Endogenous Risk-Exposure and Systemic Instability

Chong Shu
University of Southern California

8th Annual CIRANO-Sam M. Walton College of Business Workshop on Networks in Trade and Finance

Do highly connected financial networks contribute to systemic stability or systemic fragility?

Connected-Stability:

- Allen and Gale (2000) and Freixas et al. (2000)
- Provide a co-insurance mechanism against shocks.

Connected-Fragility:

- Acemoglu et al. (2015)
- Network also induces a propagation mechanism to spread the loss.

Motivation

- Network-Stability still under debate.
- Literature assumed exogenous shocks.
- They studied how shocks are propagated.

However, banks' exposure to which particular shock is an endogenous choice variable.

Motivation

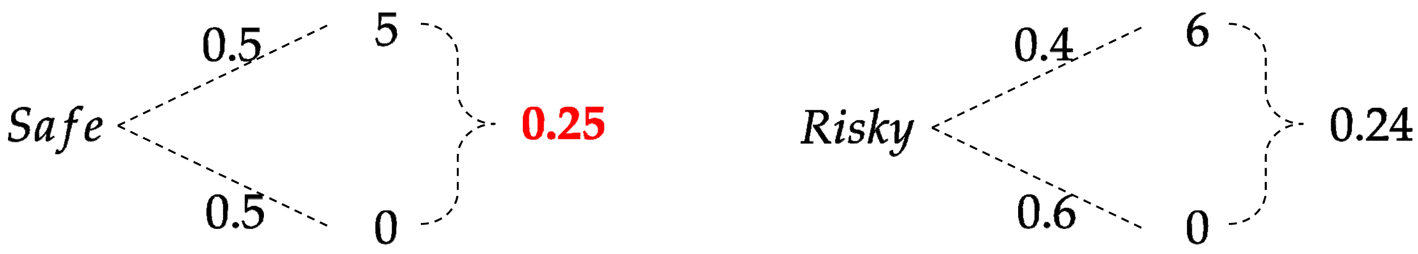
- safe borrowers vs subprime borrowers.
- exposure on asset-backed securities.

In this paper, I endogenize banks' ex-ante choice of risk exposure.

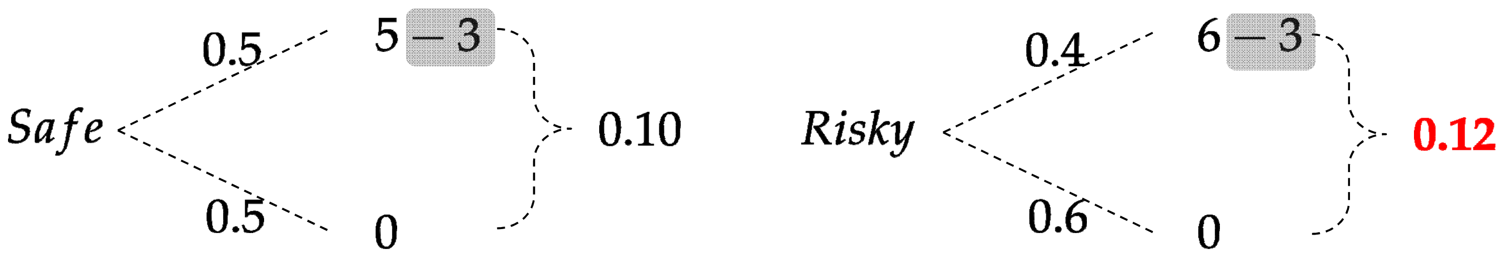
However, banks' exposure to which particular shock is an endogenous choice variable.

