Modelling and Implications of Endogenous Schumpeterian Growth for Business Cycles in a New Keynesian DSGE Model

Work in progress

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*"**DSGE models are**, of course, **not really a model of medium- to long-term growth** : that is determined by factors like the pace of innovation and the accumulation of human capital on which they provide little insight. **To understand the former,** for instance, **one needs a much more detailed analysis of technological progress, including investments in basic research and the transmission of knowledge across and within firms, than any standard macro-model can provide.**"*

– Stiglitz (2017), forthcoming, Oxford Review of Economic Policy

*"**If macroeconomics aims to address the dynamics of the last decade and the economic issues that have been central to recent political outcomes, artificial separations between modeling of business cycles and longer-term dynamics must be abandoned,** and the same must happen to similarly artificial separations between macroeconomic and trade modeling. [...] **So, yes to DSGE,** and yes to micro."*

– Ghirony (2017), forthcoming, Oxford Review of Economic Policy

- Endogenous foundation for secular growth [Romer(1986), Romer (1990), Lucas (1988), Rebelo (1991), and Aghion & Howitt (1992)]
- R&D effects do matter also at business cycle frequencies [Comin & Gertler (2006)]
- Investments in innovation are sensitive to monetary policy shocks [Barlevy (2007) and Fatas (2000)]
- Our hunch :
    - Ignoring endogenous growth considerations in business cycles DSGE may not be innocuous.
    - Introducing labour and capital augmenting innovation processes could partially endogenise technology
    - The monetary policy effects on R&D investments could even lower the contribution of the usual neutral technology shocks to business cycles
    - The exclusion of the innovation process from modern DSGE models abstracts from a propagation mechanism of the diverse shocks

- Typical New Keynesian (NK) DSGE business cycles models constructed around a classical exogenous growth model [Smets & Wouters (2007) and Justiniano *et al.* (2010)]

More recently :

- Nuno (2011) : a RBC model with Schumpeterian growth, with specific functional forms and no nominal rigidity.
- Amano *et al.* (2012) : nominal rigidities being modelled as Taylor (1980) staggered price and wage contracts, and endogenous growth from horizontal innovations in the variety of intermediate goods [to study effects on the welfare costs of inflation]
- Cozzi *et al.* (2017) : price and wage rigidities arising from specific adjustment costs in presence of Solow-neutral technology with Schumpeterian growth (Use Bayesian estimation) [to study the implication of financial conditions for innovation dynamics]

Complete literature

Introduction – Why the inclusion and treatment of Schumpeterian growth features matter

(1) The endogenous choice of investing in R&D has implications for the likelihood of advancing or not the technological boundary

(2) It provides some microfoundation to monopolistic competition that is introduced *de facto* in NK models, as differing levels of technological advancement in the intermediate sector bring a justification for existing market power.

(3) Our hybrid model highlights and addresses new challenges at the modelling and simulation stages when considering the implications of price rigidities on R&D investments, as the sluggish dynamics of prices impact directly on the expected discounted value derived from innovations.

The Model's Main Building Blocks – to account jointly for endogenous growth through creative destruction and business cycle in an extended NK model :

- Forward-looking **households**, with differentiated skills j, and market power on wage setting, maximize their expected utility with respect to their sequence of budget constraints, by making optimal decisions regarding their time-paths for consumption, labour, utilization of physical capital, private investment, and net bond holdings.
- **Employment agencies**, in a competitive environment, aggregate the households' specialized labour into homogeneous labour used by the intermediate good producers.
- The **final good producers**, in a perfectly competitive market, use intermediate goods as input.
- The **intermediate good producers** in a monopolistically competitive environment, with market power on the price of their specific good i
- Operating within the intermediate sector, **entrepreneurs/innovators** invest final goods final goods in R&D to to increase their odds of pushing the technological frontier, so that an intermediate good producer that implements the new technology takes over the incumbent producer in his respective intermediate sector.
- The **monetary authority** sets nominal interest rates according to a Taylor-type reaction function for monetary policy
- Prices and wages are subjected to nominal rigidities through contracts *à la* Calvo (1983)

The Model

The final good producer

- a representative firm operating in a perfectly competitive setting, price taker of P_t and $P_t(i)$, that aggregates a continuum of intermediate goods $i \in (0,1)$:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

  where Y_t is total final output, the input $Y_t(i)$ is the good produced by an intermediate level firm i, and $0 \leq \epsilon < \infty$ is the elasticity of substitution between intermediate goods

- its profit maximization problem : $\underset{Y_t(i)}{\text{Max}} \ \Pi_{FG} = P_t \, Y_t - \int_0^1 P_t(i) \, Y_t(i) \ di$

  $\Rightarrow$ the demand for the i^{th} intermediate good : $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$

  With zero economic profits in perfect competition, total nominal output is :

$$P_t \, Y_t = \int_0^1 P_t(i) \, Y_t(i) \, di$$

  $\Rightarrow$ the aggregate price index : $P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}}$

The Model

The Employment Agency

- a continuum of households, with different skills, offer specialized labour $L_t(j)$ for $j \in (0,1)$, $\Rightarrow$ some market power in setting wages
- intermediate firms use a combination of specialized labour
- a representative employment agency, in perfect competition, taking as given the aggregate wage rate W_t and the prevailing labour compensation specific to each labour type j, that aggregates specialized labour $\Rightarrow$ the combined labour input L_t

$$L_t = \left(\int_0^1 L_t(j)^{\frac{\gamma-1}{\gamma}} \, dj \right)^{\frac{\gamma}{\gamma-1}}$$

  where $0 \leq \gamma < \infty$ is the elasticity of substitution between each labour type
- the employment agency's optimization problem : $\underset{L_t(j)}{\text{Max}} \; \Pi_{EA} = W_t \, L_t - \int_0^1 W_t(j) \, L_t(j) \, dj$

  $\Rightarrow$ the demand for specialized labour j : $L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\gamma} L_t$

  With zero economic profits in perfect competition for the employment agency

  $\Rightarrow$ the aggregate wage rate : $W_t = \left[\int_0^1 W_t(j)^{1-\gamma} \, dj \right]^{\frac{1}{1-\gamma}}$

