

# Mathematics for Scenarios of Biodiversity

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## Sustainability in the Digital Age, Resilience facing Global Changes

Montreal, May 2019

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# Mathematics for Scenarios of Biodiversity and Ecosystem Services

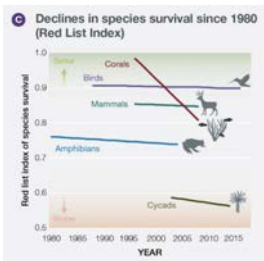
Authors

[Authors and affiliations](#)

Luc Doyen 

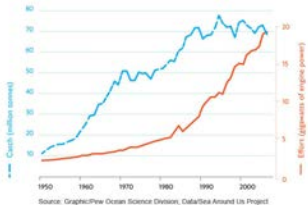


# Biodiversity and ecosystems under pressure



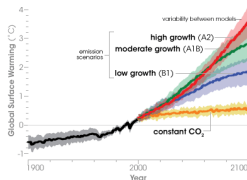
- Global changes in ecosystems
- Ecological vulnerabilities
- Economic vulnerabilities

⇒ Management of bio-economic risks

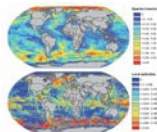


# The need for bio-economic scenarios

From

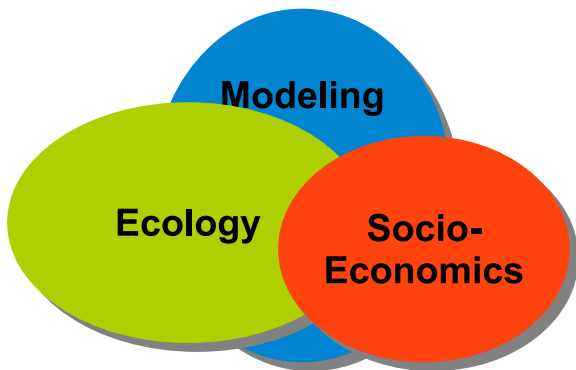


To



**FIGURE 18** PROJECTED CHANGES IN MARINE BIODIVERSITY DUE TO CLIMATE CHANGE.  
Biodiversity impact in 2100 under the IPCC A1B1.5 scenario (expressed in terms of number of new species coming from other regions (top) and local extinction events (bottom)). The projections are based on five climate scenario models for 1,000 species of fish and invertebrates. Source: authors from Cheung et al. 2006.

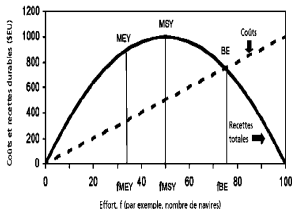
# The need for bio-economic modeling



# Important models

## Equilibria

*Gordon-Schaefer, 1954*

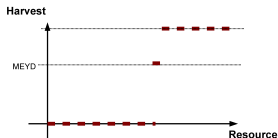


MSY  $\longleftrightarrow$  MEY

## Intertemporal

## Optimality

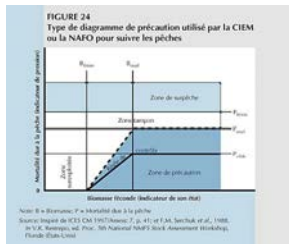
*Clark, 1976*



- Optimal closure
- Optimal extinction

## Precautionary approach

*ICES-CIEM*



# Renewable Resource Dynamics

In discrete time

$$x(t+1) = f(x(t)) - h(t)$$

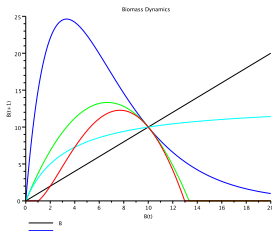
where

- $x(t)$  biomass or abundance
- $h(t)$  catches at time  $t$

Examples of  $f$  :

Logistic,

$$f(x) = x + rx\left(1 - \frac{x}{k}\right)$$



Ricker, Beverton-Holt, Gompertz

# Sustainable yield

- Equilibrium :

