Cascades and Fluctuations in an Economy with an Endogenous Production Network

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Introduction

- Production in modern economies involves a complex network of producers supplying and demanding goods from each other
- The shape of this network
  - is an important determinant of how micro shocks aggregate into macro fluctuations
  - is also constantly changing in response to micro shocks
    - For instance, after a severe shock a producer might shut down which might lead its neighbors to shut down as well, etc...
    - Cascade of shutdowns that spreads through the network

This paper proposes a

Theory of network formation and aggregate fluctuations

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Literature Review

- Endogenous network formation
  - Atalay et al (2011), Oberfield (2013), Carvalho and Voigtländer (2014)
- Network of sectors and fluctuations
  - Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2016), Acemoglu et al (2016), Lim (2017)
- Non-convex adjustments in networks
  - Bak, Chen, Woodford and Scheinkman (1993), Elliott, Golub and Jackson (2014)

I. Model

Model

- There are n units of production (firm) indexed by $j \in \{1, \ldots, n\}$
  - Each unit produces a differentiated good
  - Differentiated goods can be used to
    - produce a final good

$$Y \equiv \left(\sum_{j=1}^{n} \left(y_j^0 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

    - produce other differentiated goods
- Representative household
  - Consumes the final good
  - Supplies L units of labor inelastically

Model

- Firm j produces good j

$$y_j = \frac{A}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} z_j \left(\sum_{i=1}^{n} x_{ij}^{\frac{\epsilon-1}{\epsilon}}\right)^{\alpha\frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

- Firm j can only use good i as input if there is a *connection* from firm i to j
  - $\Omega_{ij} = 1$ if connection and $\Omega_{ij} = 0$ otherwise
  - A connection can be *active* or *inactive*
  - Matrix Ω is *exogenous*
- A firm can only produce if it pays a fixed cost f in units of labor
  - $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
  - Vector θ is *endogenous*

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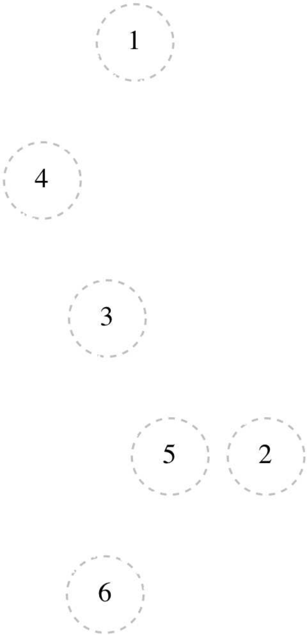
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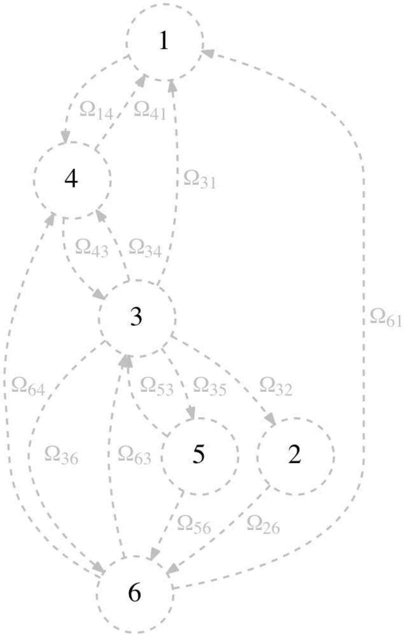
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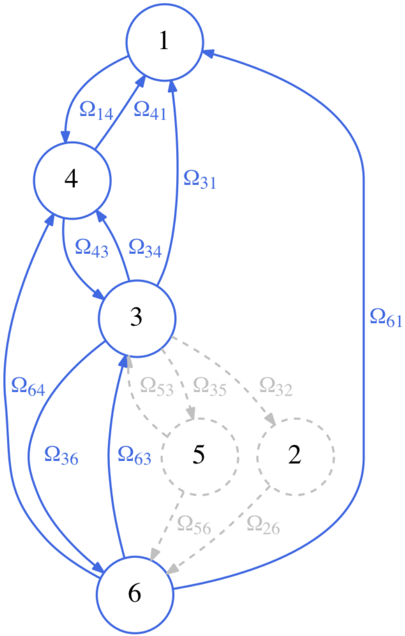
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Social Planner