The Model

The Households – the budget constraint

- At date t, the representative type-j household faces a sequence of budget constraints

$$P_t\, C_t + P_t\, I_t + P_t\, a(u_t)\tilde{K}_t + \frac{B_t}{1+R_t} \leq W_t(j)\, L_t(j) + q_t\, u_t\tilde{K}_t + B_{t-1} + D_t - T_t$$

The nominal value for the uses of funds :

- the nominal value of consumption in the final good, $P_t\, C_t$
- the desired level of investment in capital goods, $P_t\, I_t$
- the resources devoted to adjust the utilization rate of physical capital (if applicable), $P_t\, a(u_t)\tilde{K}_t$

  The utilization of the existing stock of physical capital, $\tilde{K}_t$, may be less than 100% and be time-varying, provided that the household bears a real increasing cost in the varying capital utilization u_t. [Note : with $u_t = 1$, $a(1) = 0$]
- the end-of-period nominal net holdings of a one-period discount bond $\frac{B_t}{1+R_t}$

The Model

The Households – the budget constraint (cont'd)

$$P_t\,C_t + P_t\,I_t + P_t\,a(u_t)\tilde{K}_t + \frac{B_t}{1+R_t} \leq W_t(j)\,L_t(j) + q_t\,u_t\tilde{K}_t + B_{t-1} + D_t - T_t$$

The nominal after-tax sources of funds :

- the labour income, $W_t(j)\,L_t(j)$
- the nominal payments received from supplying capital services to intermediate firms at a gross capital rental rate q_t, $q_t\,u_t\;\bar{K}_t$
- the nominal face value of the net discount bond holdings carried from the previous period
- **the nominal dividends, D_t, received from its ownership of shares in the intermediate production sector that operates in monopolistic competition**

less

- the value of lump-sum taxes, net of government transfers (with a government following a Ricardian fiscal policy)

The Model

The Households – the budget constraint (cont'd)

- to assess dividends, $\boldsymbol{D_t}$, received from the intermediate production firms
- monopolistic competition amongst intermediate firms $\Rightarrow$ positive economic rents or profits paid back as dividends to households for each firm i, arising from monopoly profits and from innovation profits (if there is a takeover from an innovator who may fail or succeed)

$$\Pi_{i,t} = P_t(i)\, Y_t(i) - w_t\, L_t(i) - q_t\, K_t(i)$$

- need to account for the innovation process in the representative household's budget constraint and the investment in R&D, investing $P_t\, X_t$ to reach the frontier (even though it is *ex-post* a sunk cost)
    - *A failed innovator's timeline for cash flows (generating no profits)*

Time period	t	t+1	t+2	t+3	t+4	t+5
Cash flow	$-P_t X_t$	0	0	0	0	0

    - *A successful innovator's timeline for cash flows (generating profits)*

Time period	t	t+1	t+2	t+3	. . . .	$t+\tau$	$t+\tau+1$
Cash flow	$-P_t X_t$	Π_{t+1}	Π_{t+2}	Π_{t+3}		$\Pi_{t+\tau}$	0

- the overall dividends paid to households : $D_t = \int_0^1 \Pi_{i,t} - P_t\, X_t(i)\, di$

The Households – utility maximization

$$\underset{C_{t+s},L_{t+s}(j),u_{t+s},\tilde{K}_{t+s+1},I_{t+s},B_{t+s}}{\text{Max}} E_t^j \sum_{s=0}^{\infty} \beta^s \left(ln(C_{t+s} - h\, C_{t+s-1}) - \theta \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right)$$

subject to

$$P_{t+s}\, C_{t+s} + P_{t+s}\, I_{t+s} + P_{t+s}\, a(u_t)\tilde{K}_{t+s} + \frac{B_{t+s}}{1+R_t}$$
$$\leq W_{t+s}(j)\, L_{t+s}(j) + q_{t+s}\, u_{t+s}\tilde{K}_{t+s} + B_{t+s-1} + D_{t+s} - T_{t+s}$$

$$\widetilde{K}_{t+s+1} = \mu_{I,t+s} \bullet \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} + (1-\delta)\widetilde{K}_{t+s}$$

$$\ln \mu_{I,t+s} = \rho_I \; \ln \mu_{I,t+s-1} + \epsilon_{I,t+s}$$

$$K_{t+s} = u_{t+s}\tilde{K}_{t+s}$$

The Model

The Households – wage setting

$$\underset{W_{t+s}(j)}{\text{Max}} \; E_t^j \left(\sum_{s=0}^{\infty} \xi_w^s \, \beta^s \left(-\,\theta \, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right) + \Lambda_{t+s} \, W_t(j) \, \pi^w_{t,t+s}, L_{t+s}(j) \right)$$

subject to $L_{t+s}(j) = \left(\frac{W_t(j) \, \pi^w_{t,t+s}}{W_{t+s}} \right)^{\gamma} L_{t+s}$

$\Rightarrow$ the optimal reset wage :

$$W_t^*(j)^{-\gamma\nu-1} = \frac{\gamma-1}{\theta\gamma} \frac{\sum\limits_{s=0}^{\infty} \xi_w^s \beta^s \Lambda_{t+s} (\pi_{w\ t,t+s})^{1-\gamma} W_{t+s}^{\gamma} L_{t+s}}{\sum\limits_{s=0}^{\infty} \xi_w^s \beta^s (\pi_{w\ t,t+s})^{-\gamma(1+\nu)} W_{t+s}^{\gamma(1+\nu)} L_{t+s}^{1+\nu}} = \frac{\gamma-1}{\theta\gamma} \frac{Aux_{bc,t}}{Aux_{dis,t}}$$

where we define auxiliary variables associated respectively with the household's budgets constraint in the numerator, and the disutility of labour in the denominator, and we exploit the relevant recursions built in the summations, which, in turn, will be useful for subsequent numeric simulation

$$Aux_{bc,t} = \Lambda_t \, W_t^{\gamma} L_t + \xi_p \beta (\pi_{w\ t,t+1})^{1-\gamma} Aux_{bc,t+1}$$

$$Aux_{dis,t} = \Lambda_t W_t^{\gamma(1+\nu)} L_t^{1+\nu} + \xi_p \beta (\pi_{w\ t,t+1})^{-\gamma(1+\nu)} Aux_{dis,t+1}$$