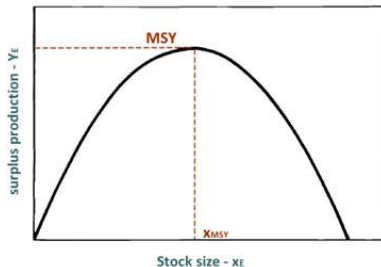
$$x = f(x) - h$$

→ Sustainable yield

$$\sigma(x) = f(x) - x$$

- Example Logistic :

$$\sigma(x) = rx\left(1 - \frac{x}{k}\right)$$





# MSY vs MEY

- MSY : Optimal catches at equilibrium

$$\max_x \sigma(x)$$

Logistic :  $x_{MSY} = \frac{k}{2}$

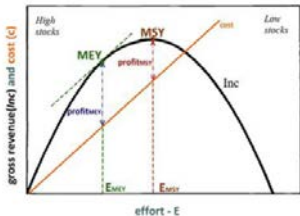
- MEY : Optimal rent at equilibrium

$$\max_{x, h=\sigma(x)} \pi(x, h)$$

where  $\pi(x, h) = ph - c \frac{h}{qx}$

$p$  price,  $c$  costs of effort

Logistic :  $x_{MEY} = \frac{k}{2} + \frac{c}{2pq}$



Bio-economic synergies

# A discounted MEY

The optimal control problem

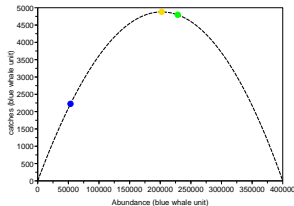
$$\max_{h(0), h(1), \dots} \sum_{t=0}^{\infty} (1 + r_f)^{-t} \pi(x(t), h(t))$$

The long term equilibrium  $x_\rho$  (Conrad-Clark, 1987)

$$r_f = \sigma_x(x_\rho) + \frac{\pi_x}{\pi_h}$$

Particular cases :

- $r_f = 0 : x_\rho \implies x_{\text{MEY}}$
- $r_f = \infty : x_\rho \implies x_{\text{OA}}$



# Where extinction is optimal (Clark 1990)

Let us consider the illustrative case where

- Logistic dynamics
- rent is of the form  $\pi_x = 0$

**Result :** Then if  $r \leq r_f$  then extinction is optimal  $x_p \leq 0$ .

More general conditions in Grafton et al. (2010)

# New bio-economic challenges

- How to operationalize the **ecosystem approach** ?
- How to operationalize **sustainability** ?
- How to operationalize **resilience** ?
- Which **gouvernance** for bio-economic public policies ?

# Eco-viability : an original and fruitful approach



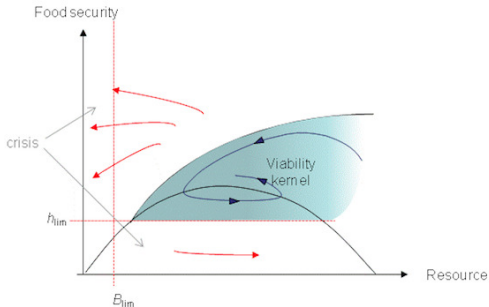
Ecological Economics  
Volume 145, March 2018, Pages 346-367



## A Survey of Applications of Viability Theory to the Sustainable Exploitation of Renewable Resources ☆

Aichouche Oubrahim, Georges Zaccour, R. B.

### Safety of dynamic systems through constraints over time

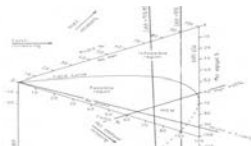
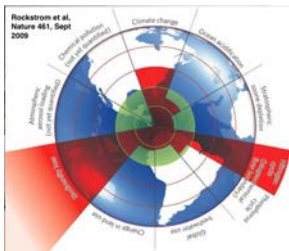


# Links with many approaches

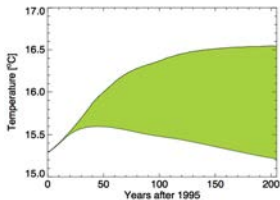
Doyen et al., *Ecological Economics*, 2019

