiscal sustainability. *Japan and the World Economy*, 15(2):185–210, 2003.

Andrea Ferrero. A structural decomposition of the us trade balance: Productivity, demographics and fiscal policy. *Journal of Monetary Economics*, 57(4):478–490, 2010.

Kristin J Forbes. Why do foreigners invest in the united states? *Journal of International Economics*, 80(1):3–21, 2010.

Luca Fornaro and Federica Romei. The paradox of global thrift. *American Economic Review*, 109 (11):3745–3779, 2019.

Eric French. The effects of health, wealth, and wages on labour supply and retirement behaviour. *Review of Economic Studies*, 72(2):395–427, 2005.

Shigeru Fujita and Ippei Fujiwara. Declining trends in the real interest rate and inflation: The role of aging. Technical report, FRB of Philadelphia Working Paper, 2016.

Frédéric Gannon, Gilles Le Garrec, and Vincent Touzé. The South's demographic transition and international capital flows in a financially integrated world economy. *Journal of Demographic Economics*, 86(1):1–45, 2020.

Mark Gertler. Government debt and social security in a life-cycle economy. *Carnegie-Rochester Conference Series on Public Policy*, 50:61–110, 1999.

Atish R Ghosh. Fiscal policy, the terms of trade, and the external balance. *Journal of International Economics*, 33(1-2):105–125, 1992.

Pierre-Olivier Gourinchas and Olivier Jeanne. Capital flows to developing countries: The allocation puzzle. *Review of Economic Studies*, 80(4):1484–1515, 2013.

Pierre-Olivier Gourinchas and Hélène Rey. International financial adjustment. *Journal of Political Economy*, 115(4):665–703, 2007.

Gary D Hansen and Selahattin İmrohoroğlu. Consumption over the life cycle: The role of annuities. *Review of Economic Dynamics*, 11(3):566–583, 2008.

Matthew Higgins. Demography, national savings, and international capital flows. *International Economic Review*, 39(2):343–369, 1998.

Nir Jaimovich and Henry E Siu. The young, the old, and the restless: Demographics and business cycle volatility. *American Economic Review*, 99(3):804–26, 2009.

Takehiro Kiguchi and Andrew Mountford. Immigration and unemployment: A macroeconomic approach. *Macroeconomic Dynamics*, 23(4):1313–1339, 2019.

Juha Kilponen, Helvi Kinnunen, and Antti Ripatti. Population ageing in a small open economy-some policy experiments with a tractable general equilibrium model. *Bank of Finland Research Discussion Paper*, (28), 2006.

Dirk Krueger and Alexander Ludwig. On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics*, 54(1):49–87, 2007.

David Lagakos, Benjamin Moll, Tommaso Porzio, Nancy Qian, and Todd Schoellman. Life cycle wage growth across countries. *Journal of Political Economy*, 126(2):797–849, 2018.

Philip Lane. External imbalances and fiscal policy. *European Economy*, 14:19, 2010.

Philip R Lane and Gian Maria Milesi-Ferretti. Long-term capital movements. *NBER Macroeconomics Annual*, 16:73–116, 2001.

Noëmie Lisack, Rana Sajedi, and Gregory Thwaites. Population ageing and the macroeconomy. Technical Report Working Paper No. 745, Banque de France, Paris, 2019.

Alexander Ludwig and Edgar Vogel. Mortality, fertility, education and capital accumulation in a simple OLG economy. *Journal of Population Economics*, 23(2):703–735, 2010.

Alexander Ludwig, Dirk Krüger, and Axel Börsch-Supan. Demographic change, relative factor prices, international capital flows, and their differential effects on the welfare of generations. In *Social Security Policy in a Changing Environment*, pages 385–414. University of Chicago Press, 2009.

Nicole Maestas, Kathleen J Mullen, and David Powell. The effect of population aging on economic growth, the labor force, and productivity. *American Economic Journal: Macroeconomics*, 15(2): 306–332, 2023.

Julia Niemeläinen. External imbalances between China and the United States: A dynamic analysis with a life-cycle model. *Journal of International Economics*, 131:103493, 2021.

Owen Nyang'oro and Githinji Njenga. Pension funds in Sub-Saharan Africa. WIDER Working Paper 2022/95, UNU-WIDER, Helsinki, 2022.

Maurice Obstfeld and Kenneth Rogoff. *Foundations of international macroeconomics*. MIT press, 1996.

Andrea Papetti. Demographics and the natural real interest rate: Historical and projected paths for the euro area. *Journal of Economic Dynamics and Control*, 132:104209, 2021.

Lukasz Rachel and Lawrence H Summers. On secular stagnation in the industrialized world. *Brookings Papers on Economic Activity*, 50(1):1–76, 2019.

Stephanie Schmitt-Grohé and Martín Uribe. Closing small open economy models. *Journal of International Economics*, 61(1):163–185, 2003.

Matthias Schön and Nikolai Stähler. When old meets young? germany's population ageing and the current account. *Economic Modelling*, 89:315–336, 2020.

Mr Abdelhak Senhadji. Sources of debt accumulation in a small open economy. Technical report, International Monetary Fund, 1997.

Zheng Song, Kjetil Storesletten, and Fabrizio Zilibotti. Growing like china. *American Economic Review*, 101(1):196–233, 2011.

Zheng Song, Kjetil Storesletten, Yikai Wang, and Fabrizio Zilibotti. Sharing high growth across generations: Pensions and demographic transition in china. *American Economic Journal: Macroeconomics*, 7(2):1–39, 2015.

Michael Sposi. Demographics and the evolution of global imbalances. *Journal of Monetary Economics*, 126:1–14, 2022.

Kjetil Storesletten. Sustaining fiscal policy through immigration. *Journal of Political Economy*, 108 (2):300–323, 2000.

United Nations, Department of Economic and Social Affairs, Population Division. World population prospects 2024: Summary of results. Un desa/pop/2024/dc/no.9, United Nations, 2024.

Philippe Weil. Overlapping families of infinitely-lived agents. *Journal of Public Economics*, 38(2): 183–198, 1989.

Peter Whiteford and Edward Whitehouse. Pension challenges and pension reforms in oecd countries. *Oxford Review of Economic Policy*, 22(1):78–94, 2006.

Menahem E Yaari. Uncertain lifetime, life insurance, and the theory of the consumer. *The Review of Economic Studies*, 32(2):137–150, 1965.

A Additional long-run results

A.1 Sign threshold for the retiree-productivity gap

Result 1 in the main text signs the calibrated net foreign asset position by directly resolving the full nonlinear steady-state system at Table 3's parameter values. The figure below traces that sign across the full range of the retiree-productivity gap $\mu^\star \in [0, 0.70]$, other parameters held at Table 3, and locates the exact threshold at which the North switches from net debtor to net creditor.

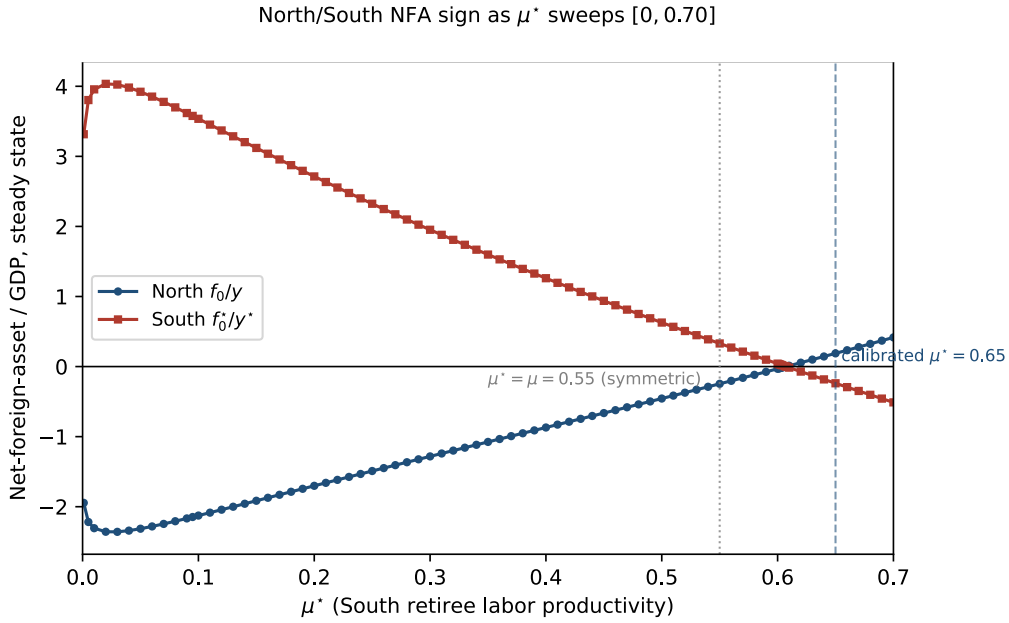


Figure A.1: North's net foreign assets f_0/y and South's $f_0^\star/y^\star$, steady state, as $\mu^\star$ sweeps $[0, 0.70]$, other Table 3 parameters fixed ($\gamma = 0.9333$, $\gamma^\star = 0.9263$, $\zeta = 0.95$, $\zeta^\star = 0.10$, $\chi_0 = 0.018$; each point resolved to a residual below 10^{-13}). The North is net creditor ($f_0 > 0$) for $\mu^\star$ above a threshold $\mu^{\star\star} \approx 0.607$, marked alongside the calibrated $\mu^\star = 0.65$ and the symmetric $\mu^\star = \mu = 0.55$: at the symmetric point the longevity gap and the ζ, χ_0 asymmetries alone do not make the North a creditor, confirming that $\mu^\star > \mu$ is what crosses the threshold in Result 1.

The calibrated creditor sign is also sensitive to the intertemporal elasticity of substitution σ. The figure below sweeps σ over $[0.25, 1.5]$, other parameters held at Table 3, and locates the value at which the North's creditor status flips sign, close to the calibrated $\sigma = 0.5$.

