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R&D Spillovers and Location Choice under Cournot Rivalry*

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Résumé / Abstract

Nous étudions le problème de localisation sous l'hypothèse que le débordement technologique est fonction de la distance entre les entreprises oligopolistiques qui font concurrence à la Cournot. La concentration géographique s'avère optimale dans certains cas. Dans d'autres cas, l'équilibre implique la dispersion géographique.

A model of location choice by Cournot oligopolists is presented, under the assumption that R&D spillovers depend on the distance between firms. We show that a variety of patterns emerge. Agglomeration is optimal under certain assumptions. Geographical dispersion in a two-dimensional plane is another possible outcome.

Mots Clés : Localisation, débordement technologique, oligopole

Keywords : Location Choice, R&D Spillovers, Oligopoly

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1 Introduction

Ever since Marshall (1890), it has been well recognized that an industry may be geographically concentrated because agglomeration tends to generate various "externalities" such as (i) mass-production (the so-called internal economies which are similar to the scale economies), (ii) the formation of a pool market for specialized labor, (iii) the development of specialized input services, and (iv) the existence of modern infrastructures. The so-called Marshallian externalities play a central role in the new economic geography literature (see Fujita and Thisse, 1997, for a recent survey).[1]

However, very little attention has been paid in theory to the interaction between innovation activity and the spatial clustering of firms. This is rather surprising since *the localized nature of business and academic research is a well-documented fact* (Audretsch and Feldman, 1996; Feldman and Florida, 1994; Jaffe *et al*, 1993). In particular, the results obtained by Jaffe et al. suggest that "these effects are quite large and quite significant statistically" (p. 595). Approximately 60 percent of citations come from the primary patent class (p. 596). Citations to domestic patents are more likely to be domestic, and more likely to come from the same state and metropolitan statistical areas as the cited patents. In this case, we may expect the spillover functions to be convex in distance.

If firms do learn from each other through knowledge spillovers, which depend on the geographical proximity, they will have an incentive to agglomerate in order to internalize the R&D externalities generated by the others. However there is another force that works in the opposite direction. Since firms are rivals in the product market, geographical proximity makes competition fiercer. Hence we may not expect a general result to hold. The outcome depends on the relative strenghts of these two forces.

The primary purpose of this paper is to show that a small number of firms competing strategically are likely to agglomerate when the spillover effect is convex in distance. This result is in accordance with the empirical findings mentioned above. Our secondary purpose is to show that agglomeration does not necessarily occur when spillovers are concave in distance.

It is worth noticing that our results are consistent with some other models developed in economic geography. For example, in a different

[1]For some recent contributions, see Carlton (1995), David and Rosenbloom (1990), Glaeser et al. (1992), Head et al (1995), Kim (1995), Wheeler and Mody (1992), Woodward (1992).

context where consumers have a dispersed shopping behavior, agglomeration of firms occurs when the shopping function is concave in distance (Papageorgiou and Thisse, 1985). In the same vein, Ogawa and Fujita (1980) show that firms agglomerate in a Central Business District when the face-to-face communication field is linear in distance. When this function is convex, several clusters are likely to emerge (Fujita and Ogawa, 1982). Finally, in a spatial competition model à la Hotelling where R&D costs are a decreasing and convex function of the interfirm distance, Mai and Peng (1997) emphasize the existence of a tradeoff between the spillover effect and the competition effect. Firms get closer and closer when the relative importance of the R&D function increases. All these results therefore suggest that concave accessibility functions or convex spillover functions favors the emergence of one center. On the contrary, convex accessibility function and concave spillover functions are consistent with several centers (see also Fujita and Thisse, 1997).

In the spirit of spatial competition models, we consider a two-stage model where firms choose first their locations and then compete in quantity. Our results are established in the Euclidean plane. Thus we departure from the "long narrow city" model used by location theorists and urban economists. In general, we cannot rule out the possibility of a dispersed location equilibrium. However, when the spillover functions are convex in distance, there is always some clustering of firms. As suggested by an example, concavity seems to be consistent with a dispersed equilibrium.

Finally, we consider the possibility that some firms choose to cooperate in R&D while remaining rivals in the product market, as in the models of semi-collusion developed by Friedmann and Thisse (1993) and Fershtman and Gandal (1994). Possible extensions are indicated in the concluding section.