Intuition

- A stand-alone bank chooses one project



- Suppose its counterparty fails



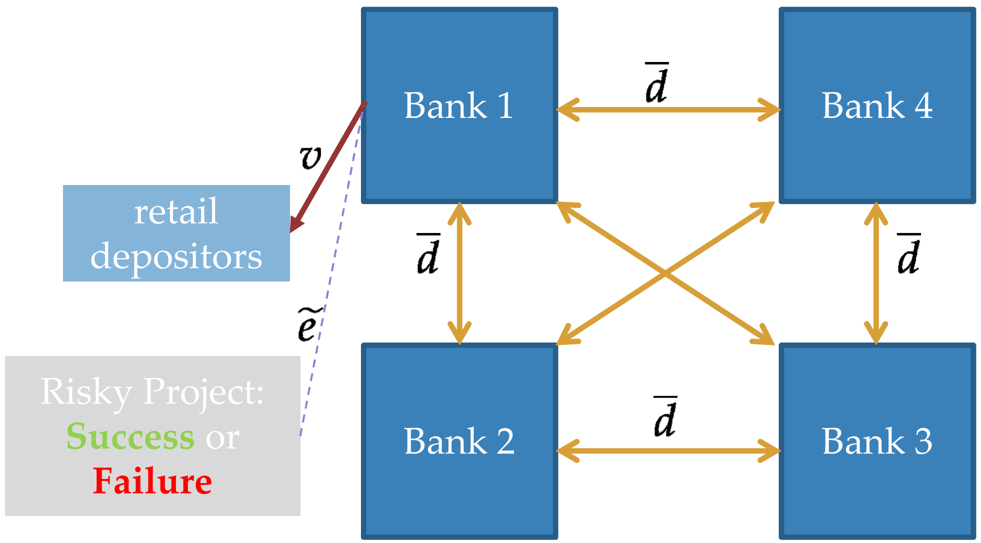
Model & Equilibrium

Model

- N risk neutral banks.
- v: retail deposits.
- $\bar{d}$: the interbank debt.
- choose one project Z_i.

$$\tilde{e}_i = \begin{cases} Z_i & \text{w.p} \quad P(Z_i) \\ 0 & \text{w.p} \quad 1 - P(Z_i) \end{cases}$$

$P(Z)$ is decreasing in Z



Model –continued

- For each state of nature $\boldsymbol{\omega} = (\omega_1, ..., \omega_N)$, the interbank payment $\boldsymbol{d^*} = (d_1^*, ..., d_N^*)$ will be determined as:

$$d_i^*(\boldsymbol{\omega};\boldsymbol{Z}) = \left\{ \min \left[\sum_j \theta_{ij} d_j^*(\boldsymbol{\omega};\boldsymbol{Z}) + e_i(\omega_i, Z_i) - v, \bar{d}, \right] \right\}^+ \quad \forall i \in \mathcal{N} \quad \forall \boldsymbol{\omega} \in \boldsymbol{\Omega}$$

payment outflow — payment inflow — profit — deposit

Limited liability: pay whatever you have or whatever you owe

Model –continued

- After the interbank payment $\boldsymbol{d}^*(\boldsymbol{\omega}, \boldsymbol{Z})$, bank i's profit at the final date is

$$\Pi_i(\boldsymbol{\omega}; \boldsymbol{Z}) = \Big(\underbrace{\sum_j \theta_{ij} d_j^*(\boldsymbol{\omega})}_{\text{payment inflow}} + \underbrace{e_i(\boldsymbol{Z}, \boldsymbol{\omega})}_{\text{profit}} - \underbrace{v_i}_{\text{deposit}} - \underbrace{d_i^*(\boldsymbol{\omega}; \boldsymbol{Z})}_{\text{payment outflow}} \Big)^+$$

- From backward induction, each bank chooses its risk exposure Z_i to maximize the expected payoff

$$Z_i^* = \text{argmax}_{Z_i} \mathbb{E}\left[\Pi_i(\boldsymbol{\omega}; Z_i, \boldsymbol{Z}_{-i}^*)\right] \quad \forall i \in \mathcal{N}$$

Timeline

choose Z_i	state $\boldsymbol{\omega} \in \boldsymbol{\Omega}$ realized	payment $\boldsymbol{d}^*(\boldsymbol{\omega}; \boldsymbol{Z})$	$\Pi(\boldsymbol{\omega}; \boldsymbol{Z})$ realized
Choose risk exposure	Project outcomes revealed	Interbank payment	Profit realized

$$Z_i^* = \text{argmax}_{Z_i} \mathbb{E}\left[\Pi_i(\boldsymbol{\omega}; Z_i, \boldsymbol{Z}_{-i}^*)\right]$$

$$\tilde{e}_i = \begin{cases} Z_i & \text{w.p} \quad P(Z_i) \\ 0 & \text{w.p} \quad 1 - P(Z_i) \end{cases}$$

$$d_i^*(\boldsymbol{\omega}; \boldsymbol{Z}) = \left\{ \min \left[\sum_j \theta_{ij} d_j^*(\boldsymbol{\omega}; \boldsymbol{Z}) + e_i(\omega_i, Z_i) - v, \bar{d}, \right] \right\}^+$$

$$\Pi_i(\boldsymbol{\omega}; \boldsymbol{Z}) = \left(\sum_j \theta_{ij} d_j^*(\boldsymbol{\omega}) + e_i(\boldsymbol{Z}, \boldsymbol{\omega}) - v_i - d_i^*(\boldsymbol{\omega}; \boldsymbol{Z}) \right)^+$$

