# Network Search: Climbing the Ladder Faster

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# Motivation

# Networks are important in labor market search

1. Significant fraction of workers search using contacts
   - SCE: $\sim \frac{1}{4}$ found their job by referral from professional-connections (Arbex et al 2018)
   - PSID: $\sim \frac{1}{2}$ found their job through social network (Corcoran, Datcher and Duncan, 1980).
2. Firms use referrals when filling a vacancy.
   - EOPP: 36% of firms filled their last vacancy through a referral (Holzer, 1987).

# Networks are “irregular”

People differ in the number of links they have, which:

- implies heterogeneity in finding rate both on and off the job
- implies heterogeneity in the quality of offers drawn.

This paper: Different people climb the ladder differently

# What we do

- Put an irregular network into a model of on/off-the-job search
  - Workers find jobs through network
  - Firms' workers become search capital
- Use mean-field approach to tractably describe the network
- Calibrate and compare vis-à-vis common empirical findings
  - New evidence from SCE

# Key results

- Use mean-field approach to reduce an $\infty$-dimensional state to 3
- Analytical results:
  - Network search draws from a “better” (FOSD) distribution than direct contact search
  - Network search reduces firms’ profit

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- Analytical results:
  - Network search draws from a “better” (FOSD) distribution than direct contact search
  - Network search reduces firms’ profit
- Calibrate to direct contact & network search. The latter:
  - Have higher wages on acquisition (Marmaros & Sacerdote, 2002)
  - Occur after a shorter unemployment spell (Goel & Lang, 2009)
  - Longer match duration (Dustmann et al 2014)
  - More likely higher on the ladder (Arbex et al. 2018)

# Basic environment

On-the-job search (as in Burdett and Mortensen 1998)

- Firms post wages that may be found via direct contact
- Workers are ex ante heterogeneous in their peers
- Employees pass offers to peers for positions just like own

Easily extensible to additional heterogeneity

# Before getting into the weeds

The mechanism is:

- Workers with more connections sample jobs more quickly
- They climb the ladder faster
- Referrals are useful for 2 reasons:
  1. Draw from the wage distribution rather than direct offer distribution
  2. Draw from friends who are better connected (*paradox of friendship*)

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Network search is done by better connected workers:

- Jobs through the network are higher paid
- Jobs through the network last longer
- Jobs through the network follow shorter unemployment

# Literature

- Network theory: Vega-Redondo (2007), Calvo-Aremengol & Jackson (2007), Calvo-Aremengol & Jackson (2004)
- Empirical finding: Cornelissen, Dustmann & Schoenberg (2015), Hellerstein, Kutzbach, Neumark (2014), Holzer (1988)
- Search and networks: Galenianos (2014), Fontaine (2008), Ioannides & Soetevent (2006), Mortensen & Vishwanath (1995)

# Model of search and networks in labor markets

# Technology, flows and types

Technologies:

- Workers heterogeneous in number of peers, $z$
  - Characterized by degree distribution $\Omega(z)$
- Workers homogeneous in non-employment flow value, $b$
- Firms are homogeneous, with productivity 1

Flows:

- Random search, matched via either *direct* or *network* search
- Jobs break up exogenously at rate $\delta$

# Vanilla direct search

- Firms post wages $w$, distributed as $F(w)$, firm offer distribution
- A worker meets vacancy at rate $\gamma^i$
- An unemployed worker exits if $w \geq R(\cdot)$
- An employed worker accepts jobs above her current wage

# Networks search

- Employed find & pass along jobs at their firm at rate $\gamma^1 \nu$
- Workers sample via their network connections, arrival rate $\rho(\cdot)$
  - Any employed peer equally likely to send a referral
  - Any peer of employed worker equally likely to receive a referral
- Connections pass jobs with the same wage (i.e. same firm)
- Same acceptance rules: reservation wage $R(\cdot)$ or $w$.