2 Location Choice in a Duopoly with R&D Spillovers

In this section, we consider a two-stage game between two rival firms. In the first stage, they choose their locations, which influence the extent of their mutual technological spillovers, and therefore their cost structures. In the second stage, they produce their outputs and compete in quantities.

Let us solve for the equilibrium of the second stage, for any given cost structure. Let θ_i denote firm i 's unit cost of production. The inverse demand function is $P = P(Q)$, with $P'(Q) < 0$, where $Q = q_1 + q_2$.

Firm i 's profit is

$$\pi_i = [P(Q) - \theta_i]q_i. \tag{1}$$

Implicit in (1) is the assumption that transport costs are zero. This allows us to focus on the role of distance in the extent of R&D spillovers. The role of proximity to markets is by now well understood, and would unnecessarily complicate our analysis.

Firm i knows the value of θ_i and θ_j, and takes q_j as given. It choose q_i to maximize profit. The first order condition for an interior maximum is

$$P'(Q)q_i + P(Q) = \theta_i \tag{2}$$

and the second order condition is

$$P''(Q)q_i + 2P'(Q) < 0. \tag{3}$$

Summing (2) over the two firms, we obtain

$$P'(Q)Q + 2P(Q) = \theta_1 + \theta_2 = \theta^s \tag{4}$$

where θ^s denotes the *sum* of the two unit costs. Equation (4) indicates that the equilibrium industry output depends only on the sum of the unit costs, and is independent of how this sum is split between θ_1 and θ_2. This result is due to Bergstrom and Varian (1985). We assume that the left-hand side of (4) is a monotone decreasing function of Q, that is,

$$QP''(Q) + 3P''(Q) < 0 \tag{5}$$

or, equivalently,

$$E < 3 \tag{6}$$

where E denotes the elasticity of slope of the demand function:

$$E = \frac{-P''(Q)Q}{P'(Q)}. \tag{7}$$

Condition (5) is the usual stability condition; see Dixit (1986), for example. Note that if the industry's marginal revenue curve is downward sloping, then $E < 2$.

From (4) and (6), we have the result that the equilibrium output is a decreasing function of θ^s:

$$\frac{dQ}{d\theta^s} = \frac{1}{(3-E)P'} < 0 \tag{8}$$

We now seek to express firm i 's profit in the equilibrium of stage two as a function of θ_i and θ^s. From (2),

$$q_i = \frac{P(Q) - \theta_i}{-P'(Q)}. \tag{9}$$

Substitute this into (1):

$$\pi_i = \frac{(P(Q) - \theta_i)^2}{-P'(Q)} = [-P'(Q)]q_i^2 \tag{10}$$

where Q is a function of θ^s, by (4) or (8).

We now describe the stage-one game. Without loss of generality, we fix the location of firm 1 and let firm 2 choose its distance d from firm 1. We assume that

$$\theta_i = \theta - r_i, (i = 1, 2) \tag{11}$$

where r_i is the reduction in unit cost due firm i 's R&D expenditure, x_i, and the spillovers it obtains from the other firm's R&D expenditure, x_j. The magnitude of these spillovers depends on the geographical distance d between the two firms:

$$r_i = f(X_i), \quad f'(X_i) > 0, \tag{12}$$

$$X_i = x_i + \beta(d)x_j, \quad \beta'(d) < 0, \quad 1 > \beta(d) > 0, \tag{13}$$

where, following Kamien, Muller and Zang (1992), we may call X_i firm i 's *effective R&D investment*, and $\beta(d)$ the spillover coefficient. Kamien, Muller and Zang postulated that β is a positive constant, while we allow β to be a positive and decreasing function of the distance d.

In order to focus on the choice of location, we assume that the firms have made a symmetric choice $x_1 = x_2 = \bar{x}$. We then show that the optimal distance is zero, independent of $\bar{x}$.

Proposition 2.1 . Assume that $E < 2$. Then in a symmetric equilibrium, the distance between the two firms is zero.

Proof: Omitted, as it can be constructed from the proof of Proposition 3.1 in section 3.

The intuition behind Proposition 2.1 is as follows. Given $x_1 = x_2 = \bar{x}$, the closer the firms are to each other, the higher is their effective R&D investment. The resulting cost reduction will increase each firm's profit, provided that the equilibrium price does not fall too much. The condition $E < 2$ guaranties that the equilibrium price does not fall significantly. It can be easily verified that $E < 2$ is the condition that the industry's marginal revenue, $P'(Q)Q + P(Q)$, is a decreasing function of Q.