- **Schumpeterian paradigm :**
  - With an endogenous probability n_{t-1} : an innovation may arise from investing in R&D in period $t-1$ and push the technology level at A_{t-1}^{max} for an intermediate firm operating at date t
  - With a probability $1 - n_{t-1}$: an intermediate firm is lagging, while still using older technology level.
  - A firm's investments in R&D depends on the discounted expected profits arising from innovating

- **Additional level of complexity arising from existing price rigidity :**
  - Even after an innovation and the implementation of a new technology, then the monopolistic rent also depends on the expected price paths that a firm will be allowed to follow (with some probability that prices may be fixed in a given period, and some probability that they may be adjusted).
  - With the implementation of a newly discovered innovation, an intermediate firm is immediately allowed to set the optimal price.
  - At subsequent dates (quarters), barring some new innovation, the same intermediate firm is stuck with a more or less older technology, and there are probabilities that its price remains sticky for some time (up to possibly some backward indexation) $\Leftrightarrow$ If not innovating, the lagging firms are in a Calvo-type environment

- **Coexistence of three categories of intermediate firms in each period :**
  - Advanced firms that reset the optimal price
  - Lagging firms that are allowed to reoptimize their respective price
  - Lagging firms with previously set prices

The Optimal Reset Price

- Intermediate firms' market power from both their diversification and the technology used in production in a monopolistically competition setting
- Prices fixed through binding Calvo contracts and set to maximize expected profits conditional on not being allowed to reoptimise
- Given an initial level of technological advancement $A_t(i)$, the intermediate firm's constrained cost minimization problem :

$$\underset{K_t(i),L_t(i)}{\text{Min}} \; W_t L_t(i) + q_t K_t(i)$$

$$\text{subject to} \;\; Y_t(i) = \mu_{Z,t} \, A_t(i)^{1-\alpha} \, K_t(i)^{\alpha} \, L_t(i)^{1-\alpha} \geq \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \;,$$

$$\text{and} \;\; \ln \mu_{Z,t} = \rho_Z \, \ln \mu_{Z,t-1} + \epsilon_{Z,t}$$

- Regardless of their individual level of technological advancement, all intermediate firms' productions are subjected to a common transitory AR(1) technological shock.

The Optimal Reset Price (cont'd)

- All intermediate firms employ the optimal capital-labour ratio : $\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{q_t}$
- The nominal marginal cost of producing an additional unit of intermediate good :

  $MC_t(i) = \exp(\mu_{Z,t})^{-1}\, A_t(i)^{\alpha-1}\,\Omega_t \qquad \Leftrightarrow \qquad \uparrow A_t(i) \Rightarrow \downarrow MC_t(i)$

  with $\Omega_t \equiv \frac{q_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$, the portion of marginal costs that is not directly dependent on the level of technology
- In traditional New-Keynesian models : all firms operate at the same level of technological advancement $\Leftrightarrow$ all firms set the same optimal reset price, when allowed
- Our model : the optimal reset price depends on the technology $\Rightarrow$ with an infinite number of intermediate firms $\Rightarrow$ an infinite number of coexisting technologies $\Rightarrow$ an infinite number of reset prices because the marginal cost is a function of the technology level.

The Optimal Reset Price (cont'd)

- Given their respective marginal cost, intermediate firms maximize their profits with respect to their price $P_t(i)$:

$$\mathrm{Max}_{P_t(i)} \;\; E_t \left\{ \sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Lambda_{t+s}}{\Lambda_t} \left[n_{t+s-1} \left(P_t(i) \pi_{p\;t,t+s} - MC_{t+s}(i) \right) Y_{t+s}(i) \right] \right\}$$

subject to $Y_{t+s}(i) = \left(\frac{P_t(i)\,\pi_{p\;t,t+s}}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}$

where $\pi_{p\;t,t+s}$: the gross rate of backward price indexation from t and $t+s$, built in the price-setting rule

- Initial technology is known and can differ between firms
- Future technology levels depend as well on future investment levels in R&D.

The Optimal Reset Price (cont'd)

- The solution for the optimal reset price $\Rightarrow \quad P_t^*(i) = A_t(i)^{\alpha-1} F_t$

  as a function of initial technology $A_t(i)$, and of a factor F_t that is not directly dependent of the technology level.

  where

$$F_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Lambda_{t+s}}{\Lambda_t} (\pi_{p\ t,t+s})^{-\epsilon} \Omega_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{\sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Lambda_{t+s}}{\Lambda_t} (\pi_{p\ t,t+s})^{1-\epsilon} P_{t+s}^{\epsilon} Y_{t+s}} = \frac{\epsilon}{\epsilon - 1} \frac{Aux_{cost,t}}{Aux_{rev,t}}$$

$$Aux_{rev,t} = P_t^{\epsilon} Y_t + \xi_p \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1 - n_t)(\pi_{p\ t,t+1})^{1-\epsilon} Aux_{rev,t+1}$$

$$Aux_{cost,t} = \Omega_t P_t^{\epsilon} Y_t + \xi_p \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1 - n_t)(\pi_{p\ t,t+1})^{-\epsilon} Aux_{cost,t+1}$$

The Innovation Process

- R&D activities conducted by entrepreneurs/innovators
- Innovation within the intermediate sector ⇒ an outward push of the technological frontier
- The implementation of an innovation in technology by an intermediate good producer conveys additional market power from producing an improved version of the intermediate good ⇒ reaching the new technological frontier
- An entrepreneur/innovator invests some amount of final goods to raise the probability of innovating, which remains an uncertain prospect.
- Outside researchers or a new successful innovator supplants or "leapfrogs" an incumbent entrepreneur. [Note : abstracting from step-by-step technological progress, that would imply both Schumpeterian and escape-competition effects ⇔ prohibitively costly to develop a knockoff]
- The innovation production function (with decreasing marginal returns, $\eta > 0$:)

$$n_t = \left(\frac{X_t}{\zeta \, A_t^{max}} \right)^{1/(1+\eta)}$$

where $\frac{X_t}{\zeta \, A_t^{max}}$: the intensity of R&D effort ; X_t : the real amount of final goods invested in R&D ; A_t^{max} is the targeted technology level (frontier), to be used in date t+1 production ; $\zeta > 1$ is a scaling factor. [Note : the specification captures the increasing complexity of further progress associated with a larger A_t^{max}.]