## Minimal Sustainable Whinge

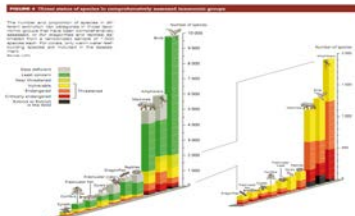
SOS



TWA



PVA



# The mathematical formulation of viability

- Uncertain controlled dynamics :

$$\begin{cases} x_i(t+1) = F_i(x(t), a(t), \omega(t)), \\ x(0) = x_0 \end{cases}$$

where

- $x(t) \in \mathbb{R}^n$  the state at time  $t$
  - $a(t) \in \mathbb{R}^p$  the control at time  $t$
  - $\omega(t) \in \mathbb{R}^q$  the uncertainty at time  $t$
- 
- Constraints :

$$I_k(x(t), a(t), \omega(t)) \geq I_k^{\text{lim}}$$

- Maximal probability of viability

$$V(t_0, x_0) = \max_{a(\cdot)} \mathbb{P} \left( \text{constraints satisfied } t = t_0, \dots, T \right)$$

- Viability kernels : level sets of  $V$

$$\text{Viab}_\beta = \{x_0 \mid V(t_0, x_0) \geq \beta\}.$$

- A dynamic programming structure

$$\boxed{V(t,x)} = \max_a \mathbb{E} \left[ \mathbf{1}_{\text{constraints}}(x, a) * \boxed{V\left(t+1, F(x, a, \omega)\right)} \right]$$

where  $\mathbf{1}_{\text{constraints}}$  boolean function of constraints.



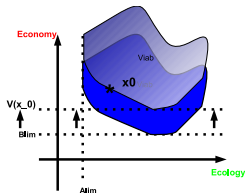
- **Maximin** Rawls (1971), Solow (1974)

$$\max_{a(\cdot)} \min_t I(x(t), a(t))$$

$$= \max(I_{lim}, x_0 \in \text{Viab}(I_{lim}))$$

where  $\text{Viab}(I_{lim})$  the viability kernel with

$$I(x(t), a(t)) \geq I_{lim}$$



- Maximin : extreme viability
- Intergenerational equity in viability
- **Inverse viability : sustainable thresholds** : Gajardo et al., 2019



Ecological Economics

Volume 124, April 2016, Pages 69-75



Methodological and Ideological Options

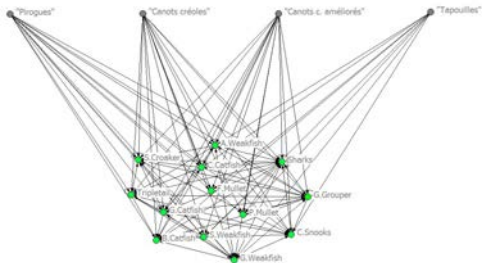
## Economic viability and small-scale fisheries — A review

Anna Schuhbauer , U. Rashid Sumaila 

# A first example : Small scale fisheries in French Guiana

Cissé et al., *Environmental Development Economics*, 2013

Cissé et al., *Ecological Economics*, 2015



# The bio-economic dynamic model

Cissé et al., Ecological Economics, 2015

A MICE model (14 species - 4 fleets)

$$x_i(t+1) = x_i(t) \left( 1 + r_i + \underbrace{\sum_{\text{species } j} s_{ij} x_j(t)}_{\text{Trophic}} - \underbrace{\sum_{\text{fleets } f} q_{if} e_f(t)}_{\text{Fishing}} + \underbrace{\epsilon_i(t)}_{\text{Stochastic}} \right)$$

where  $\epsilon_i(t) \rightsquigarrow \mathcal{N}(0, \sigma_i)$

From data 2006-2010 :

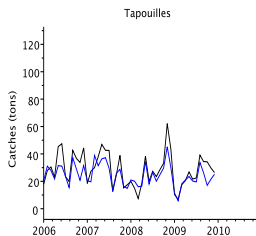
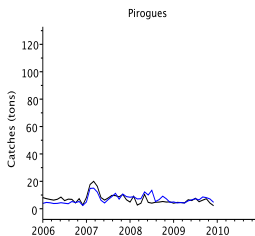
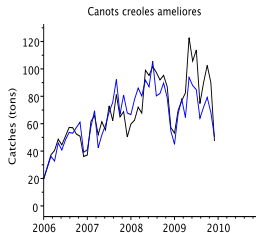
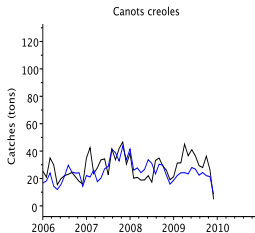
- catches + efforts

$$h_{if}(t) = q_{fi}e_f(t)x_i(t)$$

$$e_f(t)$$

We estimate

- Trophic matrix  $s_{ij}$
- Catchability  $q_{fi}$
- Initial biomass  $x_i(2006)$