Problem $\mathcal{P}_{SP}$ of a social planner

$$\max_{\substack{y^0, x, l \\ \theta \in \{0,1\}^n}} \left(\sum_{j=1}^{n} \left(y_j^0 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$y_j^0 + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor

$$\sum_{j=1}^{n} l_j + f \sum_{j=1}^{n} \theta_j \leq L$$

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subject to

1. a resource constraint for each good j **(Lagrange multiplier: λ_j)**

$$y_j^0 + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor **(Lagrange multiplier: w)**

$$\sum_{j=1}^{n} l_j + f \sum_{j=1}^{n} \theta_j \leq L$$

II. Social Planner with Exogenous θ

Social Planner with Exogenous θ

Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1-\alpha)\, y_j = q_j l_j$
- q_j is the *labor productivity* of firm j

Proposition 1

In the efficient allocation,

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}} \tag{1}$$

Furthermore, there is a unique vector q that satisfies (1).

Social Planner with Exogenous θ

Knowing q we can solve for all other quantities easily.

Lemma 1

Aggregate output is

$$Y = Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where $Q \equiv \left(\sum_{j=1}^{n} q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ *is aggregate labor productivity.*

Labor allocation

III. Social Planner with Endogenous θ

Social Planner with Endogenous θ

$$\max_{\theta \in \{0,1\}^n} Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

"Very hard problem" (MINLP — NP Hard)

- The set $\theta \in \{0,1\}^n$ is not convex
- Objective function is not concave

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Social Planner with Endogenous θ

Consider the relaxed and reshaped problem $\mathcal{P}_{RR}$

$$\max_{\theta \in \{0,1\}^n} Q\left(L - f\sum_{j=1}^{n} \theta_j\right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

Parameters $a > 0$ and $b \geq 0$ are *reshaping constants*

- Reshape the objective function *away* from optimum (i.e. when $0 < \theta_j < 1$)
  - For a: if $\theta_j \in \{0,1\}$ then $\theta_j^a = \theta_j$
  - For b: $\{\theta_i = 0\} \Rightarrow \{q_i = 0\}$ and $\{\theta_i = 1\} \Rightarrow \left\{\theta_i^b q_i^{\epsilon-1} = q_i^{\epsilon-1}\right\}$
- Parameters such that P1 and P2 are satisfied:

$$\boxed{a = \frac{1}{\sigma - 1} \quad \text{and} \quad b = 1 - \frac{\epsilon - 1}{\sigma - 1}} \qquad (\star)$$

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Proposition 2

Under some parameter restrictions and if Ω is sufficiently connected then the Karush-Kuhn-Tucker conditions are necessary to characterize a solution to $\mathcal{P}_{RR}$. Furthermore, a solution to $\theta^* \in \{0,1\}^n$ to $\mathcal{P}_{RR}$ also solves $\mathcal{P}_{SP}$.

Details

This proposition

- Only provides *sufficient* conditions
- In the paper: Test the approach on thousands of economies Tests

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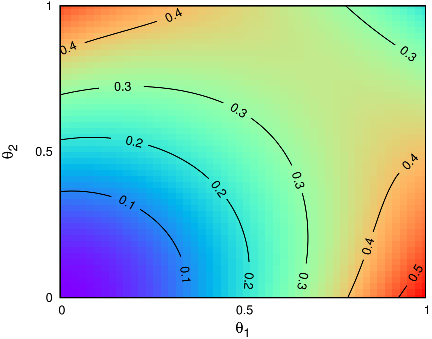
Example with $n = 2$

Relaxed problem **without** reshaping

$$V(\theta) = Q(\theta)\left(L - f\sum_{j=1}^{n}\theta_j\right) \text{ with } q_j = z_j\theta_j A\left(\sum_{i=1}^{n}\Omega_{ij}q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$



Problem: V is not concave

⇒ First-order conditions are not sufficient

⇒ Numerical algorithm can get stuck in local maxima

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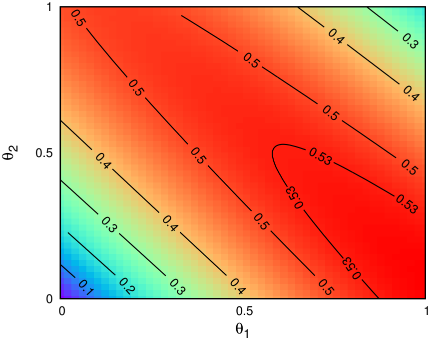