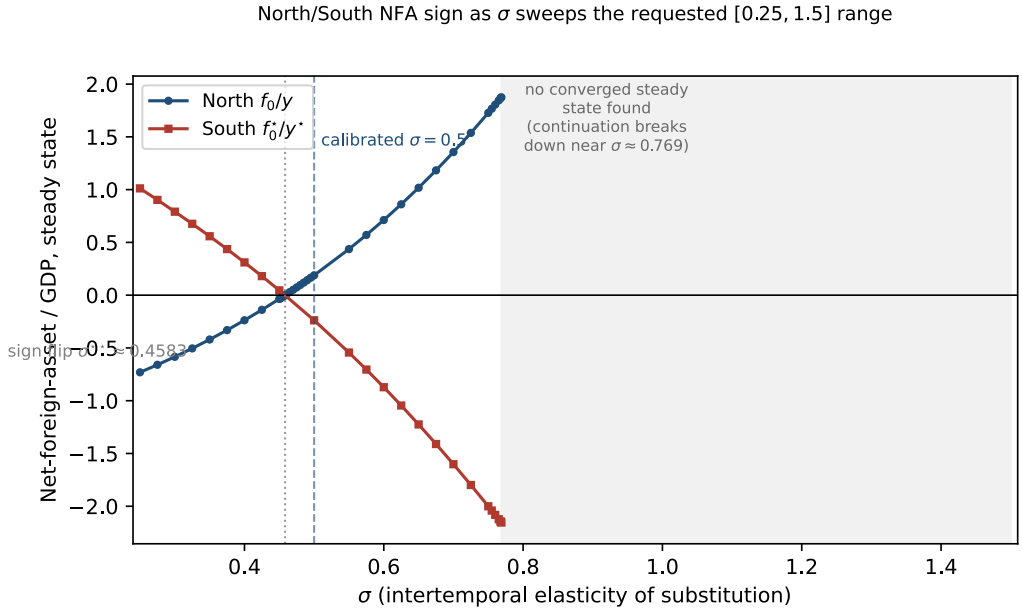


Figure A.2: North's net foreign assets f_0/y and South's $f_0^\star/y^\star$, steady state, as the intertemporal elasticity of substitution σ sweeps $[0.25, 1.5]$, other Table 3 parameters fixed at their calibrated values. The North's creditor status is not robust to σ: it flips sign at $\sigma^{\star\star} \approx 0.4583$, close to the calibrated $\sigma = 0.5$. The continuation breaks down just above $\sigma \approx 0.7688$, short of the requested upper bound of 1.5; the shaded region marks where no converged steady state was found. This is not a solver artifact: the smallest singular value of the steady-state Jacobian collapses monotonically as $\sigma \to 0.769$, the signature of an approaching fold bifurcation beyond which the branch of steady states terminates.

A.2 Cost-based premium and population aging (symmetric baseline)

The figures below give the full steady-state comparative statics summarized in Table 5 and discussed in Section 6.1 and the following subsection: a rising cost-based premium χ_0, higher Southern life expectancy $\gamma^\star$, and a decline in the North's labor-force growth rate, each swept under the symmetric baseline calibration except for the swept parameter itself.

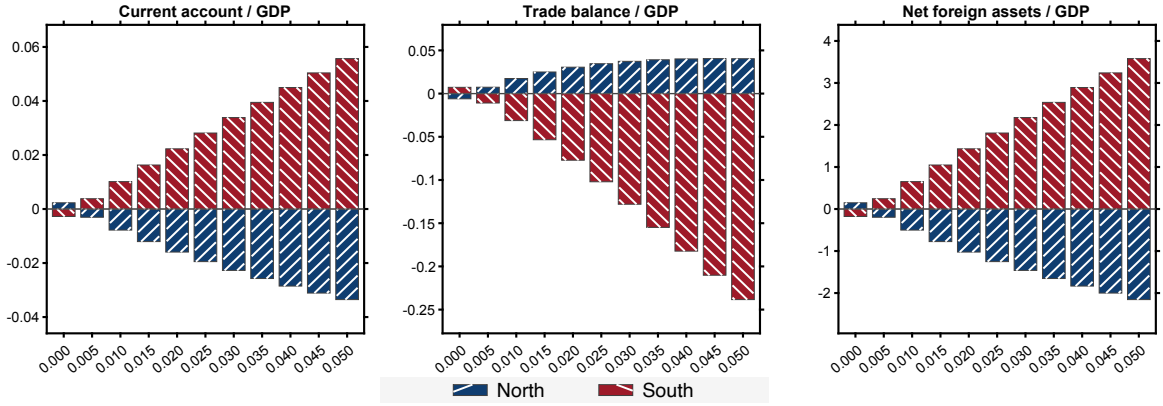


Figure A.3: Long-run effects of a rising cost-based premium on external balances (χ_0: 0 to 0.05). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

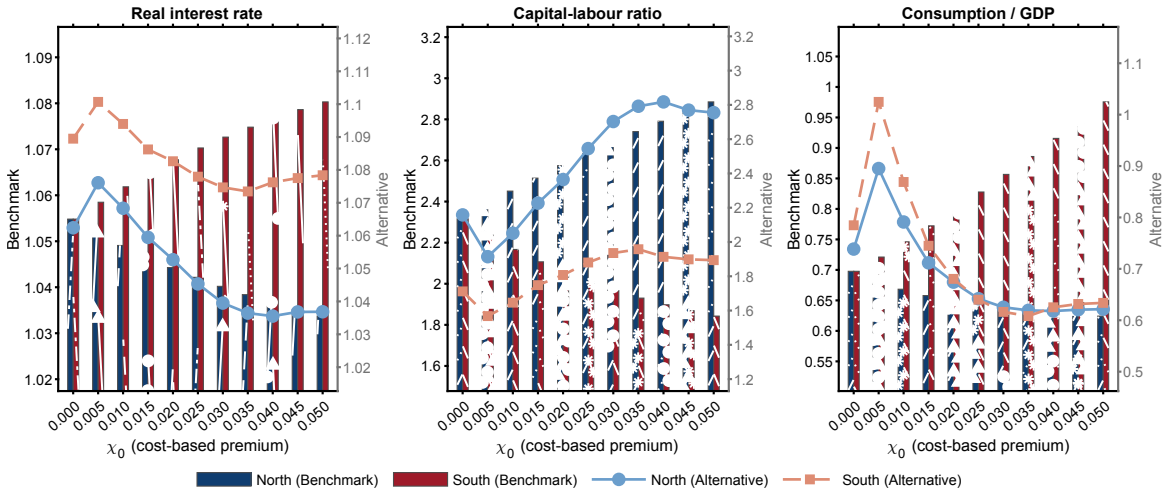


Figure A.4: Real interest rate, capital–labor ratio and consumption/GDP as χ_0 rises: Benchmark and Alternative. (Left to right: real interest rate, capital–labor ratio, consumption/GDP.)

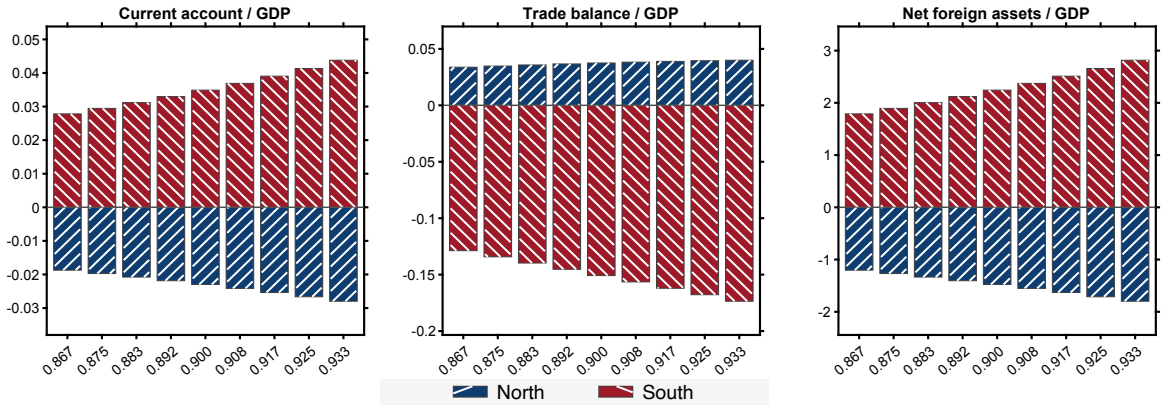


Figure A.5: Long-run effects of higher Southern life expectancy on external balances ($\gamma^\star$: 0.9263 to 0.9333). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

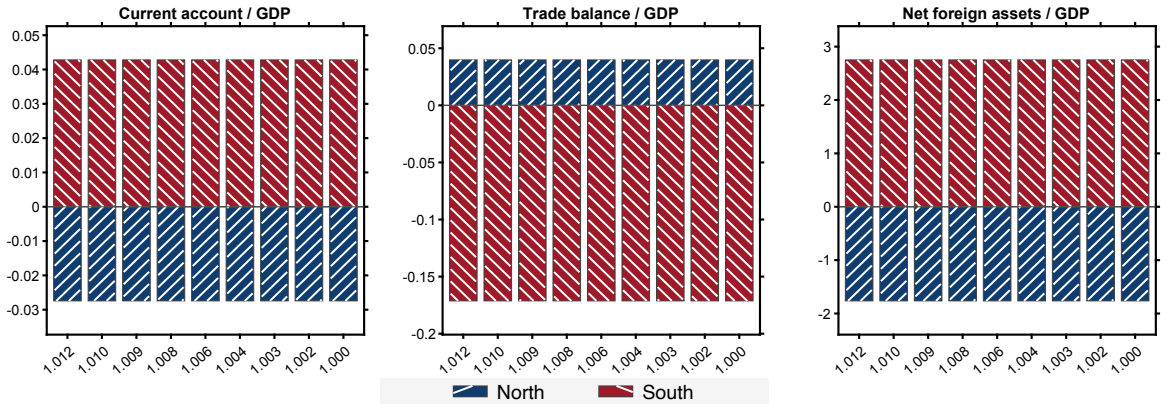


Figure A.6: Long-run effects of a declining Northern population growth rate on external balances. (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