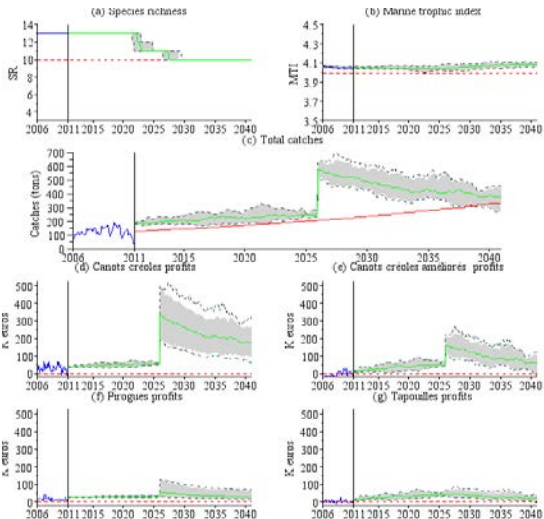
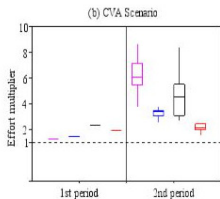
Identify fishing efforts  $e_f(t)$  such that

$$\left\{ \begin{array}{ll} \text{guaranteed biodiversity :} & \text{Species richness and MTI} \\ \text{guaranteed profitability :} & \pi_f(t) \geq 0, \quad \forall \text{ fleet } f \\ \text{food supply} > \text{demand :} & H(t) \geq D_{2008}(1+d)^t \end{array} \right.$$

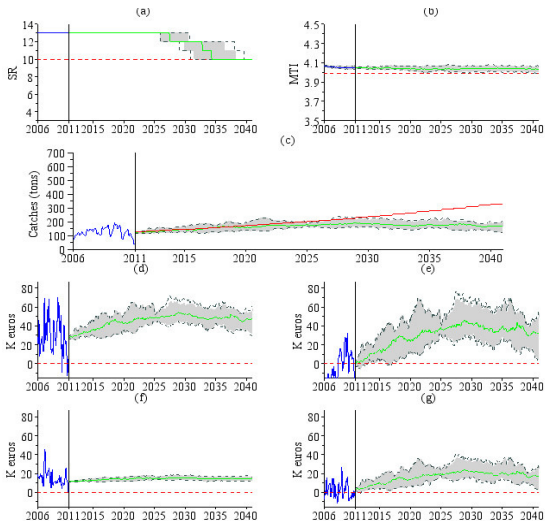
Stochastic approach

$$\max_{e_f(t_0), e_f(t_{1,..})} \mathbb{P}(\text{constraints satisfied for } t = 2011, \dots, T)$$

# Eco-viability strategy EVA



# Status Quo strategy







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## Scenario, fishEry, ecologicAI-economic modelling and Viability nEtWork (SEAVIEW)

### ▾ Funded Project Information

**Call:** Scenarios of Biodiversity and Ecosystem Services

**Lead PI:** Luc Doyen, CNRS, Pessac

**Partners:** Olivier Thebaud, IFREMER, Plouzane

L. Richard Little, CSIRO, Hobart

Martin Quaas, University of Kiel, Kiel

Astrid Jarre, University of Cape Town, Cape Town

Leo Duta, CSIRO, Dutton Park

Deborá Martins de Freitas, Technological Institute of Aeronautics, Santos

Fabian Blanchard, IFREMER, Cayenne

Claire Armstrong, University of Tromsø, Tromsø

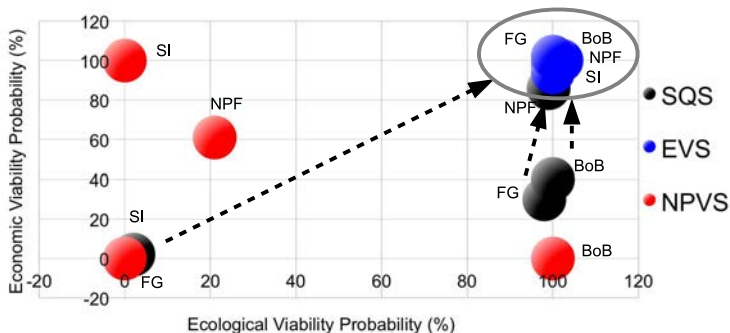
Felipe Gusmao, Federal University of Sao Paulo, Santos