The Innovation Process (cont'd)

- An innovation ⇒ a proportional increase in productivity (the technological frontier) at gross rate g_t^{max}, dictated by the probability of innovation times a spillover factor, subjected itself to some AR(1) stochastic component :

$$A_t^{max} = g_t^{max} \bullet A_{t-1}^{max} = (1 + \sigma_t \, n_{t-1}) \bullet A_{t-1}^{max}$$
$$\ln \sigma_t = \ln \sigma + \rho_\sigma \ln \sigma_{t-1} + \epsilon_{\sigma,t}$$

- σ_t : a knowledge (or technology) spillover as a positive externality derived from an innovation, since it permanently pushes the technological frontier forward
- $\epsilon_{\sigma,t}$: stochastic spillover shock from unpredictable variations and other heterogeneities in the transmission of knowledge and/or abilities to capitalize on new innovations to push the technological frontier further
- Baldwin *et al.* (2005) : Theoretical and empirical link between knowledge spillovers (from multinational corporations and foreign direct investments) and endogenous growth

  We think a similar effect can span across firms within an industry, as well as across industries to some extent (e.g. spillovers from the diffusion in information and communication technologies).

The Innovation Process (cont'd)

- Saia *et al.* (2015) :
  - Knowledge spillovers are significant for an economy's effectiveness to learn from the technological frontier and to increase productivity.
  - Sources of an economy's spillovers : the degree of international connectedness, the ability to allocate skills efficiently and the investments in knowledge-based capital (including managerial capital and R&D)
- The entrepreneurs/innovators' expected discounted profits (conditional on remaining at the helm of the monopoly) maximization problem :

$$\underset{X_t}{\text{Max}} \; \beta \, \frac{\Lambda_{t+1}}{\Lambda_t} \, n_t \, E_t V_{t+1}(A_t^{max}) - P_t \, X_t \quad \text{subject to} \quad n_t = \left(\frac{X_t}{\zeta \, A_t^{max}} \right)^{1/(1+\eta)}$$

- Assuming entrepreneurs investing in a diversified form of R&D (analogue to complete market hypothesis and successful entrepreneur not knowing *ex ante* in which sector they will end up) ⇒ all entrepreneurs will invest the same amount of final good in R&D

⇒ the optimal investment in R&D : $X_t = \beta \, \frac{\Lambda_{t+1}}{\Lambda_t} \, n_t \, \frac{E_t V_{t+1}(A_t^{max})}{P_t}$

- To complete the solution : need to write explicitly the expected value of the firm to the entrepreneur

The Innovation Process (cont'd)

- $E_t V_{t+1}(A_t^{max})$ is a function of the path that prices are expected to follow
  - Challenge #1 :

    Need **to ensure** that **the assessment of future profits follows the correct technological path**

    For instance, an innovator may reach the frontier A_t^{max} in $t+1$, and remains at that level, say until $t+3$, as it is supplanted by an advanced firm, with a lower marginal cost of operations. In this situation, from then on, the subsequent expected profits no longer matter for investing in innovation since it is now out of business : expected profits are to become zero from that date forward
  - Challenge #2 :

    Price rigidities play a crucial role in determining future profits because they **condition both the profit margin and the conditional demand** for that specific intermediate good
- To reckon the value of an innovation-implementing intermediate firm, let us consider an entrepreneur/innovator who, at date t, ponders how much to invest in R&D while seeking some returns from date $t+1$ onward $\Rightarrow$ to take into account all possible contingencies that could deliver some return from innovating, as they need to reflect
  - the probability of remaining at the helm of the monopoly
  - the appropriate stochastic discount factors
  - the probability ξ_p,, under Calvo contracts, for a firm already in operation, of not being allowed to reoptimize its price in a given period

The Innovation Process (cont'd)

- We can show that the expected value of the intermediate firm to a successful innovator is the sum of two parts :

$$E_t V_{t+1} = \Psi_{1t+1}(i) + \Psi_{2t+1}(i)$$

  - $\Psi_{1t+1}(i)$ is the expected discounted stream of profits for a new monopolist
    - taking over as of date $t+1$, that reaches the new technological frontier, so that $A_t(i) = A_t^{max}$,
    - setting the optimal price for its intermediate good i as of date $t+1$,
    - then, unable to reset its price afterwards while still in operation with what will have become an older technology.
  - $\Psi_{2t+1}(i)$ is the relevant expected discounted stream of profits for a new monopolist for all other contingent paths, weighted by the proper probabilities, for all possible l,
    - taking over as of date $t+1$, that reaches the new technological frontier, so that $A_t(i) = A_t^{max}$
    - setting the optimal price for its intermediate good i as of date $t+1$, that will prevail up to date $t+l$,
    - setting the optimal price for its intermediate good i at some future date $t+l$ with some probability $1-\xi_p$,
    - yet followed by the contingency path that price reoptimization does not occur afterwards, as there is a probability ξ_p each period of no reoptimization, even if the monopolist remains in operation.

The Innovation Process (cont'd)

- Making use of the recursion built in the summation involved in $\Psi_{1t+1}(i)$ and $\Psi_{2t+1}(i)$ and defining convenient auxiliary variable, $Aux_{rev,t+1}$, $Aux_{cost,t+1}$, and $Aux_{rem,t+l}$, we show that

$$E_t\, V_{t+1} = A_t^{max\,(\alpha-1)(1-\epsilon)} \bullet \left(F_{t+1}^{1-\epsilon} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1} + Aux_{rem,t+2} \right)$$

- The expected value of an intermediate firm for a successful entrepreneur/innovator is determined by
  - the newly reached technological frontier through $A_t^{max\,(\alpha-1)(1-\epsilon)}$,
  - the contribution from profits arising from being able to set the optimal price for good i as of period $t+1$ through $F_{t+1}^{1-\epsilon} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1}$,
  - the contribution to profits resulting from a later date optimal price setting with what will have become an older technology through $Aux_{rem,t+2}$.