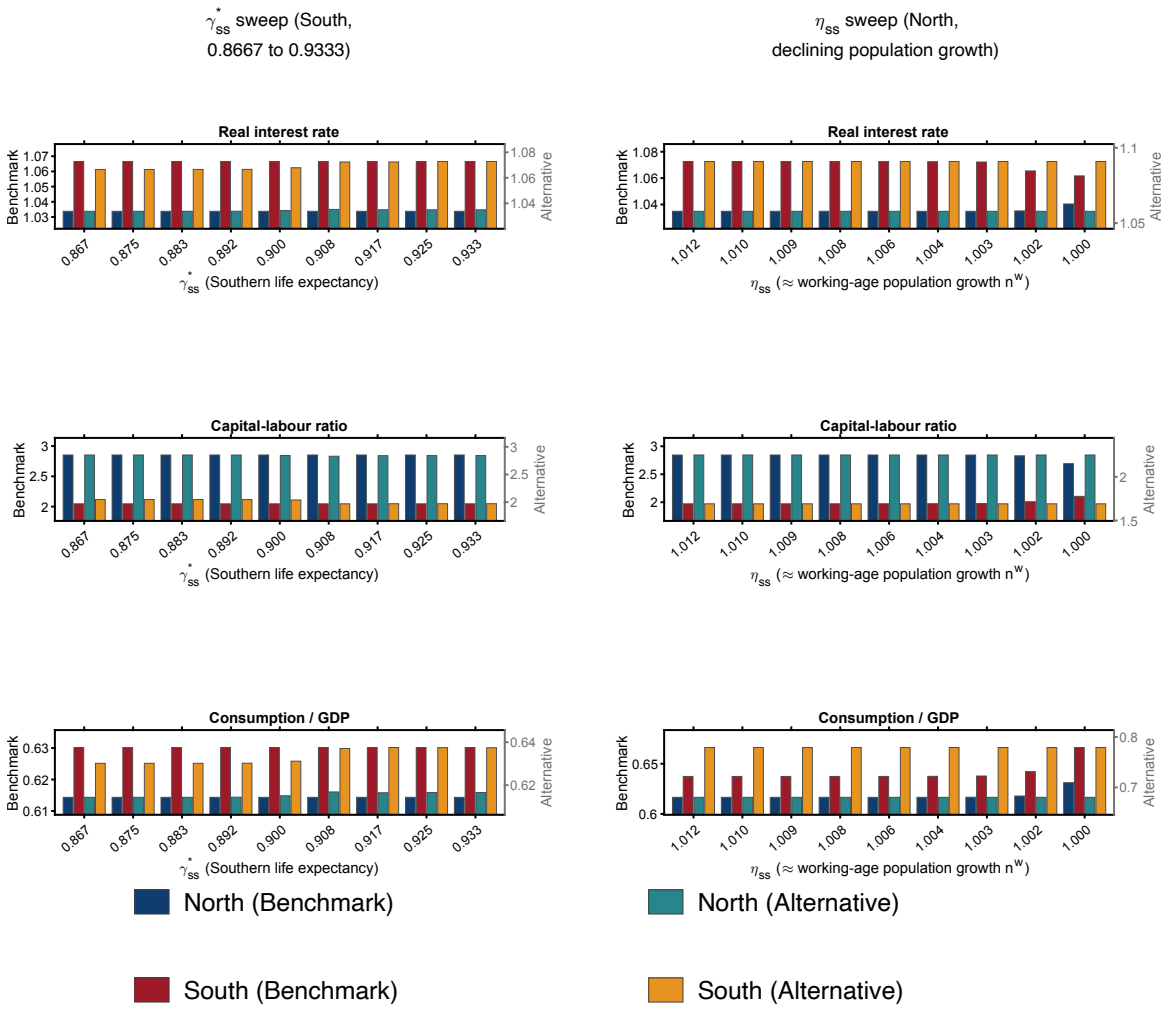


Figure A.7: Real interest rate, capital–labor ratio and consumption/GDP as $\gamma^{\star}$ rises (South) and η_{ss} (the North's steady-state fertility rate n^w, as calibrated in Table 3) declines: Benchmark and Alternative. (Left column: South ($\gamma^{\star}$ sweep); right column: North (η_{ss} sweep); top to bottom: real interest rate, capital–labor ratio, consumption/GDP.)

A.3 Asymmetric North–South calibration

A.4 Annuity-market completeness in general equilibrium

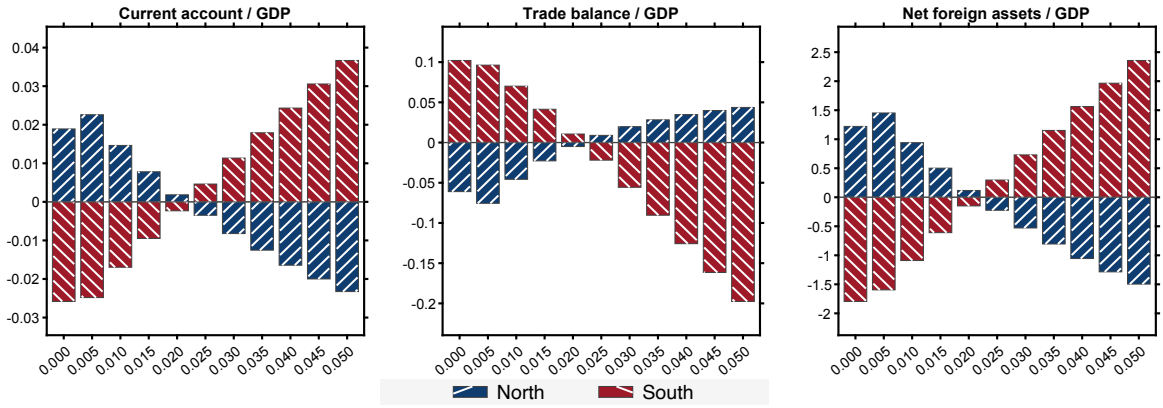


Figure A.8: Long-run effects of a rising cost-based premium χ_0 (asymmetric calibration). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

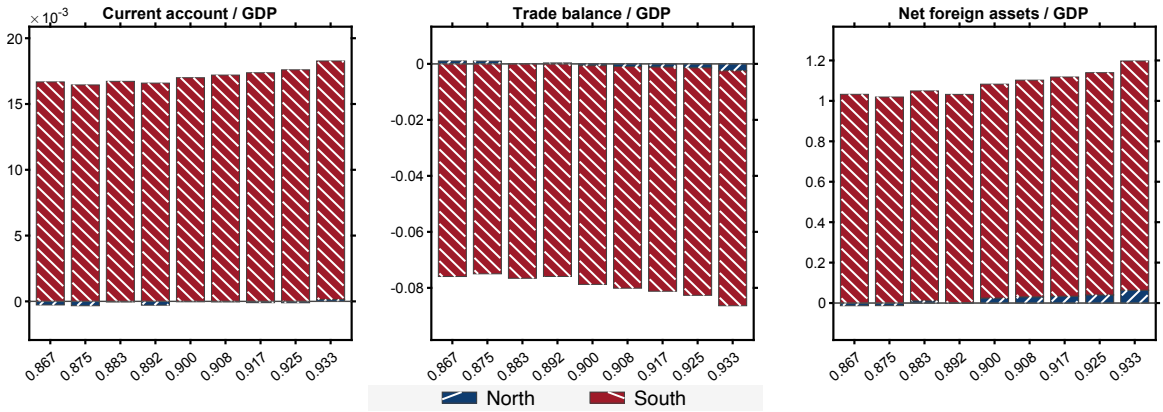


Figure A.9: Long-run effects of higher Southern life expectancy (asymmetric calibration). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

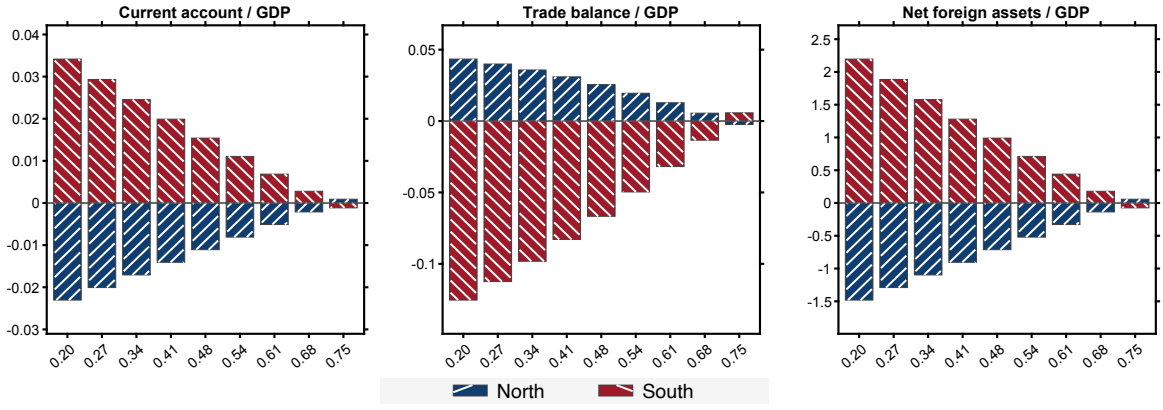


Figure A.10: Long-run effects of higher Southern working-retiree productivity $\mu^\star$ (asymmetric calibration). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