**Sponsored by:** ANR, CSIRO, DFG, FAPESP, NRF, RCN

# WP2 : Comparative analysis between case-studies

*Doyen et al., Fish and Fisheries, 2017*

FG : French Guiana ; NPF : Australian Northern Prawn ;  
SI : Solomon Islands ; BoB : Bay of Biscay

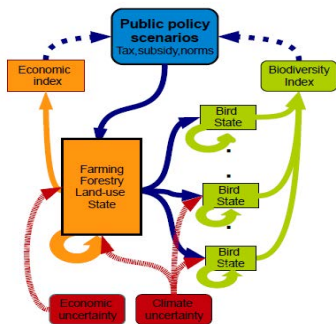


# Farming land-use and bird's biodiversity

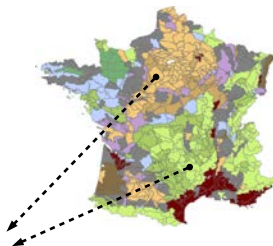


Ay et al., Climatic Change, 2014; Mouysset et al., Ecological Economics, 2014

## A MICE model



Databases 2001-2009



At SAR scale  
(small agricultural region)  
620 regions  
13 land-uses  
+ gross margins



35  
bird species



# An example of bird community dynamics

Mouysset et al., Env. Mod. Ass., 2016

Mouysset et al., Ecological Economics, 2011

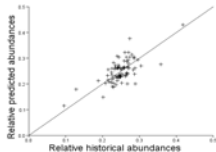
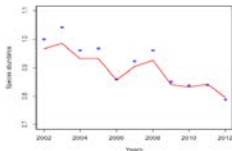
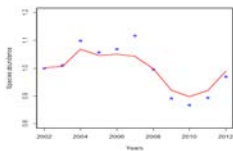
Mouysset et al., Ecological Indicators, 2012

Land-uses  $\rightarrow$  Habitat quality  $\rightarrow$  Carrying capacity of species

$$\frac{x_i(t+1)}{x_i(t)} = \frac{1 + r_i}{1 + r_i \frac{x_i(t)}{k_i(t)}} \quad \text{with} \quad k_i(t) = a_{i,0} + \sum_{\text{land-use } u} a_{i,u} \cdot A_u(t)$$

abundance  
growth of  
species  $i$

area of  
landuse  
 $u$



Bio-economic constraints :

$$FBI(t) \geq a * FBI^{SQ}(t)$$

$$CTI(t) \geq a * CTI^{SQ}(t)$$

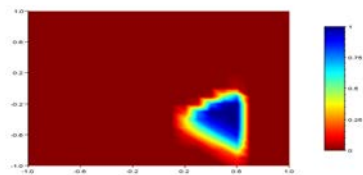
$$CSI(t) \geq a * CSI^{SQ}(t)$$

$$Inc(t) \geq a * Inc^{SQ}(t)$$

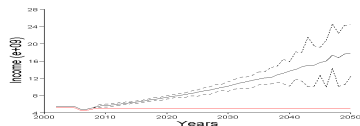
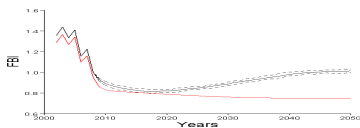
$$BUDGET(t) \leq BUDGET(t_0)$$