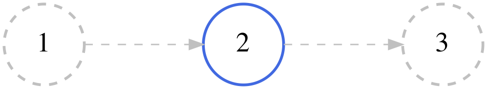
~~Problem~~: V is now (quasi) concave

- $\Rightarrow$ First-order conditions are necessary and sufficient
- $\Rightarrow$ Numerical algorithm converges to global maximum

IV. Economic Forces at Work

Complementarities

- Impact of operating 2 on the incentives to operate 1 and 3
  - Operating 3 leads to a larger q_3 because 2 is operating
  - Operating 1 increases q_2 because 2 is operating
- Complementarity between operating decisions of nearby firms



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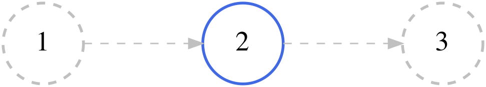

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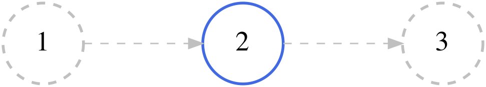

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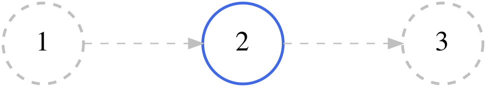

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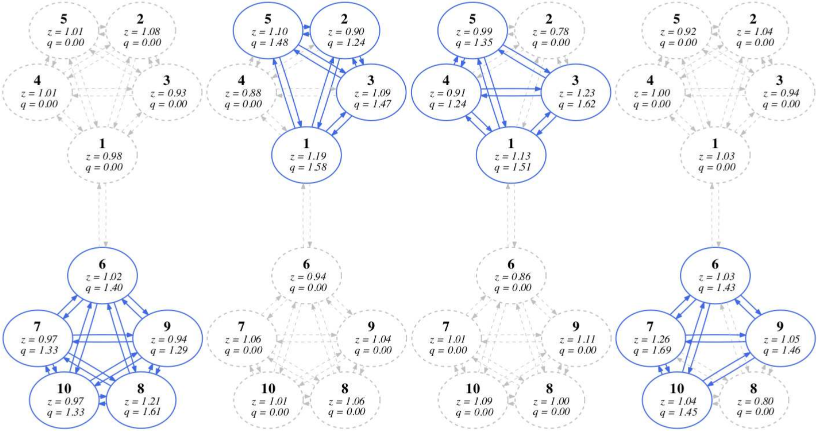
Complementarities lead to clustering



V. Quantitative Exploration

Network data

- Two datasets that cover the U.S. economy
  - Cohen and Frazzini (2008) and Atalay et al (2011)
  - Both rely on Compustat data
    - Public firms must self-report customers that purchase more than 10% of sales
    - Use fuzzy-text matching algorithms and manual matching to build networks
  - Cover 1980 to 2004 and 1976 to 2009 respectively

Parameters

Parameters from the literature

- $\alpha = 0.5$ to fit the share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \epsilon = 6$ average of estimates (Broda et al 2006)
  - Robustness with smaller ϵ in the paper
- $\log(z_{it}) \sim \mathcal{N}\left(0, 0.39^2\right)$ from Bartelsman et al (2013)
- $f \times n = 5\%$ to fit employment in management occupations
- Calibrate $n = 3000$ to match number of active firms in Atalay et al (2011)

Unobserved network Ω:

- Pick to match the *observed* in-degree distribution
- Generate thousands of such Ω's and report averages

In-degree

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Shape of the network

What types of network does the planner choose?

- Compare **optimal networks** to completely **random networks**
- Differences highlights how efficient allocation shapes the network

	Optimal networks	Random networks
A. Power law shape parameters		
In-degree	1.43	1.48
Out-degree	1.37	1.48
B. Measures of proximity		
Clustering coefficient	0.027	0.018
Average distance between firms	2.26	2.64

Efficient allocation features

- More highly connected firms
- More clustering of firms

Def. clust. coeff.

Cascades of shutdowns

Because of the complementarities between firms

- Exit of a firm makes it more likely that its neighbors exit as well ...
- ... which incentivizes the second neighbors to exit as well ...
- ...