Proposition 2.1 is not really surprising, because it has been shown (see Seade (1985)) that if $E > 2$ [respectively, $E < 2$],then a uniform increase [respectively, decrease] in unit cost across all firms will increase the profit of all firms in a *symmetric* Cournot oligopoly. Therefore if $E < 2$, the two Cournot rivals have an incentive to agglomerate to reduce cost.

We now turn to a more interesting model where there are more than two firms, and an equilibrium location choice may be *asymmetric*, at least when a subset of firms cooperate in R&D.

3 Location Choice in an Oligopoly without R&D Cooperation

We now formulate a model where the locational choice of a firm can be anywhere or a plane, and firms are not required to be symmetrically located. This is an advance over existing models where firms are either constrained to be on the same straight line, or required to be symmetrically placed on a circle.

Assume there are three firms, 1 , 2 and 3. Let d_1 (respectively d_2) denote the distance between 1 and 3, (respectively, 2 and 3). Let d denote the distance between 1 and 2. The counterparts of (13) are:

$$X_3 = x_3 + \beta(d_1)x_1 + \beta(d_2)x_2 \quad (14)$$

$$X_1 = x_1 + \beta(d)x_2 + \beta(d_1)x_3 \quad (15)$$

$$X_2 = x_2 + \beta(d)x_1 + \beta(d_2)x_3 \quad (16)$$

The three firms need not necessarily locate themselves on the same straight line. They may, for example, each occupy a corner (vertex) of a triangle. Their effective R&D investments reduce unit costs in the way described by (12), with $i = 1, 2, 3$.

In the second stage of the game, the locations have been determined, and the cost reductions r_1, r_2, r_3 have been known. Firms then compete by choosing outputs. Again, the first order conditions are:

$$P'(Q)q_i + P(Q) = \theta_i \qquad (i = 1, 2, 3) \tag{17}$$

Summing (17) over all firms to obtain

$$P'(Q)Q + 3P(Q) = \theta_1 + \theta_2 + \theta_3 = \theta^s, \tag{18}$$

where θ^s is the sum of the three unit costs. A stability condition (see Dixit (1986)) is that the left-hand side of (18) is decreasing in Q. This holds, for the 3-firm case, if and only if

$$E < 4, \tag{19}$$

where E is defined by (7). We then have

$$\frac{dQ}{d\theta^s} = \frac{1}{(4 - E)P'} < 0. \tag{20}$$

Again, firm i 's equilibrium profit is given by

$$\pi_i = \frac{(P(Q) - \theta_i)^2}{-P'(Q)} = (-P'(Q))q_i^2. \tag{21}$$

We now posit the following problem. Suppose that firms 1 and 2 have chosen their locations, and therefore d (the distance between them) is by now given. Assume $d > 0$. What is firm $3'$ s optimal location choice ? Again, for simplicity, we assume that the nominal R&D expenditure levels x_1, x_2, x_3 have been chosen. Given d, firm 3 must choose d_1 and d_2, subject to the triangle inequality,

$$d_1 + d_2 \geq d. \tag{22}$$

Without loss of generality, we require

$$d_1 - d_2 \geq 0 \tag{23}$$

$$d_2 \geq 0 \tag{24}$$

(The constraint $d_1 \geq 0$ is implied by (23) and (24) above.)

Consider the Lagrangian function

$$L = \pi_3 + \gamma(d_1 + d_2 - d) + \lambda(d_1 - d_2) + \mu d_2 \tag{25}$$

The necessary conditions are,

$$\frac{\partial L}{\partial d_1} = \frac{\partial \pi_3}{\partial \theta_3}\frac{\partial \theta_3}{\partial d_1} + \frac{\partial \pi_3}{\partial Q}\frac{\partial Q}{\partial \theta^s}\frac{\partial \theta^s}{\partial d_1} + \gamma + \lambda = 0 \tag{26}$$

$$\frac{\partial L}{\partial d_2} = \frac{\partial \pi_3}{\partial \theta_3}\frac{\partial \theta_3}{\partial d_2} + \frac{\partial \pi_3}{\partial Q}\frac{\partial Q}{\partial \theta^s}\frac{\partial \theta^s}{\partial d_2} + \gamma - \lambda + \mu = 0 \tag{27}$$

where, from (21) and (9)

$$\frac{\partial \pi_3}{\partial \theta_3} = \frac{2(P(Q) - \theta_3)}{-P'(Q)} = -2q_3 \tag{28}$$

$$\frac{\partial \pi_3}{\partial Q} = \frac{-2(P(Q) - \theta_3)(P')^2 + (P'')(P - \theta_3)^2}{(-P')^2} \tag{29}$$