# What is a worker type?

Define $\chi$ recursively:

- Each worker has $z$ peers
- $\chi$ is $z \times 4$. Element $c$ is a triple
  - $i(c)$, the labor status
  - $w(c)$, the wage
  - $k(c)$, the history of wages
  - $\chi(c)$, the position in the network
- $\chi(c)$ is also a $s \times 4$ dimensional object, s.t. $s$ is the number of peers of peer $c$

To forecast the value of a peer:

- His wage that might be passed
- His *potential* wage next period

# The mean-field approach

Goal:

- Remove local information from the state
- Instead of how particular atoms interact, use average atom effect
- Will take the position in network from $\chi$ to $z$

Requires:

1. Incomplete information about peers
2. A locally tree-like structure

# We didn't make this up

- Vega-Redondo (2007) uses this approach so that the average state of the network is replicated *locally*: **No neighborhood effects** (Vega-Redondo 2007).
- Good representation of the long-run dynamics of networks (Vega-Redondo 2007, Jackson 2008).
- This or similar idea used in network search papers: Calvo-Armengol & Zenou (2005) or Bramoulle & Saint Paul (2010)

# Assumption 1: Tree structure

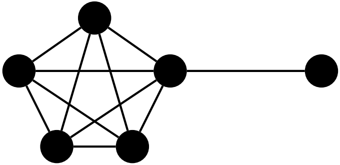
Assumption:

- The network is described *completely* by the degree distribution, $\Omega$
- As nodes $n \to \infty$, probability of a cluster $\to 0$

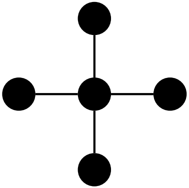
The effect:

- For any $\chi$ and $\chi'$ if $z = z'$ then $E[s|\chi] = E[s'|\chi']$
- $z$ has no information about local conditions

# Networks we rule out



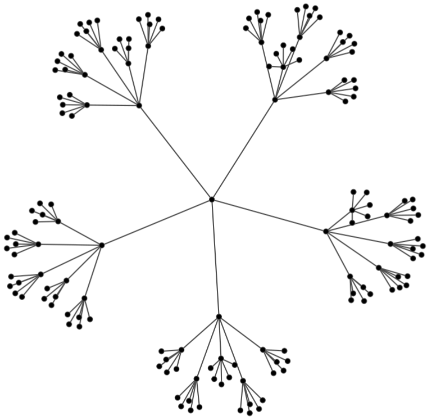
A clustered subgraph where $\langle z \rangle = 4$



A regular subgraph where $\langle z \rangle = 4$

- The clustered network has local structure
- The regular network is uninteresting

# Our network structure: A tree



No local "neighborhood," but number of connections differs

# Assumptions 2& 3: Incomplete information/memory

2. Limited observability assumption:
   - Agents do not know the state $(i(c), w(c), k(c), \chi(c))$ of peer $c$
   - Agents know $c$ exists and can use degree distribution, $\Omega$
   - Use $k$ to form beliefs $(\hat{i}(c), \hat{w}(c), \hat{k}(c), \hat{\chi}(c))\ \forall c$
3. Limited memory assumption
   - Agents know $c$ exists and can use degree distribution, $\Omega$
   - No information on which to form beliefs

# Proposition: $z$ is a sufficient statistic

Under each assumption

1. $z = z'$ can differ only in $\{i(c), w(c)\}$
2. Cannot directly observe $\{i(c), w(c)\}$
3. Cannot use $k$ to infer $\{i(c), w(c)\}$

Workers will differ in “connectedness,” but that is unidimensional

# Type-distribution of referral passer

- $\Psi(s)$: probability a worker's peer has $s$ peers herself

$$\Psi(s) = \frac{s\Omega(s)}{\langle z \rangle}$$

- $\Psi(s) < \Omega(s)$ is the *paradox of friendship*

# Type-distribution of referral passer

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$$\Psi(s) = \frac{s\Omega(s)}{\langle z \rangle}$$

- $\Psi(s) < \Omega(s)$ is the *paradox of friendship*
- Probability a peer is $s$ and passes referral:

$$\gamma^1 \nu \frac{n(s)}{s} \Psi(s)$$

- Then the distribution is

$$\tilde{\Psi}(s) = \frac{\gamma^1 \nu \frac{n(s)}{s} \Psi(s)}{\int \gamma^1 \nu \frac{n(z)}{z} \Psi(z) dz} = \frac{n(s)\Omega(s)}{\int n(z)\Omega(z) dz}$$

# Network search arrival rate

The probability a worker of type $z$ receives an offer via a peer is

$$\rho(z) = \lim_{\Delta \to 0} \left( 1 - \left[ 1 - \int_s \Psi(s) \gamma^1 n(s) \frac{\nu}{s} ds \, \Delta \right]^{z/\Delta} \right)$$
$$= \left( 1 - \exp\left( -z\nu\gamma^1 \int \frac{n(s)}{s} \Psi(s) ds \right) \right)$$