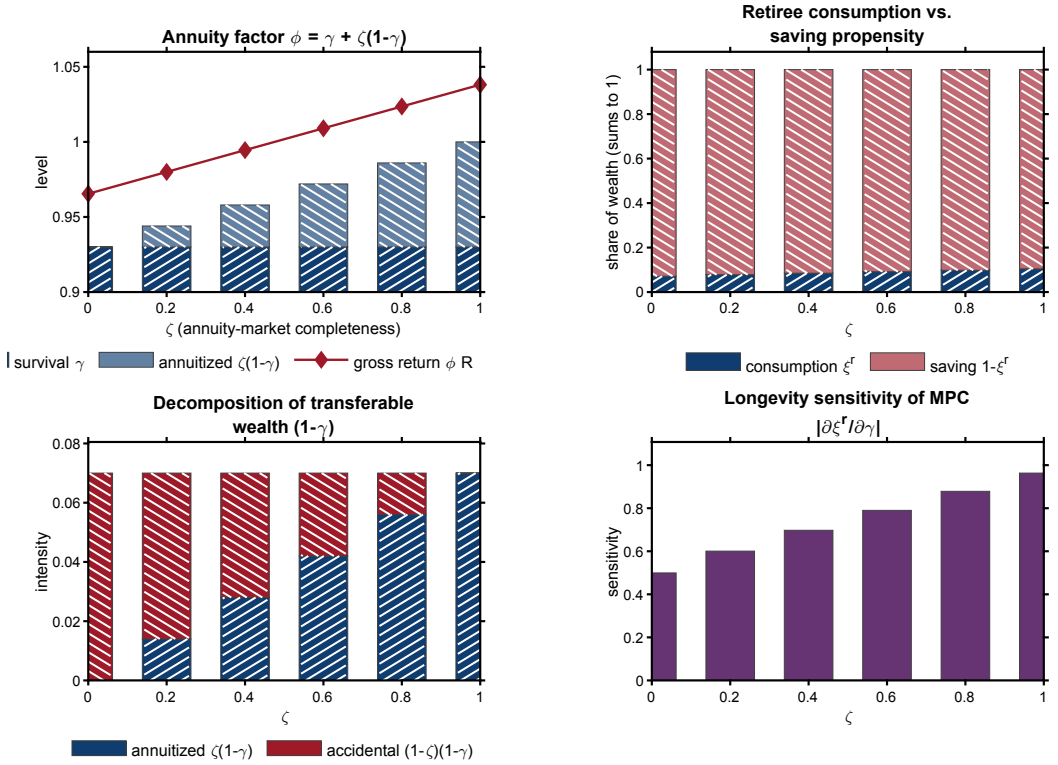


Figure A.11: Steady-state effects of annuity-market completeness ζ. Panels, top to bottom and left to right: annuity factor $\varphi = \gamma + \zeta(1-\gamma)$; retiree consumption versus saving propensity; transferable-wealth share $(1-\gamma)$; and longevity sensitivity of the MPC, $|\partial\xi^r/\partial\gamma|$.

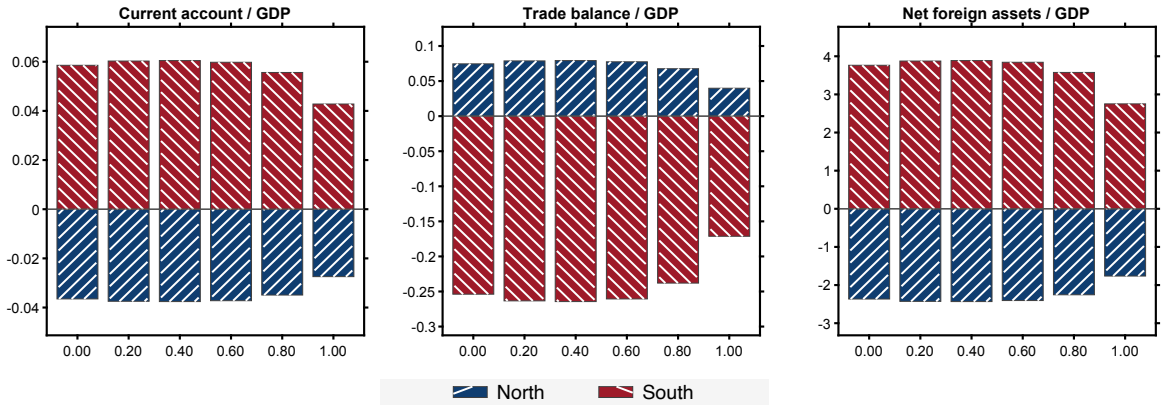


Figure A.12: Long-run effects of higher annuity-market completeness ζ (symmetric baseline). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

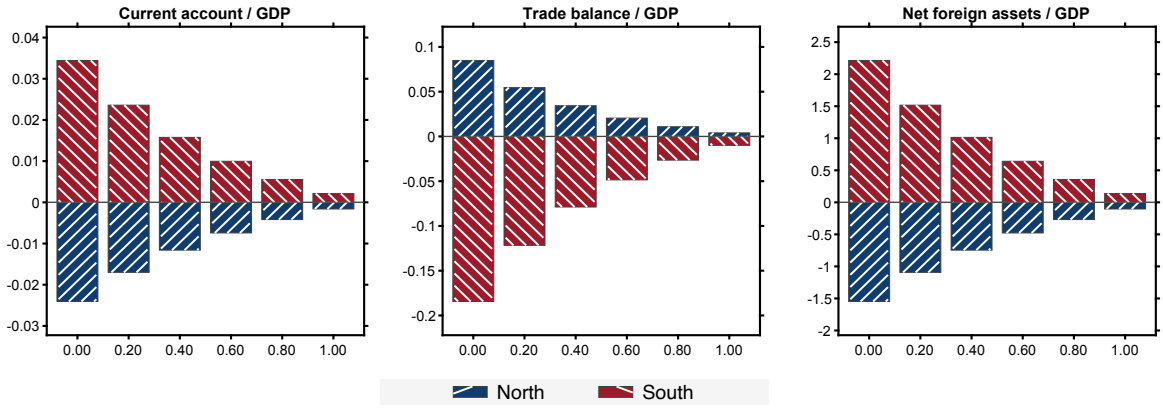


Figure A.13: Long-run effects of higher annuity-market completeness ζ (asymmetric baseline). (Left to right: current account, trade balance, net foreign assets, as a share of GDP.)

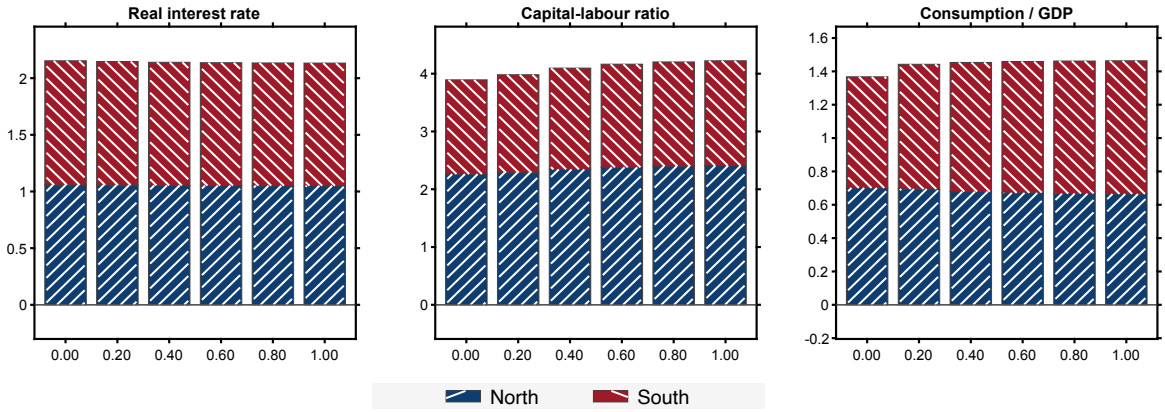


Figure A.14: Real interest rate, capital–labor ratio and consumption/GDP as the South's ζ rises (North's fixed): Alternative calibration. (Left to right: real interest rate, capital–labor ratio, consumption/GDP.)

B Simulations

The figures below complete the dynamic-response evidence discussed in Section 7. For each of the four structural shocks, one panel group (interest rate and capital) is already shown in the main text; the remaining panel groups (consumption/investment/output/assets, the labour market, and demographics) are collected here. Every figure overlays the benchmark and the alternative (asymmetric North–South) calibration on the same panel, so the two economies can be read off directly rather than compared across separate figures. The fertility and longevity shocks' demographic panels (Figs. B.5 and B.8) are the exception: because these responses are near-monotone step/decay paths, each is reported as grouped bars at four representative horizons ($t = 1, 5, 10, 20$) rather than as continuous lines, which keeps the same four-series (North/South $\times$ Benchmark/Alternative) comparison while remaining readable in bar form. Each panel also drops whichever demographic margins are structurally unresponsive to its own shock (each of the four shocks moves only its own demographic margin): the fertility panel omits immigration and the survival probability, and the longevity panel omits immigration and worker-population growth, so that every panel shown carries information. All eleven figures of this section are computed directly under Table 3, from the linearized companion solve of Section 7.3.