- The auxiliary variables :

$$Aux_{rev,t+1} = P_{t+1}^{\epsilon} Y_{t+1} + \xi_p \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1-n_{t+1}) (\pi_{p\ t+1,t+2})^{1-\epsilon} Aux_{rev,t+2}$$

$$Aux_{cost,t+1} = \Omega_{t+1} P_{t+1}^{\epsilon} Y_{t+1} + \xi_p \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1-n_{t+1}) (\pi_{p\ t+1,t+2})^{-\epsilon} Aux_{cost,t+2}$$

$$Aux_{rem,t+2} = (1-\xi_p)\, \beta^2 \, \frac{\Lambda_{t+2}}{\Lambda_{t+1}} \left(F_{t+2}^{(1-\epsilon)} Aux_{rev,t+2} - F_{t+2}^{-\epsilon} Aux_{cost,t+2} \right) + \beta \, (1-n_{t+2}) \, Aux_{rem,t+3}$$

The Model – The Monetary Authority's Policy Function

The specification of monetary policy

- The central bank's policy function modelled as a Taylor-type rule for setting the nominal interest rate :

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\, g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R} \mu_{M,t}$$

with

$$\ln \mu_{M,t} = \rho_M \ \ln \mu_{M,t-1} + \epsilon_{M,t}$$

where

R : the natural interest rate

ρ_R : the degree of smoothing of interest rate changes ;

α_π : the monetary authority's weight for deviations from its inflation target, π

α_y : the monetary authority's weight for deviations from its output growth targets, g

g : the monetary authority's output growth targets, defined as the growth rate of the average technology level in steady state

$\mu_{M,t}$: exogenous and stochastic component of monetary policy, representing deviations from the Taylor-type rule, with $\ln \mu_{M,t}$ following an AR(1) process

The Aggregate Economy

The aggregate price level

- The overall economy's aggregate price level can be inferred by weighting and combining each of the respective prices for the three categories of coexisting firms :

$$P_t^{1-\epsilon} = \xi_p(1-n_{t-1})(P_{t-1}\pi_{p\ t-1,t})^{1-\epsilon} + (1-\xi_p)\int_{n_{t-1}}^{1} P_t^*(i)^{1-\epsilon}di + \int_0^{n_{t-1}} P_t^*(i)^{1-\epsilon}di$$

  - 1rst term : firms operating some older technologies, that are not allowed to reset their prices, yet applying backward price indexation up to t
  - 2nd term : firms using older technologies that can reset their optimal price
  - 3rd term : new monopolists adopting the most recent technology, and accordingly setting the intermediate good's optimal price

- Exploiting the implicit recursion embedded in the equation above :

$$P_t^{1-\epsilon} = \xi_p(1-n_{t-1})(P_{t-1}\pi_{p\ t-1,t})^{1-\epsilon} + (1-\xi_p)(1-n_{t-1})F_t^{1-\epsilon}\,Aux_{oldtech,t-1} + n_{t-1}A_{t-1}^{max\,(\alpha-1)(\epsilon-1)}F_t^{1-\epsilon}$$

where

$$Aux_{oldtech,t-1} = n_{t-2}\,A_{t-2}^{max\,(\alpha-1)(1-\epsilon)} + (1-n_{t-2})\,Aux_{oldtech,t-2}$$

The Aggregate Economy

The aggregate wage rate

- The overall economy's wage index can be inferred from weighting the respective wages for the workers that, yet applying backward wage indexation up to t, but cannot reset their wage optimally, and those that are allowed to reset to the optimal wage :

$$W_t^{1-\gamma} = \xi_w \left(W_{t-1}\, \pi_{w\ t-1,t}\right)^{1-\gamma} + (1-\xi_w)\, W_t^{*\,1-\gamma}$$

Aggregate output

- The aggregation of output needs to account for many intermediate firms coexisting with different technology levels and, hence, specific output levels.
- Integrating over all the firms on the $[0,1]$ continuum, with the identical optimal capital-labour ratio for all firms

$$\mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} = P_t^{\epsilon} Y_t \int_0^1 A_t(i)^{\alpha-1} P_t(i)^{-\epsilon} di$$

$$\mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} = P_t^{\epsilon} Y_t \bullet \left[\int_0^{\xi_p(1-n_{t-1})} A_t(i)^{\alpha-1} P_t(i)^{-\epsilon} di \right.$$

$$\left. + \int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{\alpha-1} P_t^*(i)^{-\epsilon} di + \int_{1-n_{t-1}}^{1} A_t(i)^{\alpha-1} P_t^*(i)^{-\epsilon} di \right]$$

The Aggregate Economy

Aggregate output (cont'd)

- The first and second intervals include old-technology-running firms that are respectively non resetting, and optimally resetting their price at $P_t^*(i)$. The last one covers the innovating firms with optimal price setting at $P_t^*(i)$.

  $\Rightarrow$ aggregate output must satisfy

$$P_t^{\epsilon} Y_t = \mu_{Z,t} \frac{K_t^{\alpha} L_t^{1-\alpha}}{Aux_{output,t}}$$

  where

$$\begin{aligned} Aux_{output,t} = & \xi_p (1 - n_{t-1})(P_{t-1} \pi_{t-1,t}^{p})^{-\epsilon} Aux_{oldtechnonreset,t-1} \\ & + (1-\xi_p)(1-n_{t-1}) F_t^{-\epsilon} Aux_{oldtech,t-1} \\ & + n_{t-1} F_t^{-\epsilon} (A_{t-1}^{max})^{(1-\epsilon)(\alpha-1)} \end{aligned}$$

The aggregate resource constraint

- Accounting for investment in R&D, the aggregate resource constraint satisfies

$$C_t + I_t + a(u_t)\widetilde{K}_t + X_t = Y_t$$

Detrending and Model Solution

Detrending

- Output, consumption, physical capital, investments in physical capital and investments in R&D fluctuate around a balanced growth path because of labour augmenting technological growth.
- The variables need to be detrended before simulating the model around the steady state.
- How to detrend ?
  - Investments in R&D being a function of the technological frontier ⇒ suggesting detrending **by the technological frontier**
  - Aggregate output, investment and consumption are functions of the average prevailing technology level ⇒ suggesting **by the average technology**
- In an exogenous growth model, the frontier and the average technology are one and the same.
- Our interest being in the implications of endogenous Schumpeterian growth for business cycles ⇒, we prefer detrending with respect to the average technology level.
- The average technology level $\overline{A}_t$:

$$\overline{A}_t \equiv \int_0^1 A_t(i)di$$

$$\overline{A}_t = n_{t-1}A_{t-1}^{max} + (1-n_{t-1})n_{t-2}A_{t-2}^{max} + (1-n_{t-1})(1-n_{t-2})n_{t-3}A_{t-3}^{max} + ...$$

$$\overline{A}_t = n_{t-1}A_{t-1}^{max} + (1-n_{t-1})\overline{A}_{t-1}$$

Detrending and Model Solution

Detrending (cont'd)

- All nominal variables with a trend, including the auxiliary variables, are made stationary, generally requiring the nominal variables to be divided by the product of P_t and some appropriate power of $\overline{A}_t$.
- Consequently, the detrending of many relevant variables involves the distance of a firm i's technology level relative to the average technology level prevailing in the economy, i.e. $d_t(i) \equiv A_t(i)/\overline{A}_t$.