- $n(s)\gamma^1$ is the probability this peer is employed and hears of an vacancy
- $\nu/s$ is the probability this information is passed along

# Network offer distribution/earnings distribution

The earnings distribution among agents of type $z$

$$G(w, z)$$

Earnings distribution in the population:

$$G(w) = \int_z G(w, z)\Omega(z)dz$$

Network offer distribution:

$$\tilde{G}(w) = \int_s G(w, s)\tilde{\Psi}(s)ds$$

Offers through the network are drawn from $\tilde{G}(w)$

# Model of search and networks in labor markets:

## Workers' value functions

# Unemployed Worker's Value Function

The value function of an unemployed worker of type $z$ is

$$rV^0(z) =$$

$$b + \underbrace{\gamma^0 \left\{ \int_{R(z)}^{\bar{w}} \left[ V^1(z,x) - V^0(z) \right] dF(x) \right\}}_{\text{The value of direct search}}$$

$$+ \underbrace{(1-\gamma^0)\rho(z) \int_{R(z)}^{\bar{w}} \left[ \left( V^1(z,x) - V^0(z) \right) \right] d\tilde{G}(x)}_{\text{The value of network search}}$$

# Employed Worker's Value Function

The value of an employed worker with $z$ connections and wage $w$ is

$$rV^1(z,w) =$$
$$w + \underbrace{\delta\left[V^0(z) - V^1(z,w)\right] + \gamma^1\left\{\int_w^{\bar{w}}\left[V^1(z,x) - V^1(z,w)\right]dF(x)\right\}}_{\text{The value of direct search}}$$
$$+ \underbrace{(1-\gamma^1)\rho(z)\int_{R(z)}^{\bar{w}}\left[\left(V^1(z,x) - V^1(z,w)\right)\right]d\tilde{G}(x)}_{\text{The value of network search}}$$

# Reservation Wage

At the reservation wage $R(z)$, we have that $V^1(z, R(z)) = V^0(z)$.

$$R(z) - b = \left(\gamma^0 - \gamma^1\right)\left\{\int_{R(z)}^{\bar{w}} \left[V^1(z,w) - V^0(z)\right] dF(w)\right\}$$
$$+ \left[\begin{array}{c}(1-\gamma^0)\rho(z) \\ -(1-\gamma^1)\rho(z)\end{array}\right]\left\{\int_{R(z)}^{\bar{w}} \left[\theta(w)\left(V^1(z,w) - V^0(z)\right)\right] dw\right\}$$

$$= \left(\gamma^0 - \gamma^1\right)\left\{\int_{R(z)}^{\bar{w}} V_w^1(z,w)\left[1 - F(w)\right] dw\right\}$$
$$+ \left[(\gamma^0 - \gamma^1)\rho(z)\right]\left\{\int_{R(z)}^{\bar{w}} V_w^1(z,w)(1 - \tilde{G}(w)) dw\right\} \quad (1)$$

# Wage Distribution and Workers per Firm

- $\ell(w, z)$: Labor force of type z per firm at a firm paying wage $w$
- $L(w)$: Total labor input per firm paying wage $w$:

$$L(w) = \int_1^\infty \ell(w, z) dz \tag{2}$$

- Each employer offers a wage that gives steady state profit:

$$\pi(w) = (1 - w)L(w) \tag{3}$$

# Steady state equilibrium and analytic results

# Steady State Employment of Workers

Flows in and out of unemployment must balance, give the steady state employment rate:

$$n(z) = \frac{\overbrace{\gamma^0 \left[1 - F\left(R(z)\right)\right]}^{\text{Recruiting from direct search}} + \overbrace{(1-\gamma^0)\rho(z)\left[1 - \tilde{G}\left(R(z)\right)\right]}^{\text{Recruiting from network search}}}{\delta + \gamma^0 \left[1 - F\left(R(z)\right)\right] + (1-\gamma^0)\rho(z)\left[1 - \tilde{G}\left(R(z)\right)\right]}, \tag{4}$$

The economy's employment rate is given by

$$n = \int_1^\infty n(z)\Omega(z)dz \tag{5}$$

# Steady State Earnings Distribution

$$G(w,z) =$$

$$\frac{[1-n(z)]\left\{\overbrace{\gamma^0\left[F(w)-F(R(z))\right]}^{\text{Direct search effect}}+\overbrace{(1-\gamma^0)\rho(z)\left\{\tilde{G}(w)-\tilde{G}(R(z))\right\}}^{\text{Network search effect}}\right\}}{n(z)\left[\delta+\gamma^1\left[1-F(w)\right]+(1-\gamma^1)\rho(z)(1-\tilde{G}(w)\right]}$$