From (29) and (20),

$$\frac{\partial \pi_3}{\partial Q}\frac{\partial Q}{\partial \theta^s} = \frac{-q_3(2 - s_3 E)}{4 - E} \tag{30}$$

where s_3 is firm **3** 's market share:

$$s_3 = \frac{q_3}{Q} \tag{31}$$

Finally,

$$\frac{\partial \theta_3}{\partial d_i} = -f'(X_3)\beta'(d_i)x_i, (i = 1, 2) \tag{32}$$

$$\frac{\partial \theta^s}{\partial d_i} = -\beta'(d_i)[x_3 f'(X_i) + x_i f'(X_3)], (i = 1, 2) \tag{33}$$

Conditions (26) and (27) become:

$$2q_3 f'(X_3)\beta'(d_1)x_1 - \frac{q_3(2 - s_3 E)}{(4 - E)}\left[x_3 f'(X_1) + x_1 f'(X_3)\right]\beta'(d_1) + \lambda + \gamma = 0 \tag{34}$$

$$2q_3 f'(X_3)\beta'(d_2)x_2 - \frac{q_3(2 - s_3 E)}{(4 - E)}\left[x_3 f'(X_1) + x_2 f'(X_3)\right]\beta'(d_2) + \mu - \lambda + \gamma = 0 \tag{35}$$

In addition, we have

$$\lambda \geq 0, \lambda(d_1 - d_2) = 0, \gamma \geq 0, \gamma(d_1 + d_2 - d) = 0, \mu \geq 0, \mu d_2 = 0 \quad (36)$$

The following proposition follows immediately:

Proposition 3.1: Given the distance between firms 1 and 2, assume that R&D expenditures are the same for all three firms, and $f(X)$ is linear. If $E < 2$, then firm 3 will choose to be on the straight line segment joining firm 1 and firm 2. In other words, $d_1 + d_2 = d$.

Proof : Suppose firm 3 is not located on the straight line segment joining firm 1 and firm 2. Then $d_1 + d_2 > d$ and $\gamma = 0$.

Then from (34) and (35)

$$\frac{2q_3 f'(X)x}{(4-E)}(2 - (1 - s_3)E)\beta'(d_1) = -\lambda \quad (37)$$

$$\frac{2q_3 f'(X)x}{(4-E)}(2 - (1 - s_3)E)\beta'(d_2) = \lambda - \mu \quad (38)$$

There are three mutually exclusive (and exhaustive) possibilities: (a) $d_1 - d_2 > 0$, $d_2 > 0$, (b) $d_1 = d_2 > 0$ and (c) $d_2 = 0$. Case (a) implies $\lambda = \mu = 0$, which is not possible because the left-hand side of (37) is negative. Case (b) implies $\mu = 0$. Adding (37) and (38) then gives an equation with two identical negative terms on the left-hand side, and zero on the right- hand side, which is not possible. Case (c) implies that firm 3 and firm 2 occupy the same location which is consistent with what we want to prove (i.e, $d_1 + d_2 = d$). This completes the proof of Proposition 3.1.

Having proved that firm 3 is located on the straight line segment joining firm 1 and firm 2, we now wish to find out whether it is located exactly in between them. The following Proposition provides an answer.

Proposition 3.2: Given the assumptions stated in Proposition 3.1,

(a) If β is a strictly concave function, then firm 3 will be located exactly in between firm 1 and firm 2 (so that $d_1 = d_2 = \frac{d}{2}$).

(b) If β is a strictly convex or linear function, then firm 3 will choose to be as close to firm 2 as possible.

Proof:

(a) Assume β is strictly concave. Suppose $d_2 = 0$ and $d_1 = d$, then $\lambda = 0$. Subtracting (38) from (37):

$$\frac{2q_2 f'(X)x}{(4-E)}[2-(1-s_3)E][\beta'(d)-\beta'(0)] = \mu \geq 0, \tag{39}$$

which is not possible because $\beta'(d)-\beta'(0) < 0$ when β is strictly concave. It follows that $d_2 > 0$. With $d_2 > 0$, we have $\mu = 0$ and $\lambda \geq 0$. In this case, subtracting (38) from (37) yields

$$\frac{2q_2 f'(X)x}{(4-E)}[2-(1-s_3)E][\beta'(d_1)-\beta'(d_2)] = -2\lambda \tag{40}$$

If $d_1 > d_2$ then $\lambda = 0$ while $\beta'(d_1) - \beta'(d_2) < 0$ due to the concavity of β ; therefore (40) would be violated. It follows that $d_1 = d_2$, in view of the constraint (23).