The Calibration of the Parameters and the Characteristics of the Various Shocks

The share of capital in the production function α, the discount rate β and the depreciation rate of physical capital δ are set to standard values in the literature. The steady-state gross trend inflation π is set at 1 (i.e. the inflation rate is zero in the steady-state). We also assume full capacity utilization in steady-state, i.e. $u = 1$. Market power in labour and intermediate goods leads to, generally agreed upon, wage and price markups of around 20% which is the benchmark in Christiano *et al.* 2005. It is equivalent to an elasticity of substitution of 6 between intermediate goods, as well between labour types. The 0.66 Calvo parameter amounts to a 3 quarter average duration of price/wage contracts.

The standard parameters

Parameter	**Value**	**Meaning**
α	1/3	Share of capital
β	.99	Discount rate
δ	.025	Depreciation rate of physical capital
π	1	Steady state inflation
u	1	Steady state capacity utilization
ϵ	6	Elasticity of substitution of intermediate goods
γ	6	Elasticity of substitution of labor
ξ_p	.66	Calvo parameter of prices
ξ_w	.66	Calvo parameter of wages
θ	5	Weight on the disutility of labour
ν	1	Utility parameter that determines the Frisch elasticity of labour, $(\frac{1}{\nu})$

Calibration and Simulation of the Model

Its Specificities :

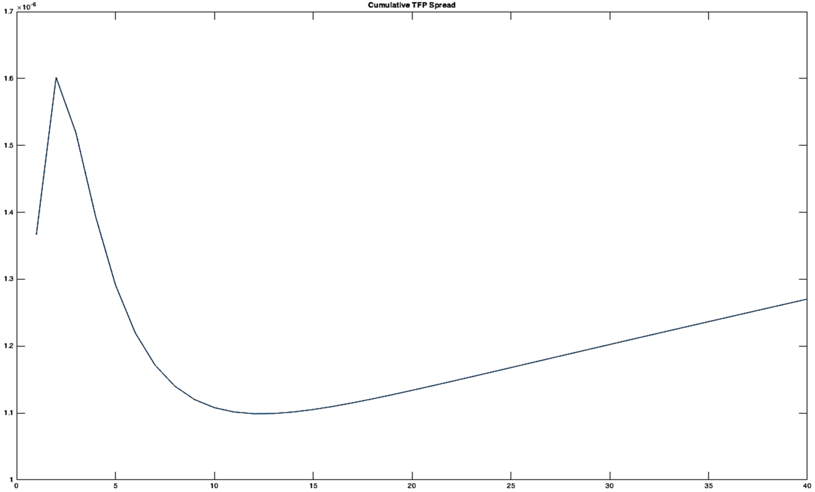
- To set the steady-state value of the R&D-to-GDP ratio : from 1960 to 2016, the average share : 2.56%.
- Innovation translates into a permanent technological shift forward of the production frontier : we set the innovation probability and spillover effect at the steady-state to mimic a steady-state quarterly growth rate of the frontier to 1.06%, in accordance with the HP trend component of U.S. TFP, with a somewhat persistent (0.9) stochastic process for technical knowledge-spillover, to match the observed autocorrelation and volatility of TFP (from Fernald, 2017).

Calibration of the Parameters and Characteristics of the Various Shocks

Persistence and variance of shocks

- Transitory technological shock
- Spillover shock $g_t = 1 + \sigma_t n_{t-1}$
- Investment shock
- Monetary policy shocks

The compounded (cumulative) impact on TFP of a transitory technology shock with endogenous growth vs exogenous growth



The Business Cycle Analysis

Key features of the model

- Sticky prices
- Sticky wages
- No indexation
- No trend inflation
- Variable capacity utilization

The impulse response functions

Note : The IRFs are that of real detrended variables, as they express deviations from trend, so they should be analyzed as such. A negative response associated with an aggregate variable would not necessarily entail a decrease in that variable, but may rather represent a smaller rise, when taking the trend into account.

- Transitory Technological Shock in NK endogenous growth vs. NK exogenous growth models Figure 1
- Transitory Technological Shock in NK endogenous growth vs. RBC endogenous growth models Figure 2
- Efficiency of Investment Shock in NK endogenous growth vs. NK exogenous growth models Figure 3
- Monetary Policy Shock in NK endogenous growth vs. NK exogenous growth models Figure 4
- Spillover Shock in a NK endogenous growth model : cyclical impact Figure 5 ; level impact Figure 6

Main remarks

- Similar impulse response function in both cases as expected
- Differences in amplitude linked to the additional propagation mechanism
- Strong relation between R&D and interest rates
  - Spillover $\uparrow \implies$ Output $\uparrow$ and Inflation $\uparrow$
  - Monetary authority reacts to deviations from its inflation and output targets $\implies R \uparrow$
  - $\frac{\Lambda_{t+1}}{\Lambda_t} \downarrow \implies$ the value of the firm $\downarrow$ and R&D $\downarrow$
- **Optimal monetary policy in the presence of endogenous growth**

Key correlations

	$\rho(\hat{Y},\hat{C})$	$\rho(\hat{Y},\hat{I})$	$\rho(\hat{C},\hat{I})$
Data	0.8549	0.7534	0.5796
Exogenous growth	0.3962	0.1444	-0.0582
Endogenous growth	0.8857	0.8821	0.5628

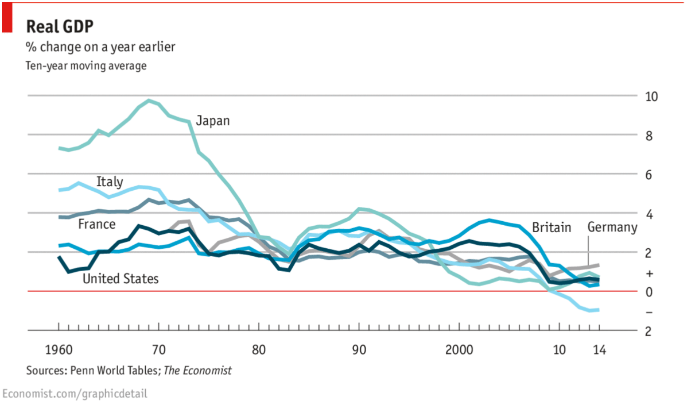
Note : A ˆ-variable expressed percentage deviations from trend. This table shows selected correlations from an exogenous growth New Keynesian model, from an endogenous growth New Keynesian (both models include trend inflation, variable capacity utilization and no price or wage indexation) and data from the Federal Reserve Bank of St Louis. We apply a logarithmic transformation to the data before extracting the cyclical component using the Hodrick-Prescott filter. Simulated data is already detrended so we use the logarithmic difference to compute percentage deviations from the trend. Moments in the data are computed for the sample 1960q1-2016q4.