Network Distortion

▷ We can rewrite the expected payoff as

$$\mathbb{E}\Big[\Pi_i(\boldsymbol{\omega};\boldsymbol{Z})\Big] = \underbrace{P(Z_i)(Z_i - v)}_{\text{stand-alone E}(\Pi)} - P(Z_i)\underbrace{\mathcal{D}(\boldsymbol{Z}_{-i})}_{\text{network distortion}}$$

▷ The network distortion has a clear interpretation

$$\mathcal{D}(\boldsymbol{Z}_{-i}) \equiv \sum_{\boldsymbol{\omega}_{-i}} \Big(\bar{d} - \sum_j \theta_{ij} d_j^*(\boldsymbol{\omega}^{i=s})\Big) \cdot \Pr(\boldsymbol{\omega}_{-i}) > 0$$

net interbank payment (bailout amount)

▷ The network distortion $\mathcal{D}$ is the -3 in the previous example

Supermodularity

Proposition

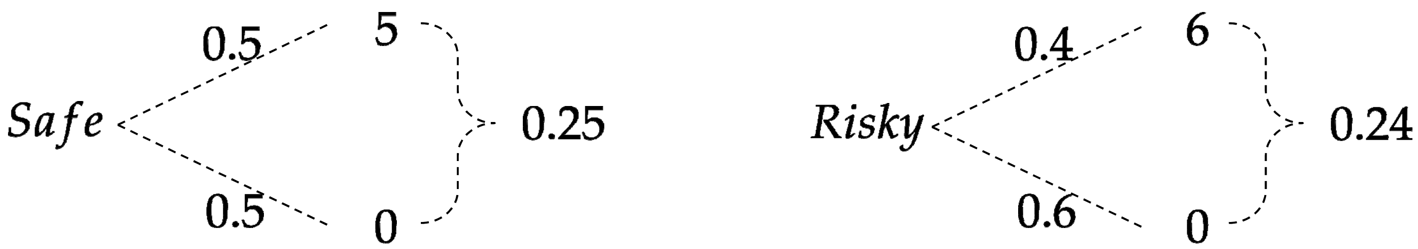
The choice of risk exposure $\mathbf{Z}$ *is supermodular (strategically complementary) among all banks in the same financial network.*

The intuition is as follows:

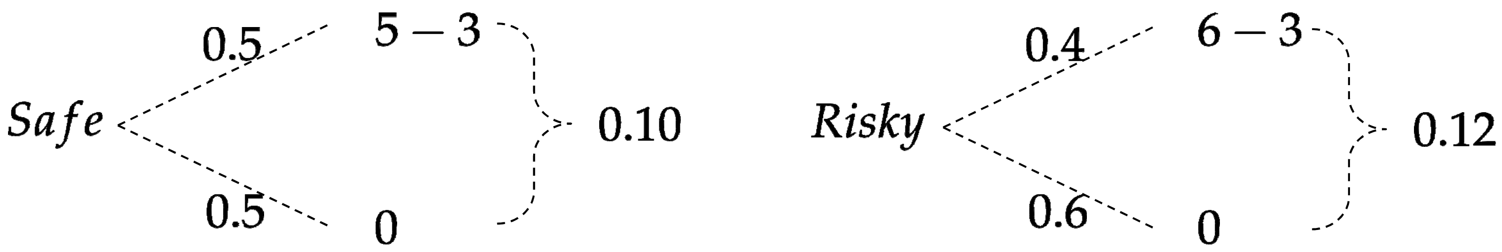
- ▷ if bank j chooses a greater exposure to risk, its project will be more likely to fail.
- ▷ When bank j's project fails, bank i's net interbank payments (bailout) to other banks will increase.
- ▷ To compensate this increased distortion, bank i will rationally choose a greater risk exposure.

Appendix - Proof

- When bank j succeeds (with probability p_j)



- When bank j fails (with probability $1 - p_j$)



- Bank i will choose safe project if

$$0.25 \cdot p_j + 0.10 \cdot (1 - p_j) > 0.24 \cdot p_j + 0.12 \cdot (1 - p_j)$$

$$p_j > 2/3$$

Risk-taking Externality

Proposition

Banks in any network structure will choose greater exposure to risks than stand-alone banks.