B.1 The dynamic effects of increasing the North's productivity

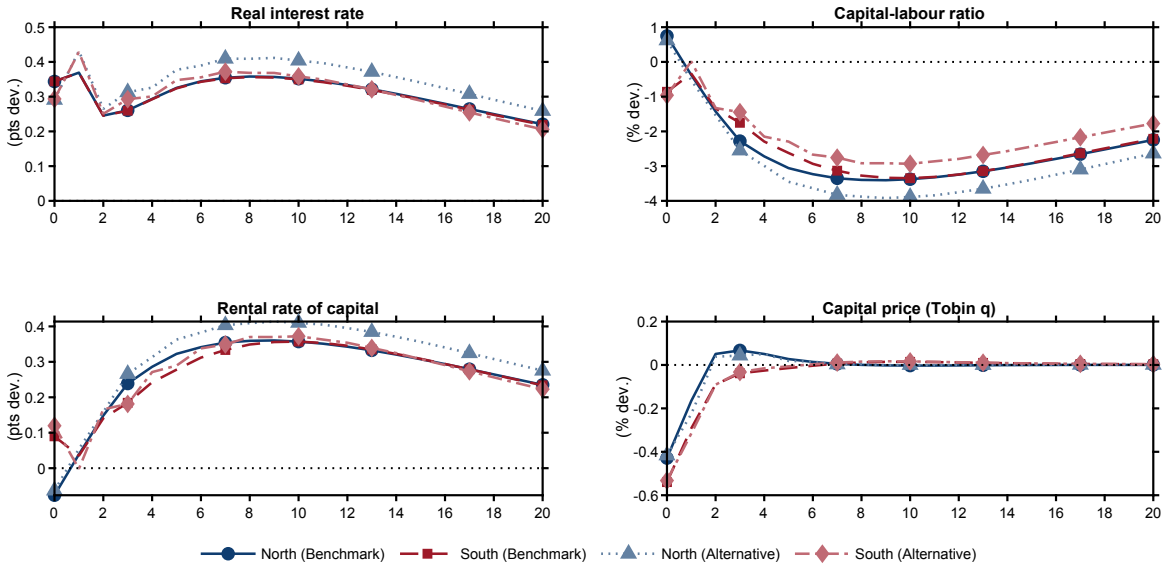


Figure B.1: Productivity shock: interest rate, capital, and profits, Benchmark and Alternative. (Panels, top to bottom, left to right: real interest rate; capital–labor ratio; rental rate of capital; capital price (Tobin's q).)

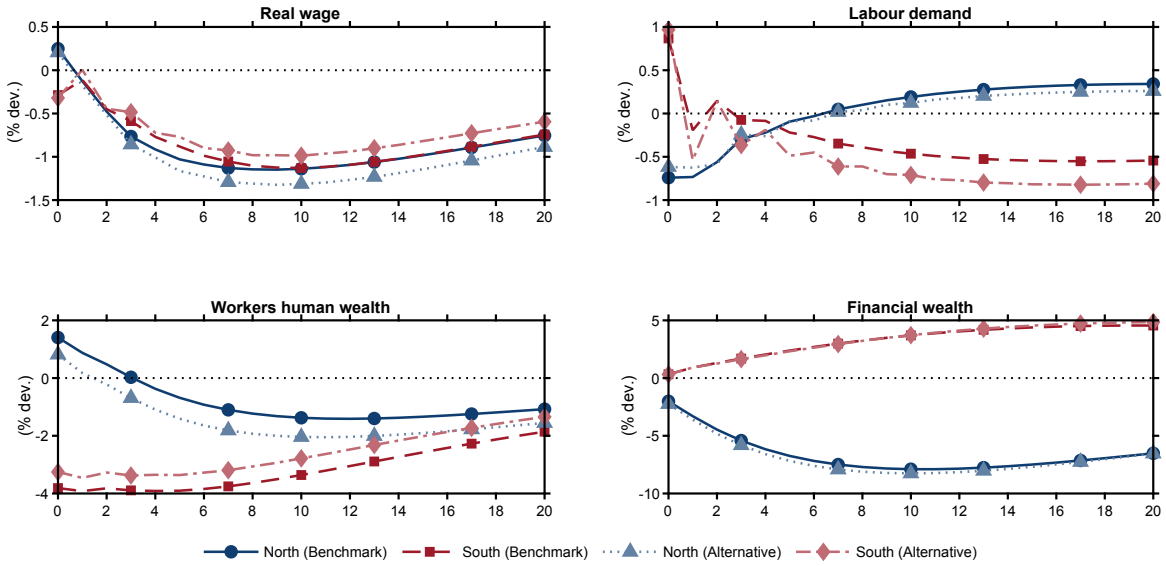


Figure B.2: Productivity shock: labour market and wealth, Benchmark and Alternative. (Panels, top to bottom, left to right: real wage; labor demand; workers' human wealth; financial wealth.)

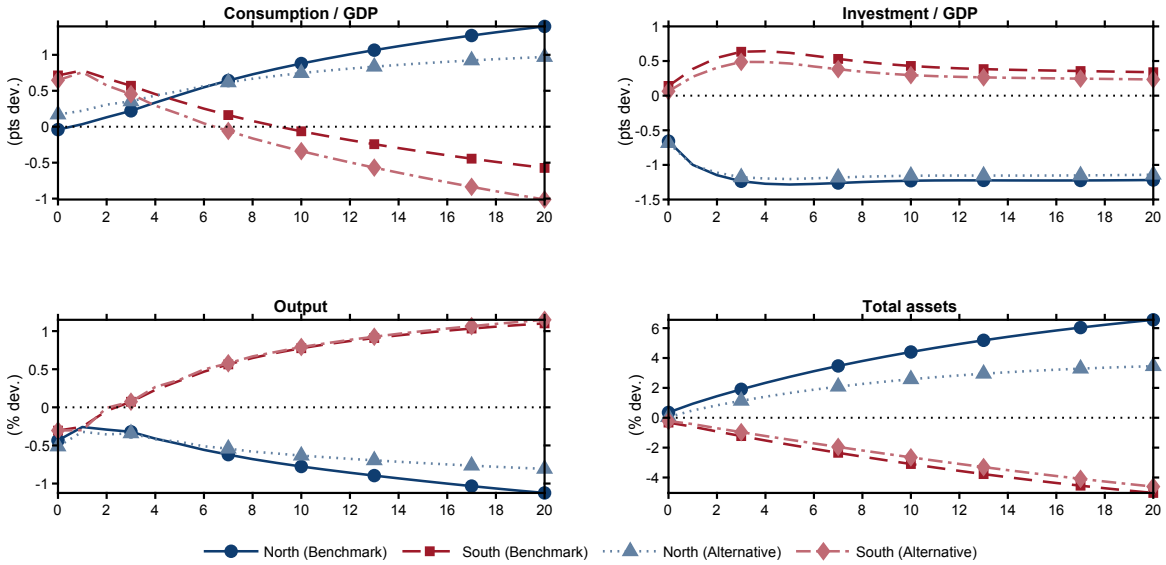


Figure B.3: Fertility shock: consumption, investment, output, and assets, Benchmark and Alternative. (Panels, top to bottom, left to right: consumption/GDP; investment/GDP; output; total assets.)

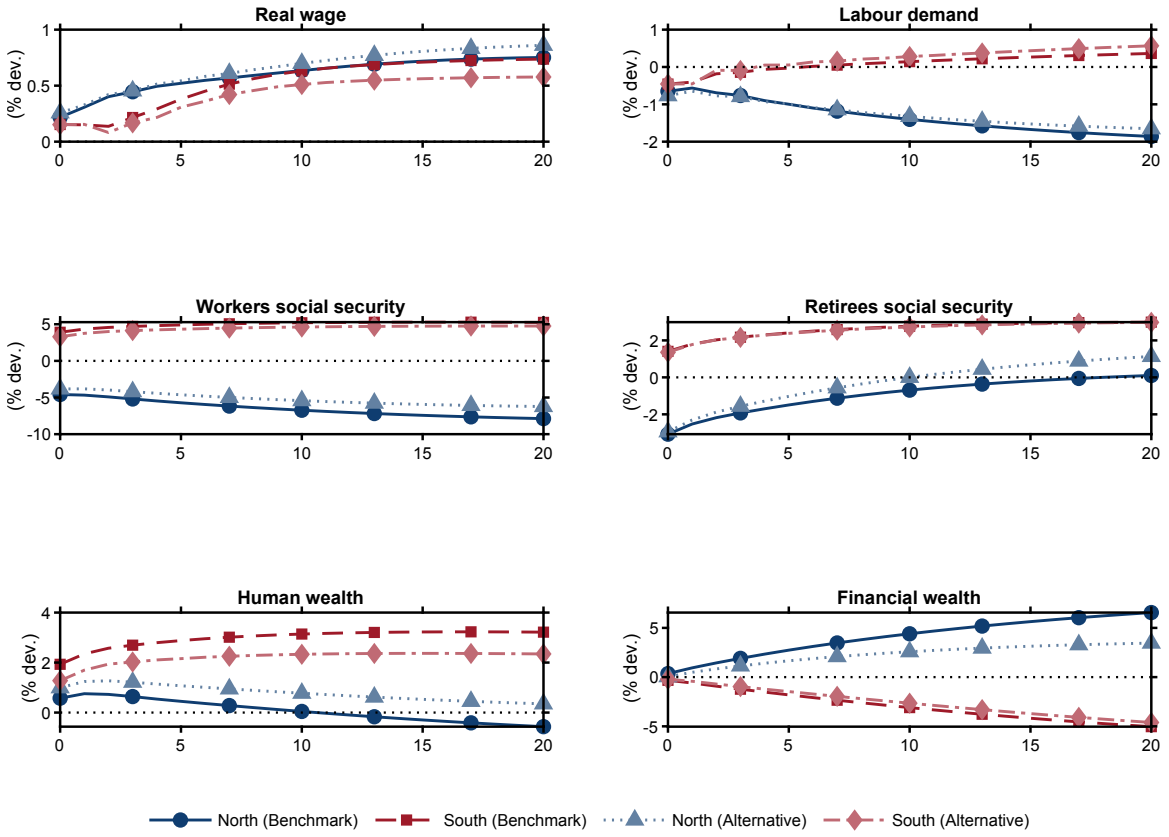


Figure B.4: Fertility shock: labour market, social security, and wealth, Benchmark and Alternative. (Panels, top to bottom, left to right: real wage; labor demand; workers' social security; retirees' social security; human wealth; financial wealth.)