Key correlations

	$\rho(\hat{X},\hat{I})$	$\rho(\hat{X},\hat{C})$	$\rho(\hat{Y},\hat{X})$
Data	0.5329	0.5152	0.5796
Endogenous growth	0.6577	0.9034	0.8857

Note : This table shows selected correlations from an endogenous growth New Keynesian(which includes trend inflation, variable capacity utilization and no price or wage indexation) and data from the Federal Reserve Bank of St Louis. We apply a logirthmic transformation to the data before extracting the cyclical component using the Hodrick-Prescott filter. Simulated data is already detrended so we use the logarithmic difference to compute percentage deviations from the trend. Moments in the data are computed for the sample 1960q1-2016q4.

Preliminary conclusions - Secular stagnation (1)



Preliminary conclusions - Secular stagnation (2)

- Did we shift from a steady state to another ?
  $\rightarrow$ deterministic simulation
- Can these shifts allow us to better match the data through changes in :
  - the innovation probability / average duration of patents
  - the frontier growth rate
  - the average growth rate
  - the spillover effect

Preliminary conclusions - Policy implications

- Budgetary/fiscal policy's effects on innovation and growth
- Optimal monetary policy
  - Effects of the spillover shock
  - Effects of persistently high interest rates
  - Monetary authority's reaction should vary based on the source of the fluctuations

Merci pour votre attention !

Literature

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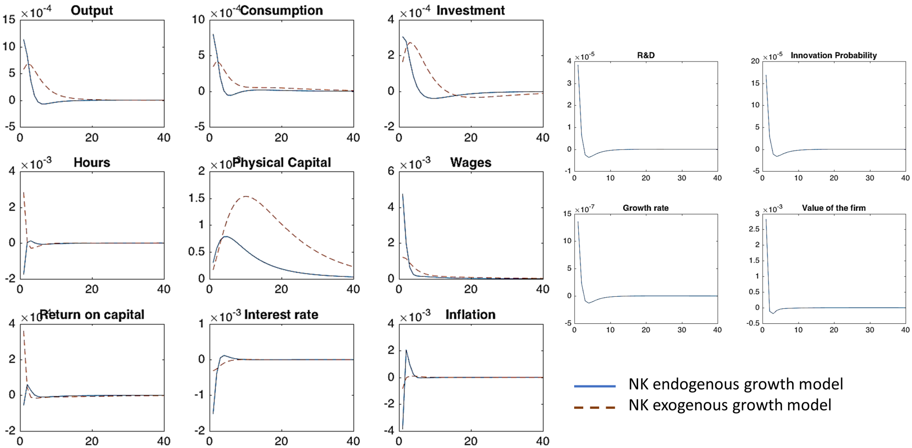
- Typical New Keynesian (NK) DSGE business cycles models constructed around a classical exogenous growth model [Smets & Wouters (2007) and Justiniano *et al.* (2010)]

More recently :

- Nuno (2011) : a RBC model with Schumpeterian growth, with specific functional forms and no nominal rigidity.
- Annicchiarico *et al.* (2011) : Calvo staggered prices and wages, and endogenous growth operating through non-rival access to knowledge [to analyze monetary volatility and growth]
- Amano *et al.* (2012) : nominal rigidities being modelled as Taylor (1980) staggered price and wage contracts, and endogenous growth from horizontal innovations in the variety of intermediate goods [to study effects on the welfare costs of inflation]
- Annicchiarico & Rossi (2013) : Calvo staggered prices, and endogenous growth from IRS knowledge externalities [to study optimal monetary policy]
- Annicchiarico & Pelloni (2014) : one-period ahead preset prices and wages, with CRS innovation production technology, and labour as sole endogenous allocated to produce either goods or R&D [to study the effect of nominal rigidities on the uncertainty of long-term growth]
- Cozzi *et al.* (2017) : price and wage rigidities arising from specific adjustment costs in presence of Solow-neutral technology with Schumpeterian growth (Use Bayesian estimation) [to study the implication of financial conditions for innovation dynamics]
- Anzoategui *et al.* (2017) : staggered Calvo contracts driving sluggish adjustments of wages and final-good prices, with endogenous growth *via* the expanding variety of intermediate goods resulting from public learning-by-doing in the R&D process and an endogenous pace of technology adoption (estimation of the model) [to evaluate the sources of the productivity slowdown following the Great Recession]

Transitory Technological Shock : NK endogenous growth vs. NK exogenous growth



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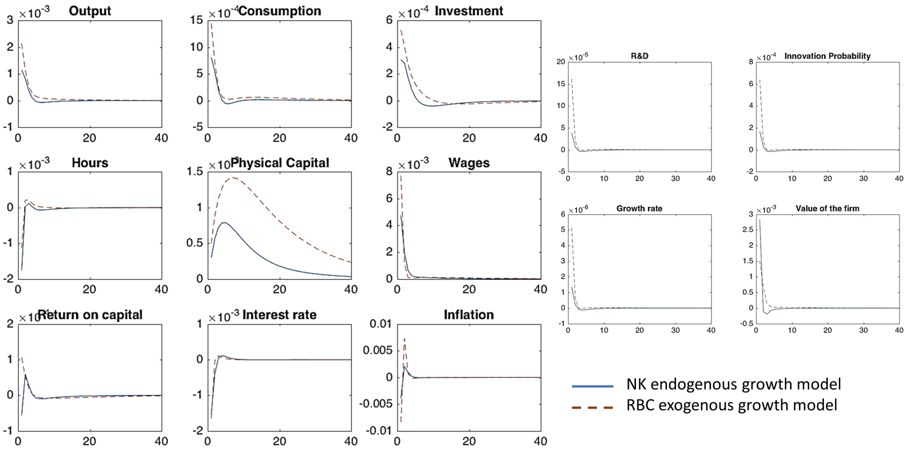
NK endogenous growth vs. RBC endogenous growth