2050



Viability probability





- Baumgartner S., Quaas M.F., 2009, Ecological-economic viability as a criterion of strong sustainability under uncertainty, *Ecological Economics*, 68 (7), 2008-2020.
- Schuhbauer, A., Sumaila, U.R. (2016). Economic viability and small-scale fisheries : a review. *Ecological Economics*, 124, 69-75.
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- Mouysset et al (2014), Co-viability of farmland biodiversity and agriculture, *Conservation Biology*.
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- Doyen L., Pereau J.C., 2012, Sustainable coalitions in the commons, *Mathematical Social Sciences*, 63, 57-64.
- Gourguet S. et al., 2013, Managing mixed fisheries for bio-economic viability, *Fisheries Research*, 140, 46-62.
- Hardy P.Y. et al., 2013, Food security - environment conservation nexus : a case study of Solomon Islands' small-scale fisheries, *Environmental Development*
- Doyen L. et al. 2012, A stochastic viability approach to ecosystem-based fisheries management, *Ecological Economics*

## Earth's Future

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### From resistance to transformation: a generic metric of resilience through viability

Christophe Béné, Luc Doyen 



## Resilience a concept to **be less vulnerable to shocks**

SPORT | Rugby

### Rugby: Hansen praises All Blacks' resilience

10 Nov, 2013 3:22pm

3 minutes to read

- **Successful :**  
...in Rugby



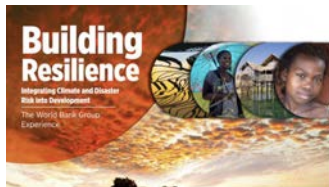
France's Brice Dulin is tackled by New Zealand's All Blacks players during their international rugby match at the Stade de France stadium in Saint Denis. Photo / AP





## Resilience a concept to **be less vulnerable to shocks**

- **Successful** : FAO, EU, World Bank, ...
- **Successful in sciences** :  
psychology (Glantz & Johnson, 1996)  
engineering sciences (Grimm and Wissel, 1997),  
ecology (Holling, 1973; Gunderson - Folke, 2005)  
economics (Derisen et al, 2011)



## However

- Generic metrics across disciplines?
- Bad resilience
- Resilience with respect to what?

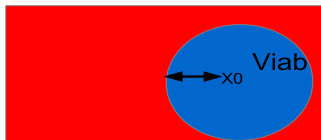


- 1 **Viability probability**  $\approx$  reliability
- 2 **Magnitude of acceptable shocks**

distance to non viable zone :

$$\min_{x \notin \text{Viab}} \|x_0 - x\|$$

Béné & Doyen, Earth's Future, 2018



- 3 **Recovery after a shock :**

Doyen-StPierre, 1997 ; Martin, 2006 ; Deffuant Gilbert, 2011; Rougé et al. 2013, Hardy et al, 2016

Minimal time of crisis

$$\min_{a(\cdot)} \sum_{t=t_0}^T \mathbf{1}_{x \setminus \text{Constraints}}(x(t), a(t))$$

# Renewable resource management example

Béné & Doyen, Earth's Future, 2018

Renewable stock  $x(t)$  harvested at rate  $e(t)$

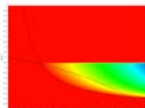
$$x(t+1) = f(x(t)) - qe(t)x(t)$$

Change in the decision  $e(t)$

$$e(t+1) = e(t) + u(t) \text{ with } |u(t)| \leq \frac{1}{\theta}$$

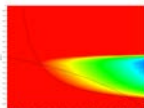
Profitability constraint and threshold :

$$\pi(x(t), e(t)) = pqe(t)x(t) - ce(t) \geq \pi^{\text{lim}}$$

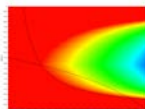


(a) Resistance

$q = 1, \pi^{\text{lim}} = 0.1, \theta = +\infty$

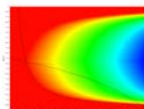


(b) Coping



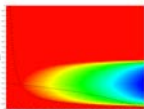
(c) Adaptive resilience

$q = 1, \pi^{\text{lim}} = 0.1, \theta = 0$



(d) Adaptive preference

$q = 1, \pi^{\text{lim}} = 0.05, \theta = 0$



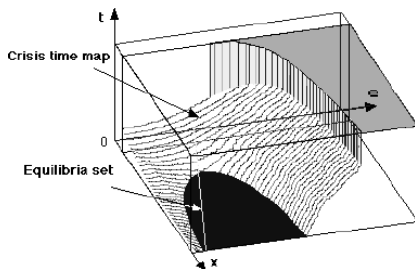
(e) Transforming

$\tilde{q} = 2, \pi^{\text{lim}} = 0.05, \theta = 0$



# Minimal time of crisis

Bene-Doyen, Ecological Economics, 2001



[Dynamic Games and Applications](#)