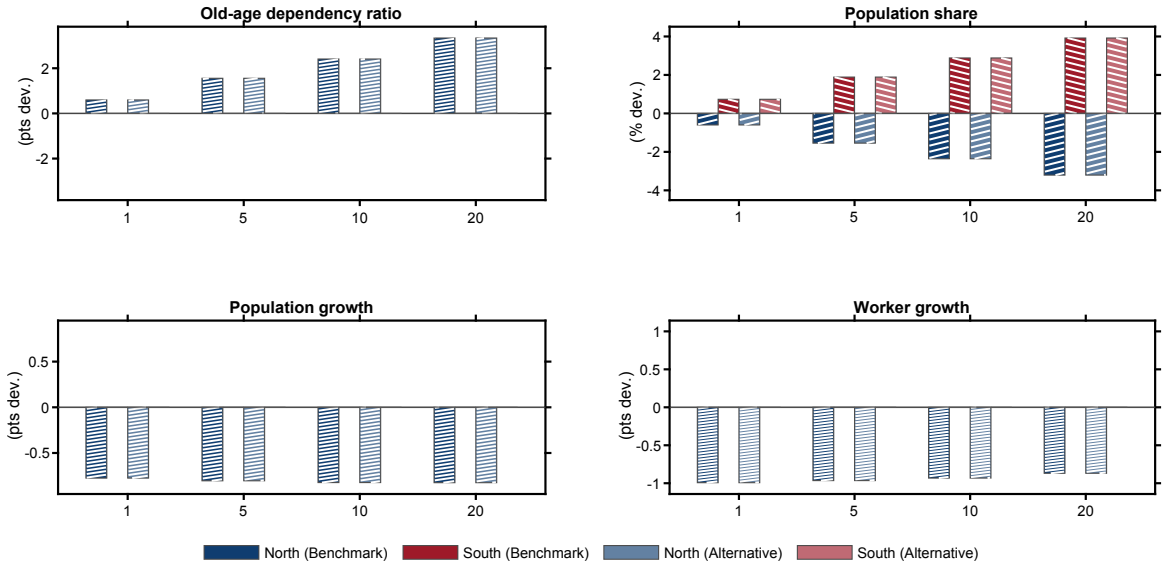


Figure B.5: Fertility shock: demographics at selected horizons ($t = 1, 5, 10, 20$), Benchmark and Alternative. (Panels, top to bottom, left to right: old-age dependency ratio; population share; population growth; worker growth.)

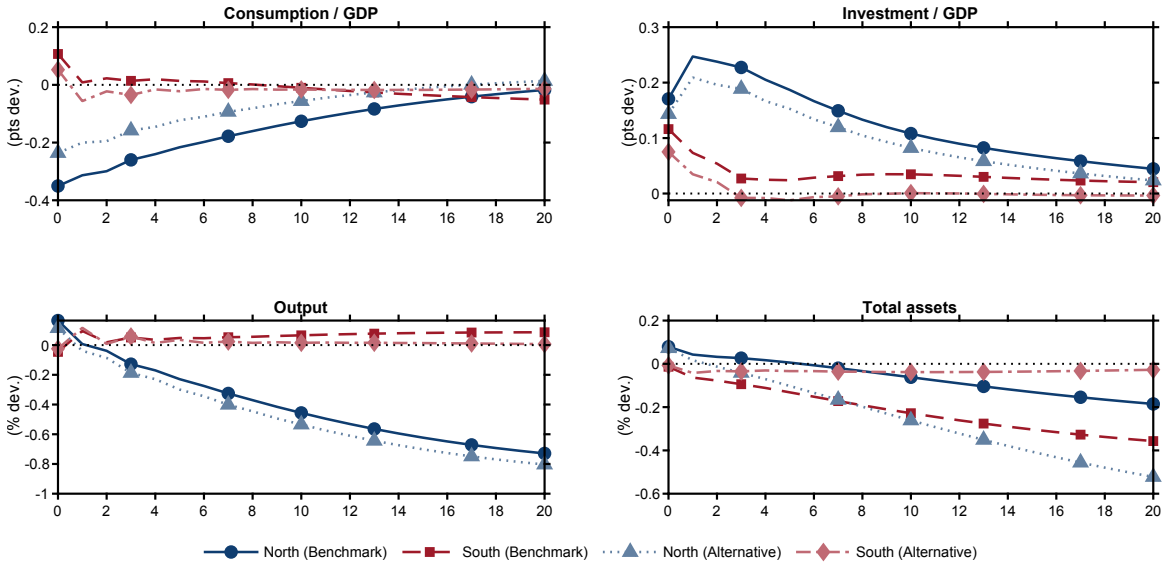


Figure B.6: Longevity shock: consumption, investment, output, and assets, Benchmark and Alternative. (Panels, top to bottom, left to right: consumption/GDP; investment/GDP; output; total assets.)

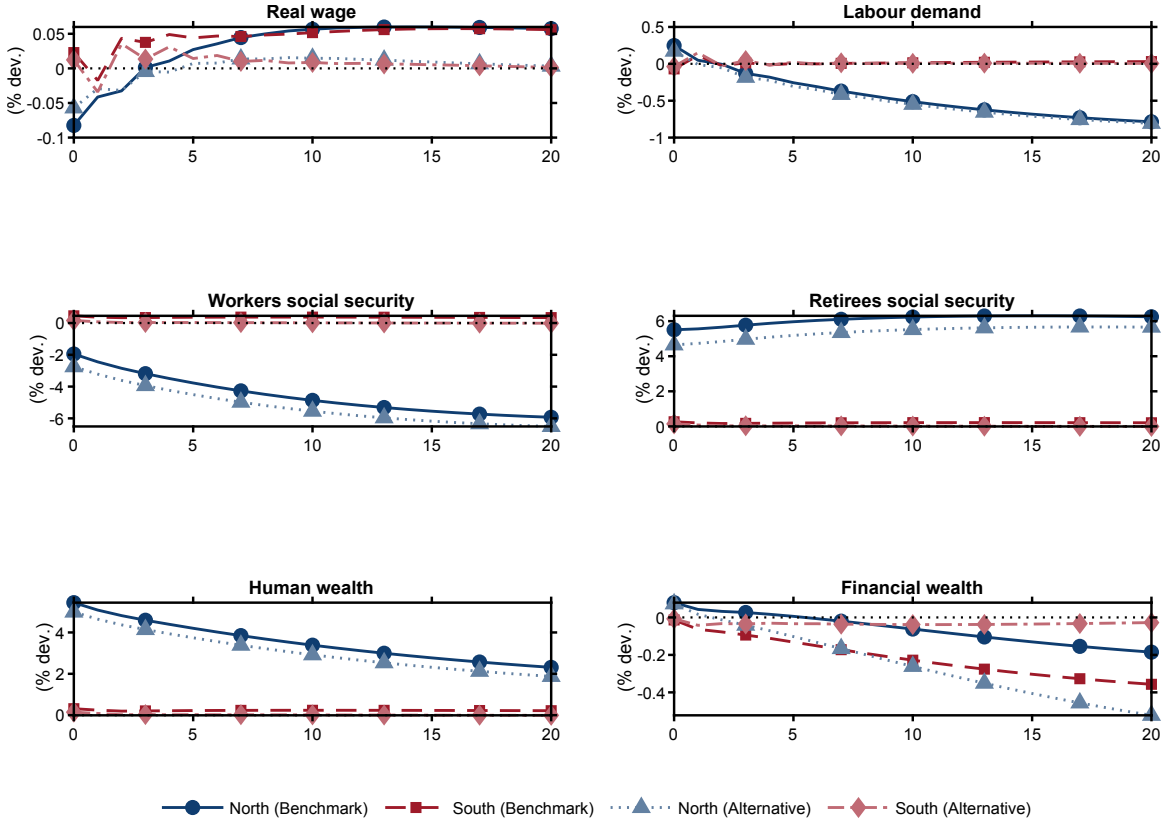


Figure B.7: Longevity shock: labour market, social security, and wealth, Benchmark and Alternative. (Panels, top to bottom, left to right: real wage; labor demand; workers' social security; retirees' social security; human wealth; financial wealth.)

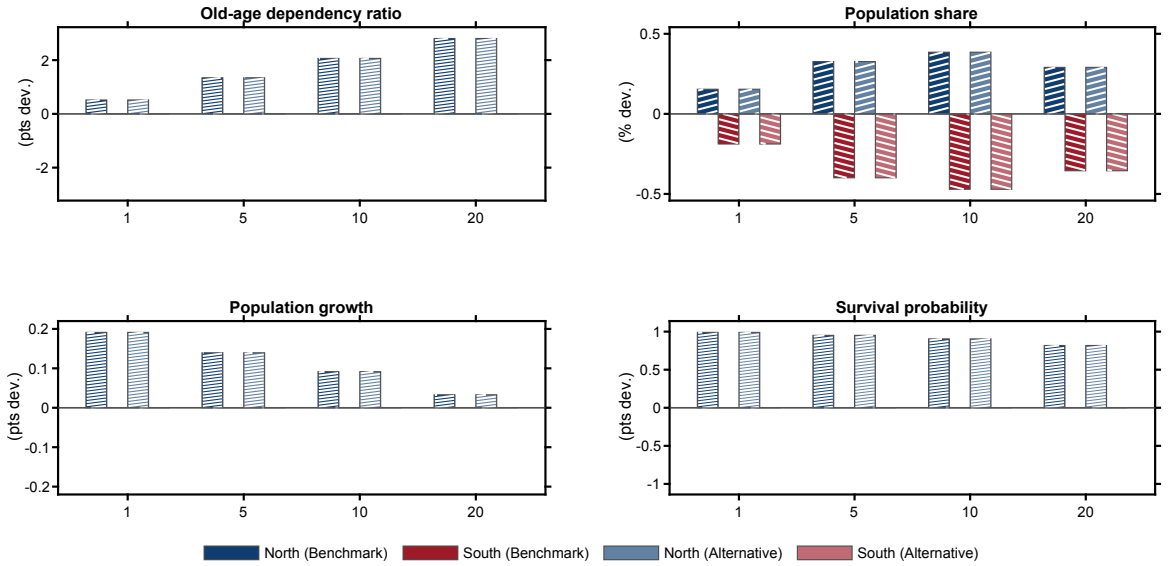


Figure B.8: Longevity shock: demographics at selected horizons ($t = 1, 5, 10, 20$), Benchmark and Alternative. (Panels, top to bottom, left to right: old-age dependency ratio; population share; population growth; survival probability.)