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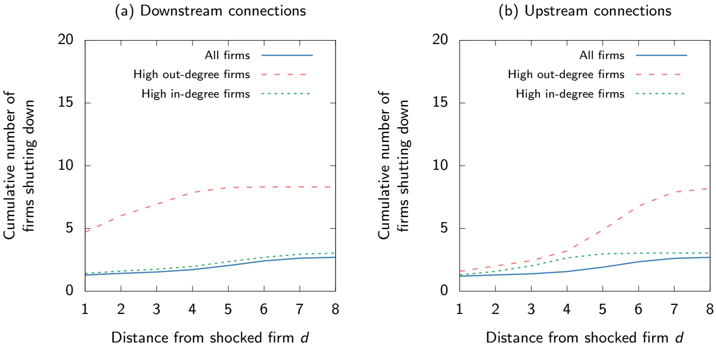

▸ Robustness

Resilience of firms

Magnitude of shock necessary to make a firm exit varies

	Probability of firm shut down after 1 std shock
All firms	92%
High out-degree firms	20%
High in-degree firms	56%

Implications:

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

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▸ Robustness

The shape of the network changes with the business cycle

	Correlation with output		
	Model	Data	
		CF (2008)	AHRS (2011)
A. Power law shape parameters			
In-degree	-0.10	-0.10	-0.21
Out-degree	-0.31	-0.24	-0.13
B. Clustering coefficient	0.47	0.70	0.15

Implications:

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

The shape of the network changes with the business cycle

	Correlation with output		
	Model	Data	
		CF (2008)	AHRS (2011)
A. Power law shape parameters			
In-degree	-0.10	-0.10	-0.21
Out-degree	-0.31	-0.24	-0.13
B. Clustering coefficient	0.47	0.70	0.15

Implications:

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

Aggregate fluctuations

Size of fluctuations

$$Y = Q\left(L - f\sum_j \theta_j\right)$$

Table: Standard deviation of aggregates

	Output Y	Labor Prod. Q	Prod. labor $L - f\sum_j \theta_j$
Optimal network	**0.039**	0.039	0.0014
Fixed network	**0.054**	0.054	0

Implications:

- Substantially smaller fluctuations in optimal network economy comes from the reorganization of network after shocks

▶ Intuition

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A given network θ^k is a function that maps $z \to Y_k(z)$

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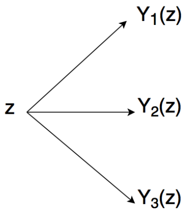

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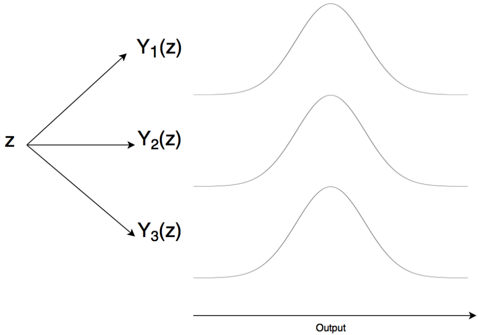

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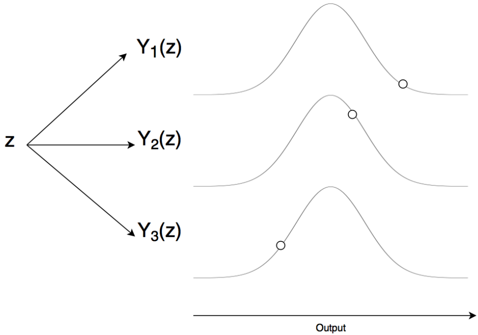

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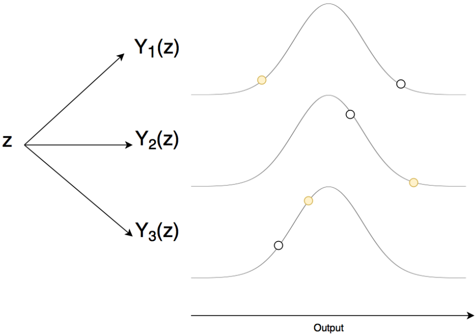