(b) If β is strictly convex or linear, then (39) can be satisfied.

This completes the proof of Proposition 3.2.

Example 3.1

Assume that the demand function is linear:

$$P(Q) = a - bQ,\ a > \theta,\ b > 0 \tag{41}$$

and

$$f(X) = X \tag{42}$$

Then, in the equilibrium in stage two, we have

$$q_3 = \frac{1}{4b}(a - 3\theta_3 + \theta_1 + \theta_2) \tag{43}$$

With $x_1 = x_2 = x_3 = x$,

$$q_3 = \frac{1}{4b}[a - \theta + x - 2\beta(d)x + 2\beta(d_1)x + 2\beta(d_2)x] \tag{44}$$

Firm $3'$ s profit is, from (21)

$$\pi_3 = bq_3^2 \tag{45}$$

Substitute (44) into (45) and maximize the resulting expression with respect to d_1 and d_2 subject to $d_1 + d_2 = d$ (in view of Proposition 3.1). Clearly, with $a - \theta + x \geq 0$, this maximization is equivalent to the maximization of $\beta(d - d_2) + \beta(d_2)$,where $0 \leq d_2 \leq d$. If β is strictly concave, then the optimum is at $d_2 = \frac{d}{2}$. If β is strictly convex, then the optimum is at $d_2 = 0$ (recall the convention that $d_1 \geq d_2$).

So far we assume that the location of firms 1 and 2 are given. Using the results of Propositions 3.1 and 3.2, it is clear that if all firms choose their locations either sequentially or simultaneously, they would end up in the same location. Thus we have shown:

Proposition 3.3: Given that all firms have the same level of R&D expenditure ($x_i = x$ for all i), they will choose to agglomerate to reap the benefits of spillovers.

It is not clear if agglomeration is the equilibrium if some subset of firms cooperate in their R&D, while others do not. This is the subject matter of the next section.

4 Location Choice with R&D Cooperation within a Subset of Firms

We now consider a model with three firms, two of which cooperate in their R&D activities. Cooperation takes the form of exchange of knowledge. This is modeled as an increase in the spillover coefficients. More specifically, suppose that firm 1 and firm 2 cooperate. Then equation (15) and (16) are replaced by

$$X_1 = x_1 + \delta\beta(d)x_2 + \beta(d_1)x_3 \tag{46}$$

$$X_2 = x_2 + \delta\beta(d)x_1 + \beta(d_2)x_3 \tag{47}$$

where $\delta > 1$. Equation (14) remains valid. We assume $\delta\beta(d) \leq 1$.

Again, in stage one the firms choose their locations and in stage two they compete in quantities. Equations (17) to (21) remain valid descriptions of the equilibrium in stage two. In stage one, we assume that firm

3 's location is given, and firms 1 and 2 collusively choose their locations to maximize the sum of their stage two Cournot equilibrium profits. This formulation is in the tradition of the theory of semi-collusion (as exemplified by the work of Friedman and Thisse (1993), Fershtman and Gandal (1994), and others). This theory reflects the stylized fact that firms cooperate on some level, while remaining rivals on some other levels. If the optimal solution is symmetric for these semi-collusive firms, then no side transfers are required. In the case of an asymmetric solution, some rules for transfers must be specified. For a more detailed discussion, see Long and Soubeyran (1995).

Given firm $3'$ s location, firms 1 and 2 must choose d_i (the distance between firm i and firm 3, $i = 1, 2$), and d (the distance between firms 1 and 2). Since the three firms must be located on three vertices of some triangle (or possibly on a straight line segment, which may be regarded as limiting case of a triangle with a vanishing area), the following triangle inequalities must be satisfied:

$$d_1 + d_2 \geq d \tag{48}$$
$$d_2 + d \geq d_1 \tag{49}$$
$$d_1 + d \geq d_2. \tag{50}$$

In addition,

$$d_1 \geq 0 \tag{51}$$
$$d_2 \geq 0 \tag{52}$$
$$d \geq 0 \tag{53}$$

Without loss of generality, we specify that

$$d_1 - d_2 \geq 0 \tag{54}$$

This condition ensures that (50) and (51) are satisfied given (52) and (53). We will therefore need to take into account only (48), (49), (52), (53) and (54).