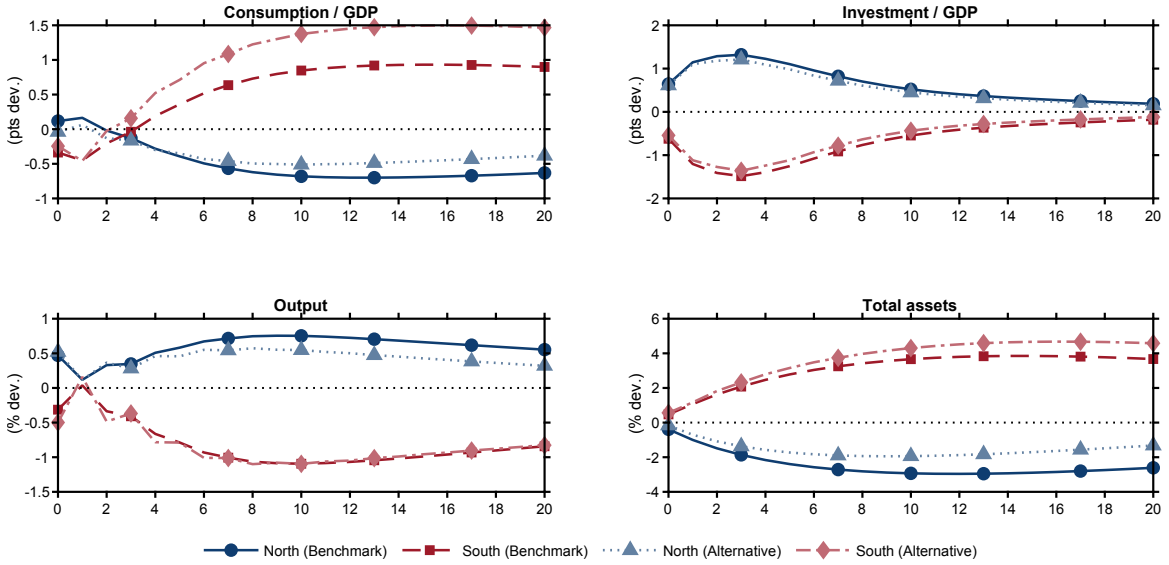


Figure B.9: Immigration shock: consumption, investment, output, and assets, Benchmark and Alternative. (Panels, top to bottom, left to right: consumption/GDP; investment/GDP; output; total assets.)

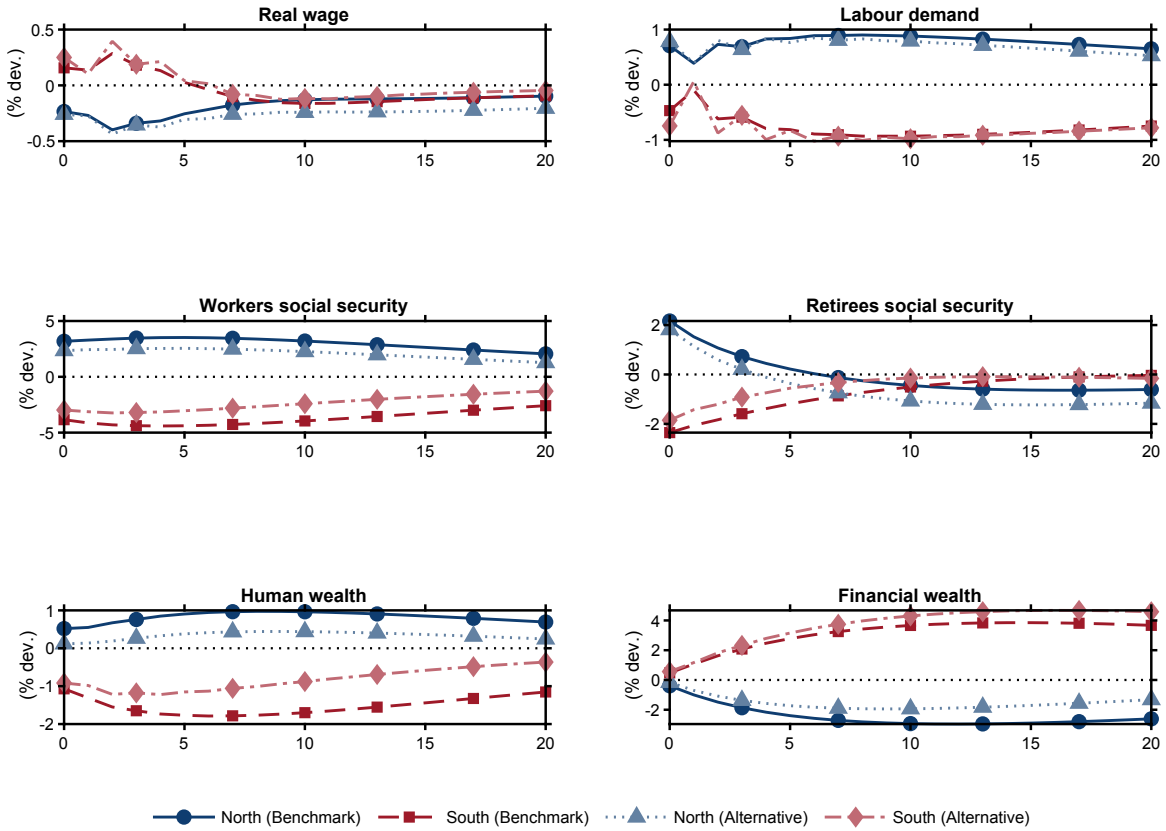


Figure B.10: Immigration shock: labour market, social security, and wealth, Benchmark and Alternative. (Panels, top to bottom, left to right: real wage; labor demand; workers' social security; retirees' social security; human wealth; financial wealth.)

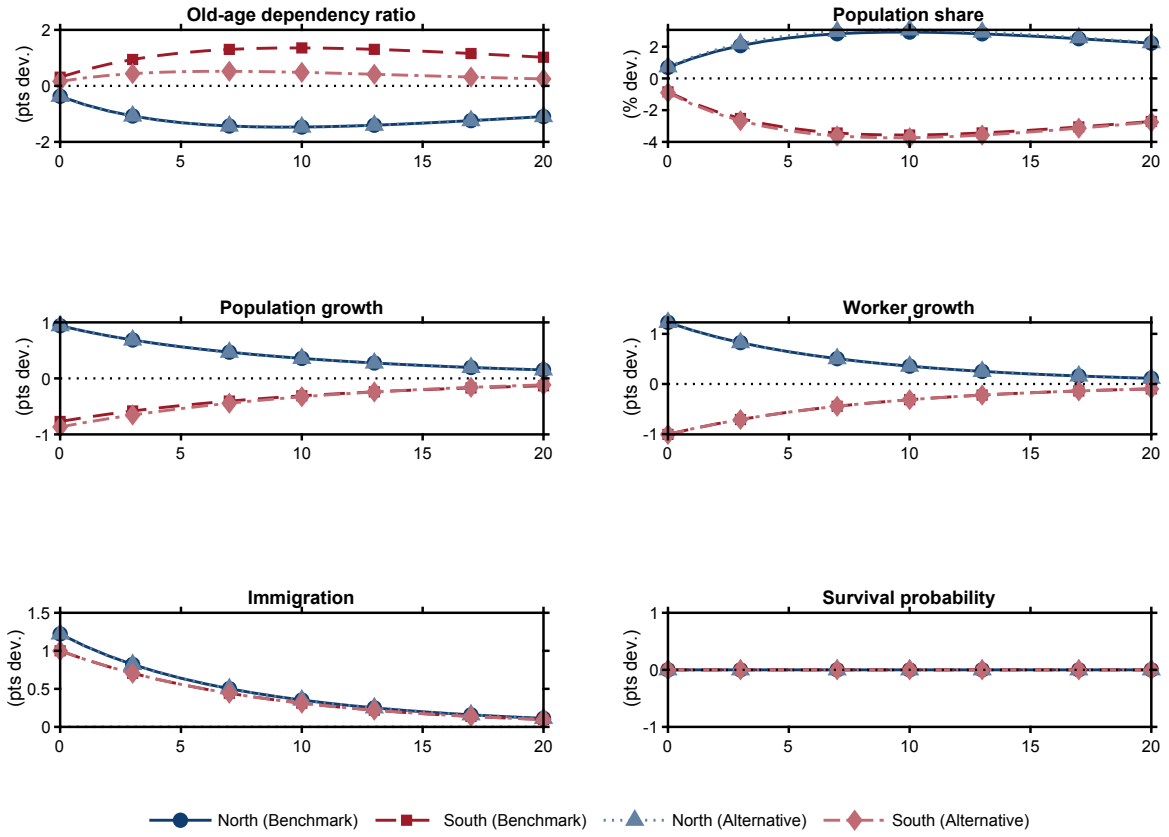


Figure B.11: Immigration shock: demographics, Benchmark and Alternative. (Panels, top to bottom, left to right: old-age dependency ratio; population share; population growth; worker growth; immigration; survival probability.)