Because $\frac{F(w)-F(R)}{(1-F(w))} \geq \frac{\tilde{G}(w)-\tilde{G}(R)}{(1-\tilde{G}(w))}$, averaging in dominating distribution

# Steady State Firm Size

Separating of $z$-type workers equal the $z$-type workers:

$$\ell(w,z)\beta(w,z) = h(w,z) \tag{6}$$

where

- $\beta(w,z) =$
  $$\delta + \underbrace{\gamma^1(1-F(w))}_{\text{Loss to poaching via direct search}} + \underbrace{(1-\gamma^1)\rho(z)\left[1-\tilde{G}(w)\right]}_{\text{Loss to poaching via network search}}$$
- $h(w,z) =$
  $$\overbrace{\frac{\Omega(z)}{M}\left\{[1-n(z)]\,\gamma^0\mathbb{I}_{R(z)\leq w} + n(z)\gamma^1 G(w,z)\right\}}^{\text{Hired via direct search}} +$$
  $$\underbrace{\gamma^1\int \ell(w,t)t\Psi(z)\left\{[1-n(z)]\,\nu\mathbb{I}_{R(z)\leq w} + n(z)\nu G(w,z)\right\}dt}_{\text{Hired via network search}}$$

# The steady state equilibrium

## Definition

A *Sufficient Recursive Equilibrium*: $V^0$, $V^1$, $R$, $\pi$ and $F(w)$, $G(w,z)$, $n(w)$, such that:

- $V^0$, $V^1$, $R$ solve household problem
- $G$, $n$ consistent with worker flows
- $F$ implies $\pi(w) = \pi \; \forall w$

# Ordering offer distributions, $\tilde{G} \leq F$

## Proposition

*$\tilde{G}$ First Order Stochastically Dominates $F$*

- As in Burdett Mortensen, $G \leq F$ because $\gamma^1 > 0$
- $\tilde{G}$ weights $G$ by $n(\cdot)$: $\int \frac{n(s)G(w,s)\Omega(s)}{\int n(z)\Omega(z)} ds$
- $n' > 0$, which is guaranteed by
  - $\rho' > 0$ by definition
  - $R' < 0$ because $V^1_{wz}(z, R(z)) > 0$

# The equilibrium effect of network search

## Proposition

*Beginning from $\nu = 0$, for sufficiently high $\gamma$*[1]

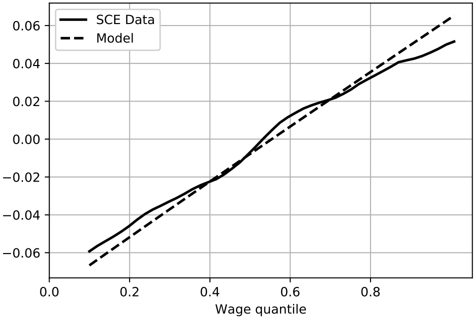
- $\frac{\partial \underline{w}}{\partial \nu} \leq 0$ *and* $\frac{\partial \bar{w}}{\partial \nu} \geq 0$
- $\frac{\partial L(\underline{w})}{\partial \nu} \leq 0$ *and* $\frac{\partial L(\bar{w})}{\partial \nu} \geq 0$
- $\frac{\partial \pi}{\partial \nu} \leq 0$

- $\frac{\partial \underline{w}}{\partial \nu} \leq 0$ because $\frac{\partial R(z)}{\partial \nu} \leq 0$
- $\frac{\partial L(\underline{w})}{\partial \nu} \leq 0$ because poaching is faster
- $\frac{\partial \pi}{\partial \nu}$ depends on $\frac{\partial L(\underline{w})}{\partial \nu}$ (Envelope condition takes care of $\frac{\partial \underline{w}}{\partial \nu}$)
- $\frac{\partial L(\bar{w})}{\partial \nu} \geq 0$ because own workers increase hiring

# Results from the calibrated economy

# SCE: Higher wage workers use networks more

- Model prediction: higher-wage workers find jobs through networks
- Survey of Consumer Expectations (SCE) asks workers their current job's finding method