Let π^J denote the joint (semi-collusive) profit of firms 1 and 2,

$$\pi^J = \pi_1 + \pi_2 \tag{55}$$

where each firm$'$ s profit is given by (21): it depends on the firm$'$ s own cost, θ_i, and on the equilibrium industry output in the Cournot game, Q, which in turn depends only on θ^s ; see (18) and (20). The distances

d_1, d_2 and d determine θ_i and θ^s (see (11), (12), (14), (46) and (47)). We assume that the nominal R&D expenditure levels have been chosen.

The Lagrangian for the semi-collusive profit maximization problem of firms 1 and 2 is:

$$L = \pi^J + \lambda(d_2 + d - d_1) + \mu(d_1 + d_2 - d) + \gamma(d_1 - d_2) + \omega d_2 + \rho d \quad (56)$$

In what follows, for simplicity, we assume

$$f(X) = X, x_1 = x_2 = x_3 = x > 0 \quad (57)$$

The necessary conditions are

$$\frac{\partial L}{\partial d_1} = 2x\delta\beta'(d_1)A_1 - \lambda + \mu + \gamma = 0 \quad (58)$$

$$\frac{\partial L}{\partial d_2} = 2x\delta\beta'(d_2)A_2 + \lambda + \mu - \gamma + \omega = 0 \quad (59)$$

$$\frac{\partial L}{\partial d} = 2x\delta\beta'(d)A + \lambda - \mu + \rho = 0 \quad (60)$$

$$\lambda \geq 0 \quad , \quad \lambda(d_2 + d - d_1) = 0,\ d_2 + d - d_1 \geq 0 \quad (61)$$

$$\mu \geq 0 \quad , \quad \mu(d_1 + d_2 - d) = 0,\ d_1 + d_2 - d \geq 0 \quad (62)$$

$$\gamma \geq 0 \quad , \quad \gamma(d_1 - d_2) = 0,\ d_1 - d_2 \geq 0 \quad (63)$$

$$\omega \geq 0 \quad , \quad \omega d_2 = 0,\ d_2 \geq 0 \quad (64)$$

$$\rho \geq 0 \quad , \quad \rho d = 0,\ d \geq 0 \quad (65)$$

where we have used the same technique as that used to derive (34) and (35) from (26) and (27), and where

$$A_1 = q_1 + \frac{(s_1E - 2)q_1 + (s_2E - 2)q_2}{4 - E} \quad (66)$$

$$A_2 = q_2 + \frac{(s_2E - 2)q_2 + (s_1E - 2)q_1}{4 - E} \quad (67)$$

$$A = \frac{q_1 + q_2}{4 - E}\left[2 - (1 - \frac{q_1 s_1 + q_2 s_2}{q_1 + q_2})E\right] \quad (68)$$

From the above necessary conditions, we obtain the following proposition.

Proposition 4.1: Assume that $f(X)$ is linear, and assume positive outputs.
(a) In equilibrium firms 1 and 2 are not located symmetrically away from firm 3 and also away from each other. In other words, the configuration $d_1 = d_2$ with $d > 0$ is not consistent with the necessary conditions
.
(b) If the three firms are located on three vertices of a non-degenerate triangle, then it must be the case that the demand curve is locally strictly convex, and that firms 1 and 2 are not symmetrically located in relation to firm 3. In other words, the configuration $d_1 + d_2 > d$, $d_2 + d > d_1$, $d > 0$ (and $d_2 \geq d_1$) implies $E > 0$ and $d_1 \neq d_2$.
(c) If firms 1 and 2 are located at the same place and away from firm 3, then it must be the case that the demand curve is locally concave (or linear). In other words, if $d = 0$ and $d_1 = d_2 > 0$, then $E \leq 0$.