B.2 A population-weighted world perspective, 2025–2100

Table 7 attributes the trade-balance path of a single bloc, the North, to the four drivers. A world observer would weight the North's and South's paths by their relative population size, exactly as the model's own world resource constraint does in the steady state, $\kappa\,\overline{\text{ca}}+(1-\kappa)\,\overline{\text{ca}}^{\star}=0$ (Proposition 10), with κ the North's calibrated population share (kap = 0.55). Along the transition this weight is not constant: over 2025–2100 the UN World Population Prospects (WPP), 2024 revision, project the population share of today's higher-income, lower-fertility economies (the model's North) to keep shrinking relative to today's lower-income, higher-fertility economies (the South), continuing a trend already underway.[48]

Concretely, let κ_t decline linearly from the model's calibrated steady-state weight $\kappa_1 = 0.55$ (an anchor for $t = 1 \sim 2025$) to an illustrative $\kappa_H = 0.50$ at $t = H = 200 \sim 2100$, and define the population-weighted world trade-balance path for driver (or driver subset) S as

$$\text{nx}_t^{\text{world},(S)} = \kappa_t\,\text{nx}_t^{(S)} + (1-\kappa_t)\,\text{nx}_t^{\star,(S)}, \tag{B.1}$$

the same convex combination as Proposition 10, applied period by period using the four drivers' impulse responses for both blocs (North nx_t and South $\text{nx}_t^{\star}$), computed here under this section's own linearized companion solve, kept methodologically separate from Table 7's nonlinear decomposition (Section 7.3). Re-running the identical Shapley procedure on $\sum_{t=1}^{H}\left(\text{nx}_t^{\text{world},(S)}\right)^2$ in place of the North-only objective gives population-weighted shares of 10.9% (longevity), 90.7% (population growth), −11.5% (migration), and 9.9% (productivity); because these figures still rest on the linearized companion solve rather than Table 7's nonlinear one, they are not directly comparable driver-by-driver to Table 7 and would need their own nonlinear rerun to be placed on the same footing.

Because the North's and South's trade-balance responses still partly offset one another once combined at population weights close to $\kappa \approx 1/2$, the population-weighted joint sum of squared deviations is smaller than the North-only figure underlying Table 7, though only moderately so, about 16.9% as large ($v(\{\gamma, n, m, x\}) = 0.00217$ against 0.01287). The population-weighted Shapley shares stay within $[0, 100]$% for three of the four drivers, and only migration turns negative. The substantive lesson is not the precise magnitudes, which inherit both the illustrative nature of κ_t (Footnote 48) and this section's own linearized companion solve, but the qualitative point that a population-weighted, two-bloc "world" perspective remains sensitive to cross-country offsetting relative to a single-bloc view: population growth remains the dominant driver of the world-weighted path, and productivity adds to it like longevity and population growth; only migration, whose emigration channel of Section 3 still moves the North and South in offsetting directions, subtracts from the world aggregate once the South is weighted in. This section's object is not a numerically exact reweighting of Table 7, which would require assigning every WPP-covered country to North or

[48] Data disclosure. We anchor κ_t below to three published headline figures from United Nations, Department of Economic and Social Affairs, Population Division (2024), robust across vintages and insensitive to the exact North/South country list, rather than to a country-by-country extraction from the UN WPP online database, since these three facts alone are sufficient to discipline what this exercise needs, the sign of the North's declining population share and an order-of-magnitude pace for it, and a full country-level extraction would not change either: world population rises from 8.2 billion in 2024 to a medium-variant peak of about 10.3 billion in the mid-2080s before easing to about 10.2 billion by 2100; population size has already peaked, as of 2024, in 63 countries and areas, including higher-income, lower-fertility economies (Germany and Japan among them) motivating this paper's North calibration, that group's total population projected to fall by roughly 14% over 2024–2054; and essentially all further growth to 2100 accrues to today's lower-income, higher-fertility South. Together these pin down the sign (declining North share) and an order-of-magnitude pace (a high-single- to low-double-digit percentage-point decline over 2025–2100) without the exact country-by-country series. A reader with that series can substitute it into the companion routine for a numerically exact version; the chosen slope in (B.1) below is conservative relative to what the headline growth-differential facts imply, so it does not overstate the exercise's conclusion.

South and is beyond the two-bloc model's intended scope, but the qualitative point that cross-bloc offsetting survives once the North's population weight is allowed to decline at a pace disciplined by Footnote 48's headline WPP facts; that conclusion is what Section 7.3's single-bloc ranking should be read alongside. Replacing the two-point linear κ_t with the full WPP 2024 medium-variant path, country by country, and re-solving the fully nonlinear transition under that time-varying weight would sharpen the magnitudes further; we do not view this as an open weakness, since Footnote 48 shows the chosen slope is conservative relative to the underlying WPP facts, but it remains a natural extension for a follow-up paper.

B.3 Welfare along the transition

The long-run comparison in Section 6.4 scores the welfare of a newborn entering each economy at its new stationary equilibrium, saying nothing about cohorts alive when the shock hits. This section measures welfare instead along the calibrated perfect-foresight transition paths underlying Section 7.

For a household of type $z \in \{w, r\}$ alive when the economy is displaced from its initial steady state 0, the relevant object is the value function evaluated on impact along the transition, V_1^z, discounting the full sequence of future prices and resources as the shock propagates and mean-reverts. The transitional consumption-equivalent variation is

$$\lambda_{\text{tr}}^z = \frac{V_1^z}{V_0^z} - 1, \tag{B.2}$$

the permanent percentage change in the type-z consumption basket leaving a household indifferent between remaining forever at the initial steady state and living through the calibrated transition ($\lambda_{\text{tr}}^z > 0$: gain). It shares the value-function ratio of (60), but where (60) sets V_0^z against V_∞^z to ask whom a shock ultimately favors once stationary, (B.2) keeps V_0^z in the denominator and the transition path's impact value in the numerator, asking how cohorts mid-adjustment fare as the impulse dies out. We evaluate (B.2) by feeding each driver its calibrated path, one at a time, jointly as the bundle $\gamma + n$, and all together, reading V^w, V^r, $\mathcal{W}$, and their Southern counterparts, in the impact period.

Three features organize Fig. B.12. The relative-productivity impulse benefits most cohorts, raising impact welfare by 4.03% for Northern workers, 2.92% for Northern retirees, and 0.27% for Southern workers, though not unambiguously, since Southern retirees lose 0.81%. The aging impulses are costly for the Northern cohorts alive when they land, as greater longevity lowers Northern welfare by 3.72% for workers and 11.12% for retirees, since longer retirement must be financed from slowly adjusting resources, while slower population growth costs Northern workers 1.20% and retirees 1.42%; fed jointly, the bundle $\gamma + n$ compounds this into a transitional loss of 7.35% in social terms and 12.54% for retirees, leaving Southern welfare close to unchanged (0.33% socially). Immigration raises welfare for both Northern cohorts on impact (0.99% workers, 1.49% retirees), a milder echo of the long-run split in Table 6, while reducing welfare throughout the South (-0.91% workers, -1.74% retirees).

The transition results point to two further regularities. Within the North, retirees swing more than workers under both aging shocks, since their shorter horizon weights the low-welfare early-transition periods more heavily than the eventual return to equilibrium; the pattern reverses for the South's longevity shock, where the small impact falls more on workers than on Southern retirees, who are inactive and shorter-lived. The productivity gain does not fully offset the demographic cost once all drivers are fed together: Northern workers turn slightly positive (0.10%), while Northern retirees remain at -8.14% and the social aggregate at -2.68%. The result is intergenerational:

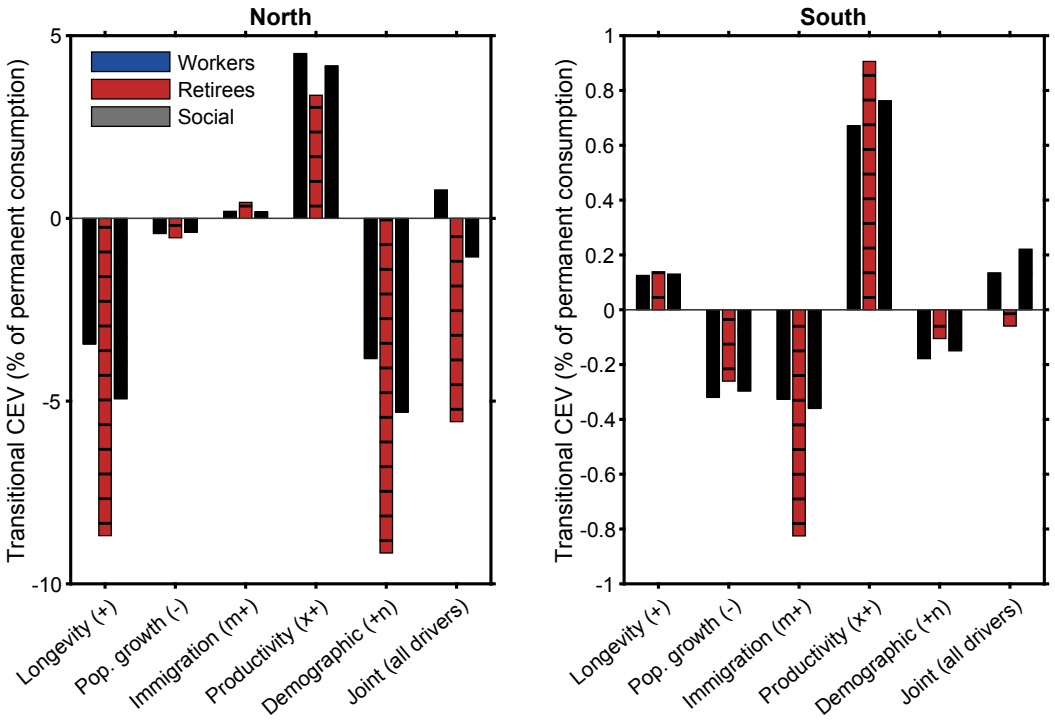


Figure B.12: Transitional consumption-equivalent welfare effects of each shock, workers versus retirees (computed directly under Table 3, from the linearized companion solve of Section 7.3). (Left: North; right: South.)

within the aging North the burden falls more heavily on retirees than workers, and it dominates the South's own joint response (-0.64% workers, -1.36% retirees, -0.78% socially).[49]

[49]The perpetual-youth structure has no age dimension within either cohort, since transition probabilities ω_t, γ_t are age-independent (Section 4.4): V^w and V^r in (50) are one representative worker and retiree, not a continuum at different distances from retirement, so the finding is about retirees versus workers as cohorts, not about which workers are "closest to retirement"; an age-structured extension would be needed for that.