- Easy to see that the only equilibrium is $(Risky, Risky)$ in the toy model.
- "**too connected to fail**": besides an ex-post loss contagion (Allen and Gale 2000; Acemoglu et al. 2015), the interbank network creates an ex-ante moral hazard for banks.
- A generalized result of Jensen and Meckling (1976), even though the net interbank liability is zero.

Appendix - Proof

Network Structure

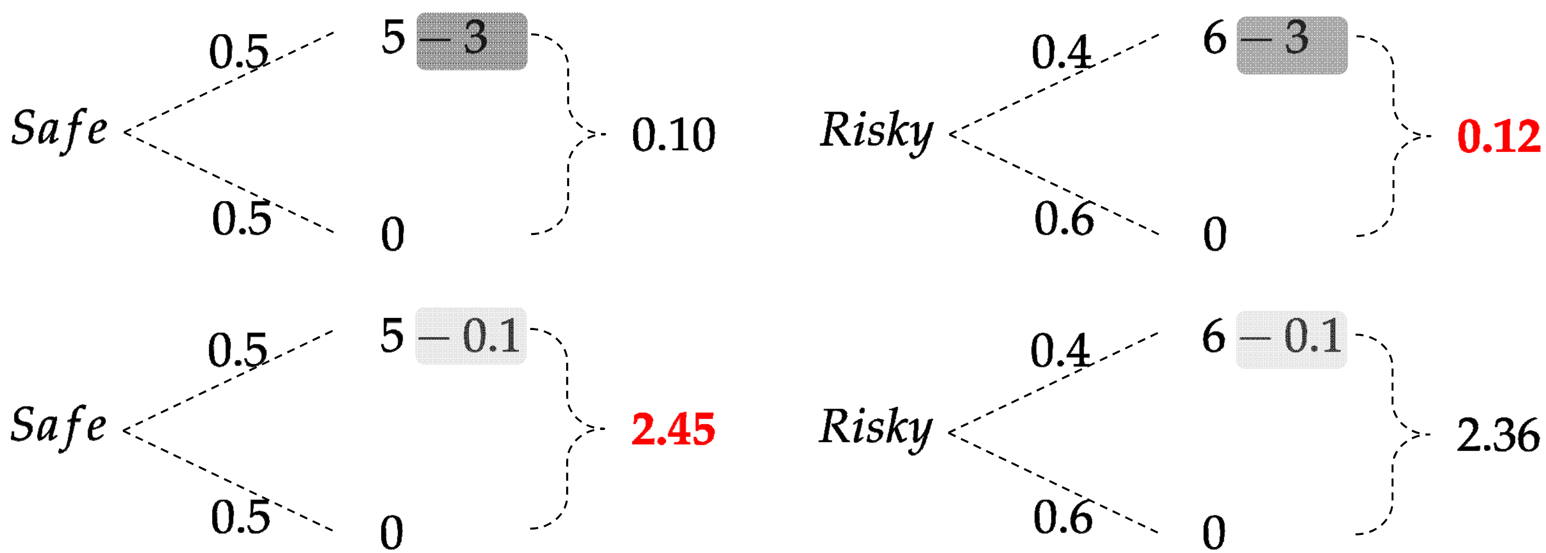
- size of interbank liabilities
- complete / ring
- number of counterparties
- central clearing counterparty

Network Structure

– size of the interbank liabilities

Proposition

Banks' choices of risk exposure Z_i^* are increasing in the size of interbank liabilities $\bar{d}$.



Appendix - Proof

Network Structure

– network completeness

Proposition

Banks' choices of risk exposure Z_i^* are larger in complete networks than in ring networks.