Proof:
(a) Suppose $d_1 = d_2$ and $d > 0$. Then $\rho = \omega = \lambda = 0$. Also, $d_1 = d_2$ implies $\theta_1 = \theta_2$, hence $q_1 = q_2$ by (17). Therefore $A_1 = A_2$. This and (58) to (59) imply $\gamma = 0$, and therefore $A_i \geq 0, i = 1, 2$. Since $q_1 = q_2$, (66) gives

$$A_1 = \frac{-q_1 E(1 - s_1)}{4 - E} \tag{69}$$

Recall that $4 - E > 0$ by (19), and that $s_1 < 1$. Hence $A_1 \geq 0$ implies $E \leq 0$. We now show that this inequality would lead to a contradiction. Take the case $E = 0$. Then (69) gives $A_1 = 0$, hence $\mu = 0$ by (58). Thus $A = 0$ by (60) with $\mu = 0$. But $E = 0$ implies $A > 0$ by (68), given that q_1 and q_2 are positive. Therefore $E = 0$ is not possible. Now take the case $E < 0$. Then by (69) $A_1 > 0$, and hence $\mu > 0$ by (58). This implies $A < 0$ by (60). But from (68), $E < 0$ implies $A > 0$. The supposition $d_1 = d_2$ and $d > 0$ has led to a contradiction.
(b) Suppose $d_2 + d > d_1$, $d_1 + d_2 > d$ and $d > 0$. Then $\mu = \lambda = \rho = 0$. Equation (60) then gives $A = 0$. Hence $E > 0$ by (68), given that q_1 and q_2 are positive. Finally, d_1 must be different from d_2 because as shown in (a) above, $d_1 = d_2$ is not possible when $d > 0$.
(c) If $d = 0$ (which implies $d_1 = d_2$) and $d_1 = d_2 > 0$ then $\omega = 0$, and $\theta_1 = \theta_2$, $q_1 = q_2$, $A_1 = A_2$. Adding (58) and (59) yields $4\delta\beta'(d_1)A_1 x + 2\mu = 0$. Hence $A_1 \geq 0$, implying, via (69), that $E \leq 0$.

This completes the proof of Proposition 4.1.

While Proposition 4.1 applies to the case of linear demand as well as non-linear demand, it is much easier to characterize the equilibrium location choice in the linear demand case by exploiting the linear structure directly. This is shown in the following example.

Example 4.1

Assume $P(Q) = a - bQ$ where $a > \theta, b > 0$, and $f(X) = X$. Then, in the equilibrium in stage two,

$$q_1 = \frac{1}{4b}(a - 3\theta_1 + \theta_2 + \theta_3),\ q_2 = \frac{1}{4b}(a - 3\theta_2 + \theta_1 + \theta_3) \tag{70}$$

With $x_1 = x_2 = x_3 = x > 0$, we have

$$4bq_1 = a - 3\theta_1 + \theta_2 + \theta_3 = a - \theta + x[1 + 2\beta(d) + 2\delta\beta(d_1) - 2\delta\beta(d_2)] \tag{71}$$

Without loss of generality we set $x = 1$. Recall that firm i 's profit is given by (21) in a Cournot equilibrium. The joint profit of firms 1 and 2 in the semi-collusive equilibrium (collusive in location choice and R&D, but rivalry in outputs) is

$$\pi_1 + \pi_2 = bq_1^2 + bq_2^2 \tag{72}$$

Therefore, maximizing $\pi_1 + \pi_2$ is equivalent to maximizing the expression W defined below:

$$W = (U + V)^2 + (U - V)^2 \tag{73}$$

where

$$U = a - \theta + 1 + 2\beta(d) \tag{74}$$
$$V = 2\delta\beta(d_1) - 2\delta\beta(d_2). \tag{75}$$

Simplifying, we obtain:

$$W = 2U^2 + 2V^2 \tag{76}$$

The firms 1 and 2 must maximize (76) by choosing d, d_1 and d_2, subject to

$$d_1 - d_2 \geq 0 \quad (77)$$

$$d_1 \geq 0 \quad (78)$$

$$d_1 + d_2 \geq d \geq d_1 - d_2 \quad (79)$$

The following result follows immediately:

Lemma 4.2: Assume that the demand function is linear, and that cost reduction is linear in effective R&D investment). Then the three firms must be located on the same straight line. Given firm 3's location, the two semi-collusive firms 1 and 2 will choose to be both to the left, or both to the right, of firm 3 (the case where all three are located at the same point is possible).

Proof: For any given pair (d_1, d_2), d must be chosen to maximize (76). This means $\beta(d)$ must be maximized subject to (79). The solution is $d = d_2 - d_1$, because β is a decreasing function.

Lemma 4.2 enables us to simplify problem (76). Without loss of generality, we can fix firm 3 at the origin of the non-negative half of the real line, and then choose two real numbers $d \geq 0$ and $d_2 \geq 0$ (where $d_1 = d + d_2$ without loss of generality).