# Parameter values

| Parameter | Value | Moment | Model | Data |
|---|---|---|---|---|
| $\gamma^0$ | 0.24 | Finding rate UE | 0.24 | 0.25 |
| $\gamma^1$ | 0.10 | Finding rate EE | 0.02 | 0.02 |
| $\nu$ | 0.04 | Hires through the network | 0.13 | 0.23 |
| $\alpha$ | 2.34 | Network finding slope | 0.26 | 0.25 |
| $\delta$ | 0.013 | Average EU | | |

# Average offer distribution by type

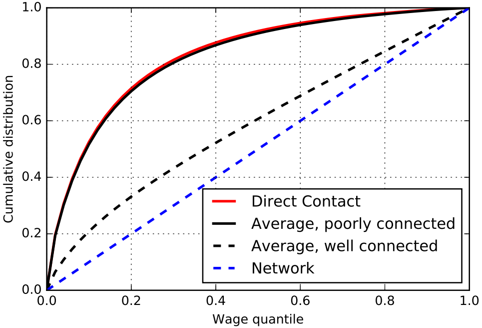



Figure: Average distribution of wage offers by contact method conditional on number of peers.

# Average hiring method by wage

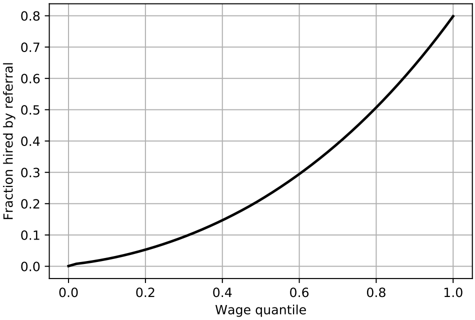



Figure: At higher wage levels, most hiring occurs through referral.

# The half-life by connections

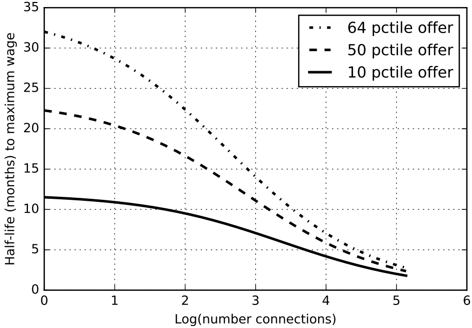



Figure: Half-life of wage growth paths to maximum wage: different starting wages and different network connections $z$.

# The effect is not just heterogeneous search

We let arrival rates differ by $z$, but not the offer distribution

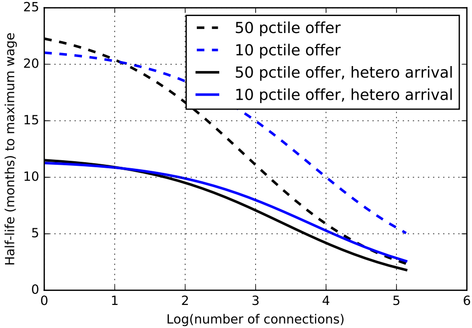



Figure: Half-life of wage growth comparing heterogeneous search rates and network search model

# Results from the calibrated economy: Relationship to empirical findings

# The different distributions of workers

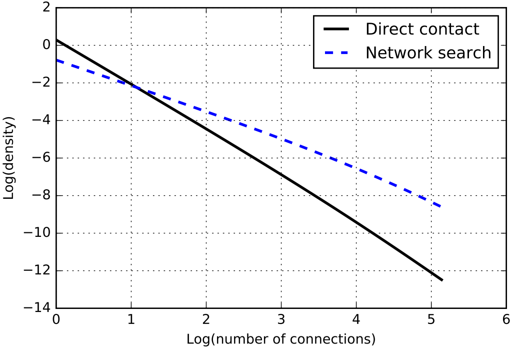



Figure: Distribution of number of peers: Direct search and network search.

# Summarizing the effects

| | Network Search | Direct Search |
|---|---|---|
| Average $z$ relative to unemployed | 4.14 | 0.93 |
| Expected wage qtile; UE | 0.251 | 0.075 |
| Search time relative to avg | 0.951 | 1.001 |
| Average $z$ relative to employed | 2.67 | 0.80 |
| Expected wage qtile; job-to-job | 0.444 | 0.224 |
| Expected duration of job match | 4.87 years | 2.70 years |

Table: Expected differences between workers finding jobs through network or directed search. Above the line describe finding from unemployment, below adds features of job-to-job transitions.

# Conclusion

# Conclusions

- We presented a model of network search
- The mean-field approach allows for tractable, irregular networks
- Highly extensible to other search frameworks
- Empirical findings on search consistent with type heterogeneity and job ladders