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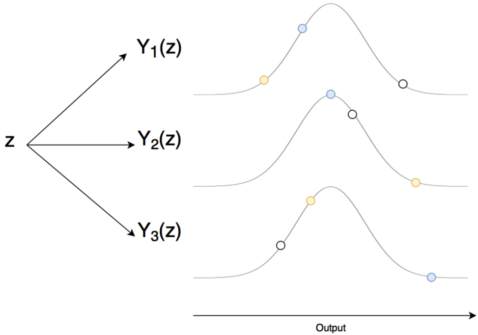

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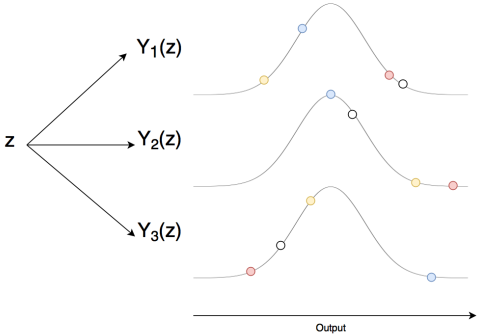

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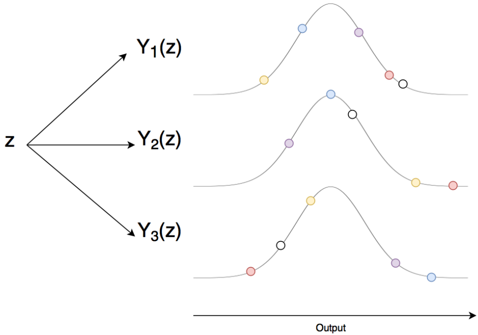

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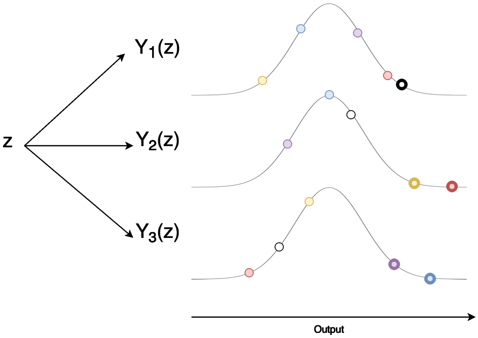

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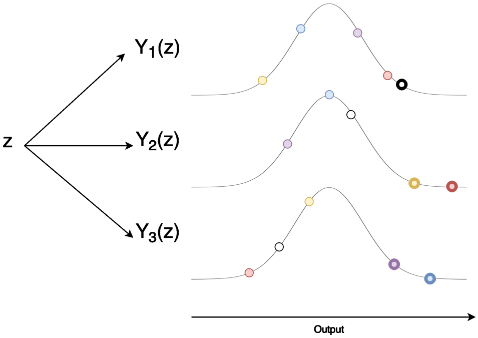

From extreme value theory

$$\mathrm{Var}(Y) = \mathrm{Var}\left(\max_{k\in\{1,\ldots,2^n\}} Y_k\right)$$

declines rapidly with n

Conclusion

Additional results in the paper:

- Impact of position in the network on firm-level characteristics
- Endogenous skewness in distribution of employment, productivity, output

Summary

- Theory of network formation and aggregate fluctuations
- Propose an approach to solve these hard problems easily
- The optimal allocation features
  - Clustering of activity
  - Cascades of shutdowns/restarts
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Labor allocation

Lemma 2

The optimal labor allocation satisfies

$$l = (1-\alpha)\underbrace{[I_n - \alpha\Gamma]^{-1}}_{(1)}\underbrace{\left(\frac{q}{Q}\right)}_{(2)}^{\circ(\sigma-1)}\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where I_n is the identity matrix and where Γ is an $n \times n$ matrix where $\Gamma_{jk} = \frac{\Omega_{jk} q_j^{\epsilon-1}}{\sum_{i=1}^{n}\Omega_{ik} q_i^{\epsilon-1}}$ captures the importance of j as a supplier to k.

Determinants of l_j

(1) Importance of j as a supplier

- Leontief inverse $\left([I_n - \alpha\Gamma]^{-1} = I_n + \alpha\Gamma + (\alpha\Gamma)^2 + \dots\right)$

(2) Relative efficiency

Return

Social Planner with Endogenous θ

P1 The alternative problem $\mathcal{P}_{RR}$ is easy to solve

Proposition 3

If $\Omega_{ij} = c_i d_j$ for some vectors c and d then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$.