Lemma 4.3: Under the assumption stated in the preceding lemma, in equilibrium firm 2 will be located at the same point as firm 3 if β is a convex function.

Proof: Given any $d > 0$, write (76) as

$$W = 2[a - \theta + 1 + 2\beta(d_2)]^2 + 8\delta[\beta(d + d_2) - \beta(d_2)]^2. \quad (80)$$

Then

$$\frac{\partial W}{\partial d_2} = 16\delta[\beta(d+d_2) - \beta(d_2)][\beta'(d+d_2) - \beta'(d_2)] \tag{81}$$

Clearly, $\beta(d+d_2) - \beta(d_2) < 0$, and $\beta'(d+d_2) - \beta'(d_2) \geq 0$, because β is a convex function. This shows that it is optimal to set $d_2 = 0$ for any $d > 0$.

Proposition 4.4: Given the assumptions in Lemma 4.3, in equilibrium all three firms must be located at the same point.

Proof: Since $d_2 = 0$ by Lemma 4.3, we can write (80) as

$$W = 2[a - \theta + 1 + 2\beta(d)]^2 + 8\delta[\beta(d) - \beta(0)]^2. \tag{82}$$

Therefore

$$\frac{\partial W}{\partial d} = 8[a - \theta + 1 + 2\beta(d) + 2\delta\beta(d) - 2\delta\beta(0)]\beta'(d) \tag{83}$$

which is equal to $24bq_1\beta'(d)$ by (71). Therefore it is optimal to set $d = 0$.

Finally, let us consider an example where $f(X)$ is not linear.

Example 4.2: Assume $f(X_i) = \ln(1 + X_i), i = 1, 2, 3$. With linear demand and $x_1 = x_2 = x_3 = 1$, we have, from (70) for $i, j = 1, 2$

$$4bq_i = a - \theta + \ln\left[\frac{(2 + \delta\beta + \beta_i)^3}{(2 + \delta\beta + \beta_j)(2 + \beta_1 + \beta_2)}\right], \tag{84}$$

where $\beta = \beta(d), \beta_i = \beta(d_i), i = 1, 2$.

We want to maximize the joint profit, $bq_1^2 + bq_2^2$. Since $\beta(0) < 1$, it is easy to see that for given β_1 and β_2, q_i is an increasing function of β. It follows that given d_1 and $d_2 \leq d_1$, the optimal d is $d = d_1 - d_2$. In other words, the three firms must be located on the same straight line, and firms 1 and 2 will chose to be both on the right, or both on the left of firm 3. We have therefore obtained the counterpart of Lemma 4.3 for this example with a non-linear $f(X)$.

Next, suppose the distance d is zero. Then $d_1 = d_2$ and the maximization of $\pi_1 + \pi_2$ is equivalent to maximizing, with respect to d_1,

$$T = a - \theta + \ln\left[\frac{(2 + \delta\beta(d) + \beta_1)^2}{(2 + 2\beta_1)}\right]. \tag{85}$$

The derivative of T with respect to d_1 is positive, given that $d = 0$. It follows that the two semi-collusive firms will want to be as far away from firm 3 as possible.

Proposition 4.4: With $f(X) = \ln(X + 1)$, the two semi-collusive firms will want to be as far away from firm 3 as possible.

5 Conclusion

In this paper, we have explored the implications of R&D spillovers on the choice of locations by Cournot oligopolists. We have obtained a variety of results. The optimal choice seems very sensitive to the specifications of the relationship between cost reduction and R&D spillovers. At one extreme, firms may want to agglomerate. At the other extreme, one subset of firms may want to be as far away from the rest as possible.

Our model can be extended in several directions. Firstly, there is an obvious need to endogenize R&D effort. It would seem natural to model location choice as preceding the R&D expenditure choice. The incentive to agglomerate would then be stronger as increased proximity is a partial substitute for R&D. In addition, if firms are allowed to coordinate their R&D efforts, there will be a tendency for asymmetric outcomes where much of the industry's R&D is concentrated in one firm. Long and Soubeyran (1995) have demonstrated this result in a non-spatial model.

The second avenue for generalization is to introduce transport costs. In this case, like in the Hotelling model, firms have an incentive to split the market into segments, so that each firm serves its own segmented market. This incentive counteracts the agglomerating tendency driven by the desire to take advantage of spillovers from the R&D expenditure of other firms.

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