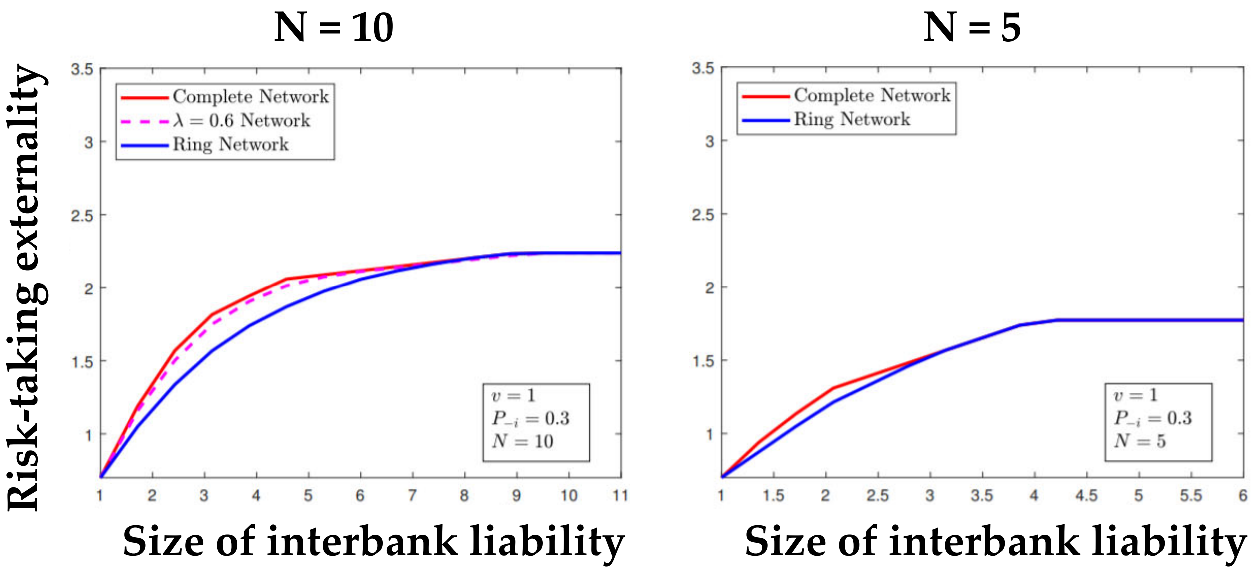
- In complete networks, each bank is exposed to the risk-taking externality of more other banks.
- The result stands in sharp contrast to the view of Allen and Gale (2000). They argue that a complete network is better at co-insurance and hence more robust.
- But because of precisely this co-insurance, solvent banks will anticipate a greater amount of interbank payments to failed banks.

Network Structure

– number of counterparties

Proposition

Banks' choices of risk exposure Z_i^* are larger in networks with more counterparties.

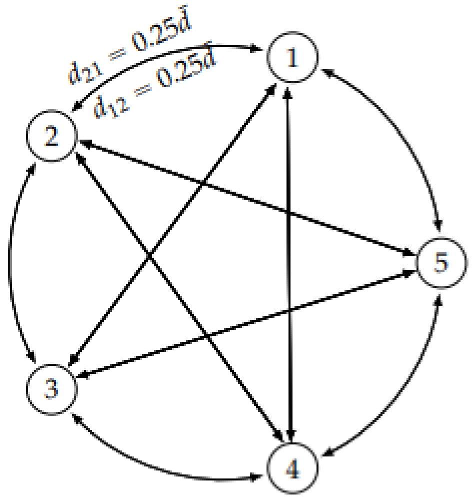


Network Structure

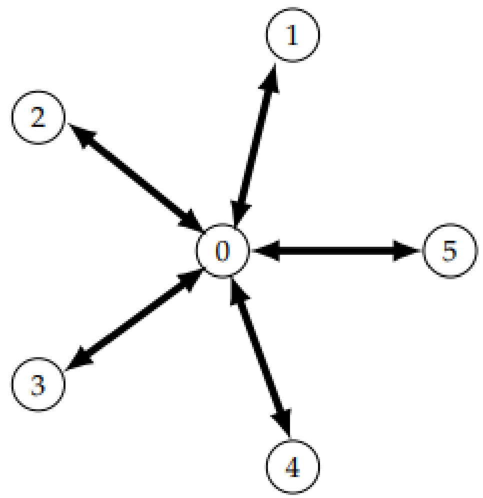
– Central Clearing Counterparties

Proposition

In any network structure with a central clearing counterparty, the risk-taking equilibrium is equivalent to that of a complete network.



(a) complete network



(d) core-periphery network

Network Structure

– Central Clearing Counterparties

Proposition

In any network structure with a central clearing counterparty, the risk-taking equilibrium is equivalent to that of a complete network.

- ▷ Through CCP, each bank is "forced" to connect to every other bank.
- ▷ Banks with a CCP hence becomes exposed to more risk-taking externalities.
- ▷ A CCP may increase originally loosely connected banks' risk-taking incentives.