Proposition 4

Let $\sigma = \epsilon$ and suppose that $f > 0$ and $\overline{z} - \underline{z} > 0$ are not too big. If Ω is sufficiently connected, then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$.

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P2 A solution to the alternative problem $\mathcal{P}_{RR}$ also solves $\mathcal{P}_{SP}$

Proposition 5

If θ^* solves $\mathcal{P}_{RR}$ and that $\theta_j^* \in \{0, 1\}$ for all j, then θ^* also solves $\mathcal{P}_{SP}$.

Solution θ^* to $\mathcal{P}_{RR}$ is such that $\theta_j^* \in \{0, 1\}$ for all j (P2) if

- the $(\star)$ condition is satisfied
- there are many firms
- the network is sufficiently connected

▸ Details

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Reshaping

Intuition:

- First-order condition on θ_j:

$$\text{Marginal Benefit}\left(\theta_j, F\left(\theta\right)\right) - \text{Marginal Cost}\left(\theta_j, G\left(\theta\right)\right) = \bar{\mu}_j - \underline{\mu}_j$$

- Under $(\star)$ the marginal benefit of θ_j only depends on θ_j through aggregates
- For large connected network F and G are independent of θ_j

Return FOCs

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Details of reshaping

Simpler to consider

$$\mathcal{P}'_{RD}: \quad \max_{\theta \in [0,1]^n, q} \left(\sum_{j=1}^{n} q_j^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \left(L - f \sum_{j=1}^{n} \theta_j \right)$$

$$q_j \leq A z_j \theta_j^a A B_j^\alpha \qquad (\text{LM: } \beta_j)$$

where $B_j = \left(\sum_{i=1}^{n} \Omega_{ij} \theta_i^b q_i^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}$.
First order condition with respect to θ_k:

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} \left(L - f \sum_{j=1}^{n} \theta_j \right) - fQ + \sum_{j=1}^{n} \beta_j \left(\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} \right) \frac{\partial q_j}{\partial B_j} = \overline{\mu}_k - \underline{\mu}_k$$

The terms are

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} = z_k a \theta_k^{a-1} A B_k^\alpha \times \left(z_k \theta_k^a A B_k^\alpha \right)^{\sigma-2} Q^{2-\sigma}$$

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} = B_j \theta_k^{b-1} \Omega_{kj} \left(\frac{z_k \theta_k^a A B_k^\alpha}{B_j} \right)^{\epsilon-1} \left(a + \frac{b}{\epsilon - 1} \right)$$

Return

Testing the approach on small networks

For small networks we can solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- Comparing approaches for a million different economies:

	Number of firms n			
	8	10	12	14
A. With reshaping				
Firms with correct θ_j	99.9%	99.9%	99.9%	99.8%
Error in output Y	0.00039%	0.00081%	0.00174%	0.00171%
B. Without reshaping				
Firms with correct θ_j	84.3%	83.2%	82.3%	81.3%
Error in output Y	0.84%	0.89%	0.93%	0.98%

Notes: Parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z \in \{0.34, 0.39, 0.44\}$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$ and $\epsilon \in \{4, 6, 8\}$. For each combination of parameters 1000 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim$ iid $\mathcal{N}(0, \sigma_z)$ and Ω is drawn randomly such that each link Ω_{ij} exists with some probability such that a firm has on average five possible incoming connections. A network is kept in the sample only if the first-order conditions give a solution in which θ hits the bounds.

The errors come from

- firms that are particularly isolated
- two θ configurations with almost same output

Testing the approach on large networks

For large networks we cannot solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- After all the 1-deviations θ are exhausted:

	With reshaping	Without reshaping
Firms with correct θ_j	99.8%	72.1%
Error in output Y	0.00028%	0.69647%

Notes: Simulations of 200 different networks Ω and productivity vectors z that satisfy the properties of the calibrated economy.