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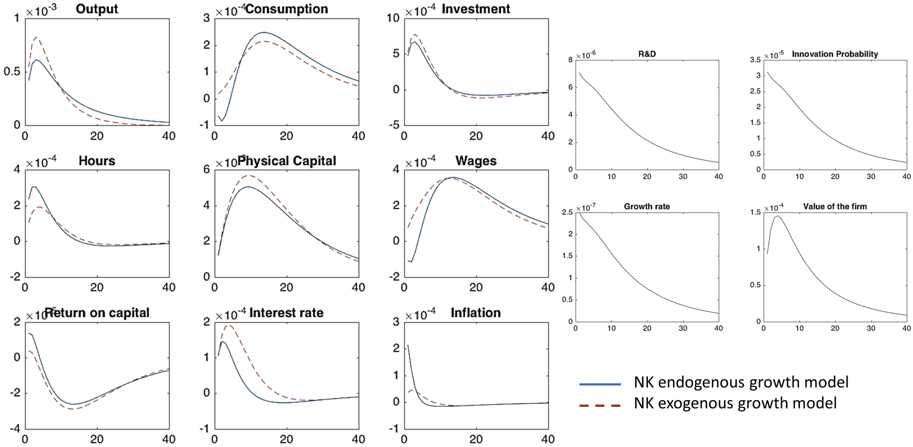
Efficiency of Investment Shock : NK endogenous growth vs. NK exogenous growth



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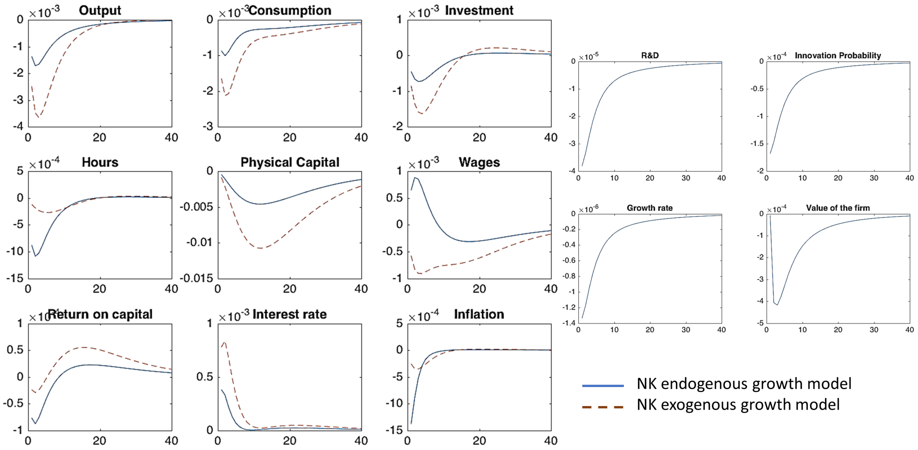
Output
Consumption
Investment
Hours
Physical Capital
Wages
Return on capital
Interest rate
Inflation
R&D
Innovation Probability
Growth rate
Value of the firm
NK endogenous growth model
NK exogenous growth model

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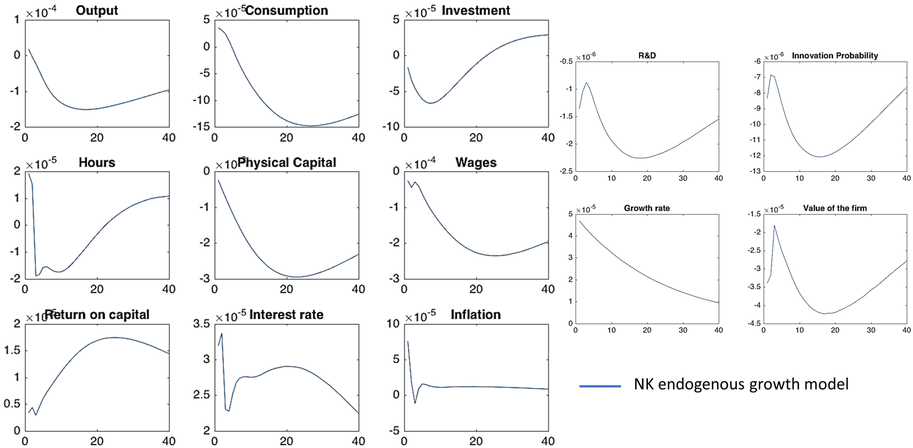
Spillover Shock in NK endogenous growth model : the cyclical impact



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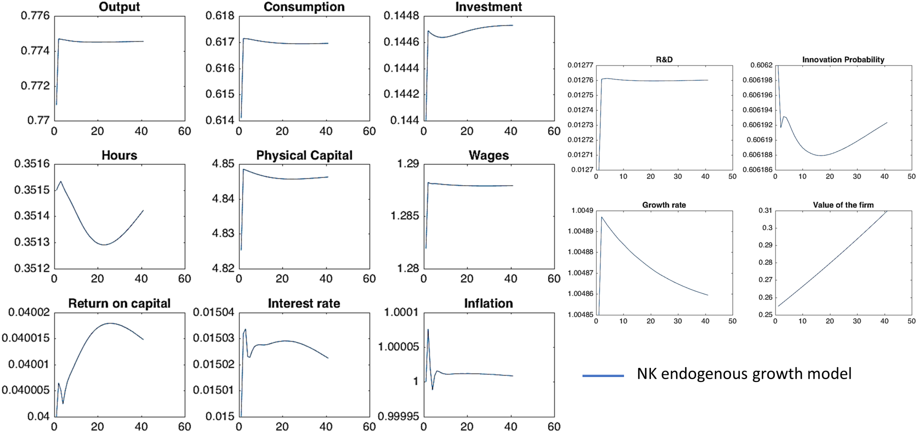
Spillover Shock in NK endogenous growth model : the cumulative impact (on level)



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