Endogenous Correlation

Endogenous Correlation

- Besides choosing the risk exposure Z_i, each bank also chooses its project's correlation with other banks, $\boldsymbol{\lambda}_i$

$$\lambda_{ij} \equiv Pr(\omega_i = s | \omega_j = s)$$

- The new equilibrium is defined as

  1. The vector of functions $\boldsymbol{d}^*(\boldsymbol{\omega};\boldsymbol{Z})$ is a payment equilibrium for any $\boldsymbol{Z}$.
$$d_i^*(\boldsymbol{\omega};\boldsymbol{Z}) = \left\{ \min\left[\sum_j \theta_{ij} d_j^*(\boldsymbol{\omega};\boldsymbol{Z}) + e_i(\omega_i, Z_i) - v, \bar{d}, \right] \right\}^+$$
  2. For each bank $i \in \mathcal{N}$, $(Z_i^*, \boldsymbol{\lambda}_i^*)$ is optimal and solves the following equation, given $\boldsymbol{d}^*(\boldsymbol{\omega};\boldsymbol{Z})$, $\boldsymbol{Z}_{-i}^*$ and $\boldsymbol{\Lambda}_{-i}^*$
$$(Z_i^*, \boldsymbol{\lambda}_i^*) = \text{argmax}_{\substack{\underline{Z} \leq Z_i \leq \overline{Z} \\ \mathbf{0} \leq \boldsymbol{\lambda}_i \leq \mathbf{1}}} \mathbb{E}\left[\Pi_i(\boldsymbol{\omega}; \boldsymbol{Z}_{-i}^*, \boldsymbol{\lambda}_{-i}^*) \right]$$
  3. The pairwise correlations are compatible among all banks. i.e. $\boldsymbol{\rho} = [\rho_{ij}]$ is symmetric and positive semi-definite.

Endogenous Correlation

Proposition

The correlated risk-taking equilibrium exists and every bank's risk exposure is perfectly correlated: $\lambda^*_{ij} = 1$ *for all* $i, j \in \mathcal{N}$.

- each bank will endogenously align their project outcomes with other connected banks
- By doing so, there will be no downward distortion when the bank's project succeeds
- a financial crisis (or simultaneous failure of several banks) will be more likely to endogenously evolve in connected banking systems.

Policies

- Equity Buffers
- Government Bailout
- Deposit Insurance (skipped)
- Transparency (skipped)

Equity Buffer

Proposition

Banks' choices of risk exposure Z_i^* are decreasing in the size of equity buffers r.

with equity buffer r, bank's expected profit becomes

$$\mathbb{E}\Big[\Pi_i(\omega;\mathbf{Z})\Big] = P(Z_i)(Z_i + r_i - v) - P(Z_i)\mathcal{D}(\mathbf{Z}_{-i}; r_j)$$

Jensen/Meckling   Network Effect

Direct effect: banks won't gamble their own equity.

Network effect: the risk taking externality gets reduced.

Appendix - Proof

Equity Buffer

Proposition

Banks' choices of risk exposure Z_i^* *are decreasing in the size of equity buffers* r.

Intuition:

- If a bank fails, its equity buffer will first be withdrawn to pay the deposits.
- The loss that may be otherwise propagated to other banks will now first be absorbed by this equity buffer.
- As a result, the network risk-taking distortion (bailout) is reduced.

Government Bailout

- I define a government bailout (n, t) as a transfer t from the government to each failed bank if and only if the number of failed banks exceeds n.

$$t_i(\boldsymbol{\omega}) \equiv t \cdot \mathbb{1}(\omega_i = f) \cdot \mathbb{1}(\#\text{ failed banks} \geq n)$$

- The payment equilibrium becomes

$$d_i^*(\boldsymbol{\omega}; \mathbf{Z}) = \left\{ \min \left[\sum_j \theta_{ij} d_j^*(\boldsymbol{\omega}; \mathbf{Z}) + e_i(\omega_i, Z_i) + t_i(\boldsymbol{\omega}) - v, \bar{d}, \right] \right\}^+$$

Government Bailout

Proposition

Each bank's network risk-taking distortion and equilibrium risk exposure is reduced if there exists a government bailout.

- In contrast to the conventional wisdom, the above proposition states that a credible government bailout will discourage the ex-ante risk taking.
- With a government bailout, the loss will be curbed before spreading to successful banks in crisis times
- Ex ante, each bank will anticipate a smaller distortion if it succeeds.

Appendix - Proof

Summary

- There exists a network risk-taking externality.
- Connected banks' choices of risk exposure are higher than stand-alone banks.
- Particularly for banks in more densely connected networks.
- A CCP may increase banks' risk taking incentives
- Connected banks endogenously expose to correlated risks.

Policy Implications

- Equity buffer has a network effect and contributes to systemic stability.
- A government bailout can reduce the network risk-taking externality.

Thanks!

chongshu@usc.edu