- Very few "obvious errors" in the allocation found by the approach

Return

Distribution of in-degree

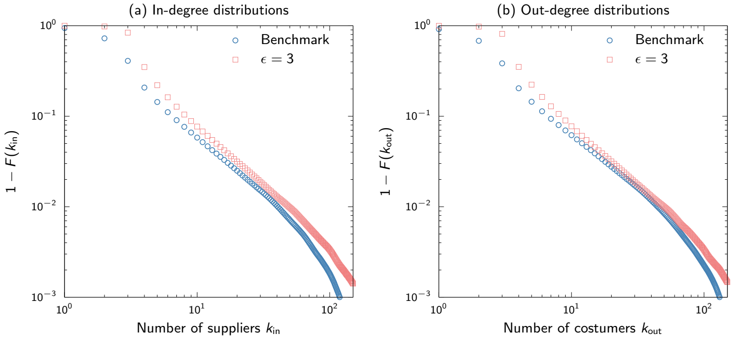

Figure: Distribution of the number of suppliers and the number of customers

In-degree power law shape parameter

- Calibration: 1.43
- Data: 1.37 (Cohen and Frazzini, 2008) and 1.3 (Atalay et al, 2011)

Return

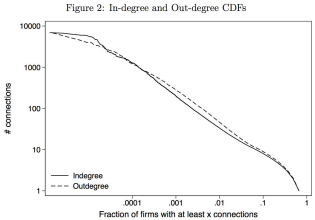

Figure: Distribution of in-degree and out-degree in Bernard et al (2015)

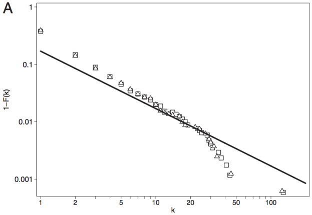

Figure: Distribution of in-degree in Atalay et al (2011)

Return

Clustering coefficient

- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

Return

Firm-level distributions

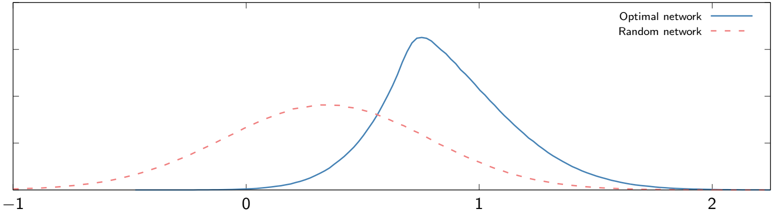

Figure: Distributions of $\log(q)$

return

Cascades of shutdowns

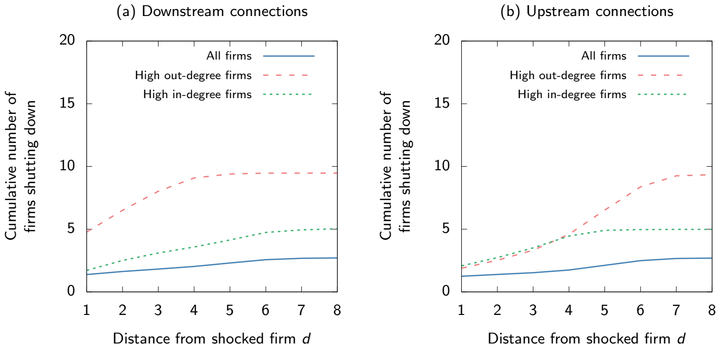

Figure: $\alpha = 0.75$

return

Cascades of shutdowns

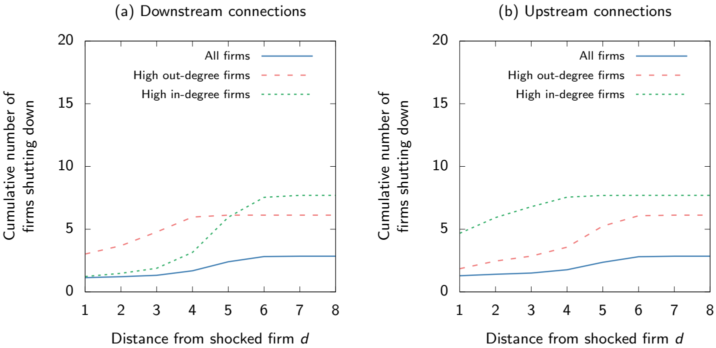

Figure: $\epsilon = 3$

return

Resilience

	Probability of firm shutdown		
	Benchmark	$\alpha = 0.75$	$\epsilon = 3$
All firms	92%	82%	32%
High out-degree firms	20%	8%	0%
High in-degree firms	56%	19%	15%

return