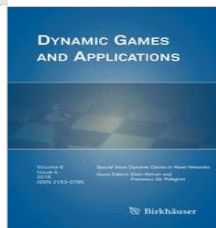
pp 1–24

## The Tragedy of Open Ecosystems

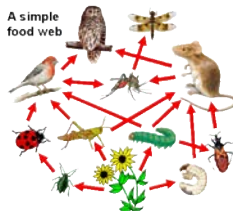
Authors

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# A MICE bio-economic model



## Multi-species Gompertz Dynamics

$$x_i(t+1) = x_i(t) \exp \left\{ r_i + \sum_{j=1}^m s_{ij} \ln(x_j(t)) \right\},$$

Harvest  $h_{if}(t)$  of agent  $f = 1, \dots, n$  on species  $i$  :

$$x_i(t+1) = \left( x_i(t) - \sum_f h_{if}(t) \right) \exp \left\{ r_i + \sum_{j=1}^m s_{ij} \ln \left( x_j(t) - \sum_f h_{jk}(t) \right) \right\},$$

# Non-cooperative vs cooperative maximization

## Non-cooperative

For each agent  $f = 1, \dots, n$

$$\max_{h_f(\cdot)} \sum_{t=0}^{\infty} \rho^t U(h_{f1}(t), \dots, h_{fm}(t)),$$

## Cooperative

$$\max_{h(\cdot)} \sum_{t=0}^{\infty} \rho^t \left( \sum_{f=1}^n U(h_{f1}(t), \dots, h_{fm}(t)) \right)$$

with  $0 < \rho < 1$  discount factor

Analytical solutions using Dynamic Programming and Cournot-Nash optimum and Utility  $U(h) = \sum_i a_i \log(h_i)$

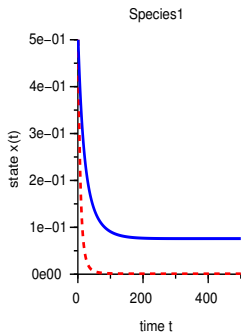


# Bio-economic gains of cooperation

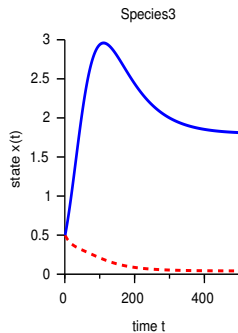
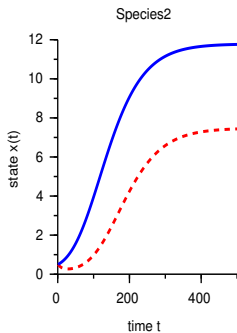
$$r = \begin{pmatrix} -0.0026 \\ 0.0392 \\ 0.0644 \end{pmatrix} \quad S = \begin{pmatrix} -0.0218 & 0.0005 & 0.0001 \\ -0.0143 & -0.0153 & 0.0003 \\ -0.0003 & -0.0085 & -0.0161 \end{pmatrix}$$

$$a = (3 \quad 2 \quad 1)', \quad \rho = 0.98, \quad n = 3$$

| harvest fractions (%)    | Species 1 | Species 2 | Species 3 |
|--------------------------|-----------|-----------|-----------|
| cooperative $f^c$        | 5.45      | 3.91      | 3.52      |
| non cooperative $f^{nc}$ | 14.79     | 10.88     | 9.87      |



$\rho = 0.98$

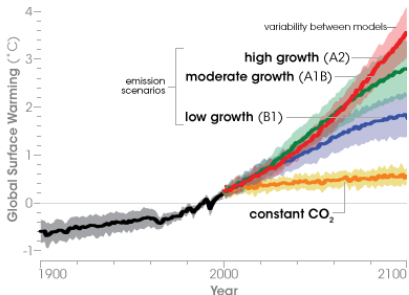


$\rho = 0.98$



# Integrate climate changes : Jean-Sauveur Ay

Sauveur-Jiguet-Leadley-Chakir-Doyen, climatic Change, 2014



<http://dentafas.free.fr/SpecialistesAgricultores/>

# NPV strategy

